RESEARCH PROGRAMME
ON SEISMIC WAVES IN COMPLEX 3–D STRUCTURES
(SW3D)

The research is focused primarily on the fundamental issues of high–frequency seismic wave propagation in complex 3–D isotropic and anisotropic structures, which go beyond the traditional approaches. The ray method and its extensions, as well as its combination with other methods are mainly applied and investigated. The emphasis is put on new, stable, more efficient and flexible algorithms for both forward numerical modelling and inversion of seismic wave fields in 3–D inhomogeneous, isotropic or anisotropic, elastic or attenuating structures. Considerable attention is also devoted to applications involving S waves, converted waves, S–wave splitting and coupling in anisotropic media, particle ground motions, etc. Much more detailed information can be obtained at "http://sw3d.cz". The research programme was begun on October 1, 1993.

RESEARCH PROGRAMME FOR THE NINETEENTH YEAR
October 1, 2011 — September 30, 2012

1. *Sample data for the program packages*
    Examples of the input data describing or approximating models delivered by the consortium members or other typical models will be prepared. Examples of the input data to perform calculations in such models will also be prepared.

2. *Ray histories and two–point ray tracing in complex 3–D structures*
    Ray histories are of principal importance not only for two–point ray tracing and for wavefront tracing, but also for the summation of Gaussian beams. Properties of the projection of ray coordinates onto Cartesian coordinates within individual ray histories will be studied. New applications of ray histories to numerical algorithms will be proposed. The two–point ray tracing code will further be tested and applied to various models.

3. *Synthetic seismograms in 3–D isotropic and anisotropic complex structures*
    Methods to calculate synthetic seismograms in complex structures will be studied, mutually compared and combined. The synthetic seismograms will also be compared with synthetic seismograms generated by non–ray methods. For models suggested by the consortium members, we are ready to perform ray–synthetic studies illustrating how the wave responses differ in heterogeneous and anisotropic media from homogeneous or isotropic media. Calculation of synthetic seismograms by coupling ray theory in layered models will further be tested. Emphasis will be put on numerical implementation of Gaussian beams.
    Gaussian–beam synthetic seismograms: Deriving the discretization error of the uneven summation of Gaussian beams. Developing an algorithm for sampling the ray–parameter domain for Gaussian beams.

4. *Gaussian beams in inhomogeneous media*
    Paraxial Gaussian beams in smoothly varying anisotropic media. Paraxial Gaussian beams in laterally varying layered anisotropic structures. Paraxial Gaussian beams in smoothly varying anisotropic attenuating media.
    Optimization of the shape of Gaussian beams.

5. *Seismic wave propagation in inhomogeneous weakly anisotropic media*

Improving the accuracy of the coupling ray theory for S waves by including the first–order perturbation of polarization vectors.

Research into the prevailing–frequency approximation of the coupling ray theory. This prevailing–frequency approximation could facilitate coupling–ray–theory migrations in the future.

Anisotropic common ray approximation of the coupling ray theory: extension of the anisotropic–ray–theory P–wave and anisotropic common S–wave ray tracing towards general initial conditions and models with structural interfaces.

Coupling ray theory along the anisotropic–ray–theory rays in smooth models without interfaces.

Derivation of coupling ray theory from the elastodynamic equation concentrated on the study of errors due to neglected terms in order to estimate the accuracy of coupling ray theory. Study of coupling ray series. Theory of coupling S–wave Gaussian packets. Definition of coupling ray theory travel times and amplitudes, and study of frequency dispersion of S–wave coupling. Quantification of the relevance of anisotropic S–wave coupling to velocity analysis and imaging.

Development of the code for modelling P– and coupled S–wave propagation based on the first–order ray tracing and dynamic ray tracing (FORT and FODRT) in layered inhomogeneous weakly anisotropic media.

Numerical comparison of various approximations of coupling ray theory with more accurate methods.

