Sensitivity of electromagnetic waves to a heterogeneous bianisotropic structure

Luděk Klimeš

Department of Geophysics
Faculty of Mathematics and Physics
Charles University in Prague
Czech Republic

Sensitivity of electromagnetic waves to a heterogeneous bianisotropic structure

Luděk Klimeš

Department of Geophysics
Faculty of Mathematics and Physics
Charles University in Prague
Czech Republic

Outline

Electromagnetic wave equation
Simplifications of the wave equation
Gabor representation of medium perturbations
Applied approximations
Sensitivity Gaussian packets
Example

Electromagnetic wave equation

Maxwell equations for a linear bianisotropic medium in the Boys-Post representation:

$$[\chi^{\alpha\beta\gamma\delta}(x^\mu)\, A_{\gamma,\delta}(x^\nu)]_{,\beta} + J^\alpha(x^\mu) = 0 \quad .$$

Einstein summation. Indices $i, j, ... = 1, 2, 3;\ \ \alpha, \beta, ... = 1, 2, 3, 4.$

Electromagnetic vector potential A_α is a covariant 4-vector.
Constitutive tensor $\chi^{\alpha\beta\gamma\delta}$ is a contravariant tensor density of weight -1.
Current density J^α is a contravariant 4-vector density of weight -1.

The constitutive tensor is skew with respect to its first and second indices, and with respect to its third and fourth indices:

$$\chi^{\alpha\beta\gamma\delta} = -\chi^{\beta\alpha\gamma\delta} = -\chi^{\alpha\beta\delta\gamma} \quad .$$

Simplifications of the wave equation

$$[\chi^{\alpha\beta\gamma\delta}(x^\mu)\, A_{\gamma,\delta}(x^\nu)]_{,\beta} + J^\alpha(x^\mu) = 0 \quad .$$

In order to simplify the application of the ray-theory approximation, we are assuming here that constitutive tensor $\chi^{\alpha\beta\gamma\delta}$ is:

Real-valued