6. *Seismic wave propagation in anisotropic attenuating media*

Investigation of plane waves propagating in anisotropic attenuating media. Both homogeneous and inhomogeneous waves will be considered. For weakly attenuating media, the perturbation methods will be used. Special attention will be devoted to the attenuation vector and to the quality factor Q, particularly to its directional dependence.

The results obtained for plane waves propagating in homogeneous media will be generalized to non–planar waves propagating in smoothly varying anisotropic weakly attenuating media using perturbation methods based on the ray theory. A special attention will be devoted to waves generated by point sources. Together with the attenuation properties, also the dispersion properties of seismic body waves propagating in attenuating media will be studied.

7. *Fast computation of multivalued ray–theory travel times and Green functions at the nodes of 3–D grids*

Controlled initial–value ray tracing is used to cover the velocity model by ray tubes and ray cells. Multivalued ray–theory travel times and other quantities at the nodes of a 3–D grid are then obtained by interpolation within ray cells. This algorithm is especially useful for the Born approximation and for Kirchhoff migrations in heterogeneous 3–D velocity models, and may be used for nonlinear hypocentre determination. The current computer code can be applied to P, S and converted waves in isotropic velocity models, and to P waves in anisotropic velocity models. It will further be tested in various velocity models, including the velocity models delivered by the consortium members.

We intend to extend the algorithm and computer code to S waves in anisotropic velocity models by including the perturbations of S–wave travel times from common S–wave rays.

Algorithms of fast calculation of ray–theory travel times in dense rectangular grids will be investigated further. The accuracy and efficiency of the interpolation of ray–theory travel times within ray cells in 3–D models will be studied, and the relevant numerical algorithms will be improved, or new ones will be proposed. Attention will also be devoted to the interpolation between different shot and receiver positions.

8. *Computation of two–point paraxial travel times*

Algorithms and codes for fast computation of approximate two–point paraxial travel times calculated in the vicinities of reference rays will be studied and tested numerically. Inhomogeneous, isotropic or anisotropic velocity models with curved interfaces will be considered. Principal attention will be devoted to the study of the accuracy and efficiency of the proposed algorithms. We shall study calculation of two–point paraxial travel times from a given source to the nodes of a dense 3–D rectangular grid, or from perturbed sources or to perturbed receivers.

9. *Perturbation derivatives and spatial derivatives of travel time*

Theoretical and numerical applications of the equations for transforming the perturbation derivatives and the third–order and higher–order spatial derivatives of travel time at structural interfaces. Generalization of spatial and perturbation derivatives of travel time to Hamilton–Jacobi equation with right–hand sides dependent on ray parameters. Spatial and perturbation derivatives of two–point travel time (characteristic function).

10. *Accuracy of seismic modelling*

The research will be concentrated mainly on the accuracy of travel–time calculations, on the accuracy of finite–difference modelling of seismic wave fields, and on the accuracy of other modelling methods designed or studied within the framework of the project. The main attention will be devoted to the estimation of the feasibility and costs of ray tracing, and to the definition and high–frequency validity of velocity models.

11. *Sensitivity of seismic waves to the structure (sensitivity Gaussian packets)*

Continuing investigation how the perturbations of a generally inhomogeneous isotropic or anisotropic structure manifest themselves in the wave field, and which perturbations can be detected within a limited aperture and a limited frequency band.

Developing the corresponding algorithm for linearized inversion based on wave–field sensitivity to structural Gabor functions.

12. *Born approximation*

Applicability of the Born approximation: Study of the nonlinearity of ray–theory seismograms with respect to perturbations of the velocity model.

Derivation of the weak–contrast reflection–transmission coefficients from the Born approximation.

13. *Lyapunov exponents and model smoothing*

Construction and smoothing of velocity macro models will further be studied, with emphasis on the application of Sobolev scalar products and Lyapunov exponents. Attention will be paid to a possible extension of the estimation of Lyapunov exponents to smooth 3–D models and to 2–D models with structural interfaces. Sobolev scalar products with spatially variable weights will also be studied.

14. *Seismic tomography and related problems*

Development of theory, algorithms and programs applicable in seismic travel–time tomography and inversion of the coherency panels, with emphasis on the estimation of the accuracy of the resulting model compared to the geological structure.

Stochastic travel–time tomography: Developing an algorithm for calculating geometrical covariances of travel times. Developing an algorithm for calculating geometrical covariances between rays and B–splines.

Determination of the medium correlation functions from well logs. Calculation of sonic–log travel times in anisotropic media. Estimation of uncertainty of sonic–log travel times. Estimation of attenuation from vertical–seismic–profiling travel times.

15. *Seismic sources and hydrofracture monitoring*

Studying the inaccuracy of the absolute source location and the inaccuracy of the relative source location due to the inaccurate velocity model.

16. *Local anisotropy parameter estimation from vertical–seismic–profiling measurements*

Local determination of elastic parameters from the vertical–seismic–profiling measurements. Use of P–wave and, possibly, S–wave data. Inversion for parameters of TI media with tilted axis of symmetry and for the angles specifying the axis. Study of possibility to use NMO or AVO data to constrain the inversion. Applications to real data sets.

17. *Decomposition of a given function into Gabor functions*

Decomposition of a given function into Gabor functions in 2–D or 3–D. The Gabor functions may depend on the wavenumber or frequency and their shape may be smoothly varying in space or time, which requires much more general decomposition than the Gabor transform. The decomposition into Gabor functions may be useful for the decomposition of the perturbations of elastic moduli into Gabor functions, or for the decomposition of a general wave field into Gaussian beams or packets.

Two alternative decompositions will be studied and compared: (a) discretized analytical integral decomposition, (b) numerical discrete decomposition. Whereas the discretized analytical integral decomposition has been applied to Gaussian–packet prestack depth migration, the numerical discrete decomposition can also be applied to linearized inversion based on wave–field sensitivity to structural Gabor functions.

18. *Migrations*

Resolution and accuracy of migrations will be studied. Attention will be paid to the physical meaning of the migrated sections and to their sensitivity to the velocity model, including its anisotropy.

Algorithm of the Gaussian–packet prestack depth migration is being developed. Gaussian packets should offer explicit correspondence between the time and depth sections. Attention is paid to the optimization of the shape of Gaussian packets.

Study of possibilities to include coupling ray theory in seismic imaging.

Amplitude preserving Kirchhoff migration in anisotropic media: Synthetic study of possibilities and limitations to recover reflection coefficients from data measured in inhomogeneous anisotropic media. Common–source Kirchhoff prestack depth migration with S waves will be generalized from isotropic to anisotropic velocity models.

19. *Concluding remarks*

In addition to this programme, we will certainly be responsive to specific technical suggestions and recommendations of the consortium members within the general framework of the project. The research in most directions listed above will continue into the future years of the project.

SW3D PROGRAM PACKAGES

Package CRT:

Model: Using package MODEL (see below).

Type of waves: Arbitrary type of elementary seismic body wave corresponding to the zero–order ray theory (P, S, converted, coupled S waves).

Computations: Arbitrary position and shape of the source, initial–value ray tracing by numerical integration of ray equations, general isotropic–ray–theory rays, anisotropic–ray–theory P–wave rays and anisotropic common S–wave rays in smooth models, two–point ray tracing by the shooting method based on ray histories, travel–time computation, dynamic ray tracing, paraxial–ray propagator matrix, geometrical spreading, vectorial amplitudes, polarization vectors. The package may be applied to the evaluation of the elastodynamic ray–theory Green function, and to the computation of synthetic seismograms, including the coupling ray theory along isotropic or anisotropic common S–wave rays and the response of fine layers at receiver sites (program package RMATRIX by C.J. Thomson, linked to the CRT package). Fast computation of multivalued ray–theory travel times, Green functions and other quantities at the nodes of 3–D grids by interpolation within ray cells (P, S and converted waves in general isotropic velocity models, P waves in smooth anisotropic velocity models). Least–square travel–time tomography with smoothing using Sobolev scalar products.

Acquisition schemes: Surface seismics (land and marine), vertical seismic profiling, cross–hole, ocean bottom.

Planned innovations: Extension of the anisotropic–ray–theory P–wave and anisotropic common S–wave ray tracing towards general initial conditions and models with structural interfaces. Tracing anisotropic–ray–theory S–wave rays in smooth models without interfaces. Coupling ray theory along the anisotropic–ray–theory rays in smooth models without interfaces. The package will be extended to solve various inverse problems, stochastic travel–time tomography in particular.

Package ANRAY:

Model: 3–D laterally varying structures containing isotropic and anisotropic non–vanishing layers. Specification of elastic parameters inside individual layers either by linear interpolation between isosurfaces of elastic parameters, or by B–spline interpolation within a 3–D rectangular grid of elastic parameters. VRML and GOCAD visualization.

Types of waves: Arbitrary type of elementary seismic body wave (P, S, any converted wave, coupled S waves).

Computations: Numerical integration of ray tracing and dynamic ray tracing equations, calculation of ray vectorial amplitudes, ray–theory Green function including the Green function in the quasi–isotropic approximation for S waves, ray synthetic seismograms, particle ground motions.

Acquisition schemes: Surface seismics (land and marine), vertical seismic profiling, cross–hole, ocean bottom.

Planned innovations: (a) Incorporation of effects of weak attenuation. (b) Ray tracing and dynamic ray tracing in media with rotated higher-symmetry anisotropy (transverse isotropy, orthorhombic symmetry). (c) First–order ray tracing of P waves and coupled S waves in layered inhomogeneous weakly anisotropic media. (d) Computation of the 6×6 propagator matrix in Cartesian coordinates. (e) Calculation of KMAH index in anisotropic media. (f) Removal of problems of P–wave reflections/transmissions in a

vicinity of S–wave singularities. (g) Further debugging, completion, and removal of inconsistencies in the description of the package.

Package SEIS:

Model: 2–D laterally varying isotropic structures composed of layers separated by curved interfaces. Any interface may form edges. It may also coincide with a neighbouring interface(s) in some region. Thus, the models with isolated bodies and pinchouts can be considered. Inside the layers, the velocities of P and S waves may vary in two directions. Weak attenuation and non–planar topography can be considered.

Types of waves: Arbitrary type of elementary seismic body wave (P,S, any converted or multiply reflected wave).

Computations: Arbitrary position of a point source, numerical integration of 2–D ray tracing and dynamic ray tracing equations, computation of ray vectorial amplitudes or Green functions of individual elementary waves, ray synthetic seismograms, particle ground motions.

Acquisition schemes: Surface seismics (land and marine), vertical seismic profiling, cross-hole.

Planned innovations: Alternative computation of synthetic seismograms in the frequency domain. Ocean bottom configuration. Extended visualization. Extension of test examples.

Package MODEL:

Model: General 3–D layered and block isotropic or anisotropic structures, containing isolated bodies, pinchouts, etc. Inside the layers and blocks, the elastic parameters may vary in all three dimensions. Attenuation and non–planar topography can be considered. Possibility of model smoothing, data fitting by inversion including fitting and smoothing GOCAD models, conversion of model parametrization, triangulation of structural interfaces, VRML and GOCAD visualization.

Package NET:

Model: Using package MODEL or using gridded velocities.

Types of waves: First arrivals, constrained first arrivals.

Computations: Arbitrary position and shape of the source. First–arrival travel times in the whole model are computed. The algorithm of computation is independent of the model's complexity.

Acquisition schemes: Surface seismics (land and marine), vertical seismic profiling, cross–hole, ocean bottom.

Package FORMS:

Computations: Subroutines used by other program packages including data input and output subroutines, management and plotting of synthetic seismograms, 2–D and 3–D graphics including 3–D virtual reality with VRML and GOCAD visualization, manipulation and calculation with gridded data (data cubes), programs for matrix and vector operations necessary for inversion, other general–purpose seismic software.

Planned innovations: Program for computation of plane–wave reflection/transmission coefficients at planar interfaces separating arbitrary anisotropic media.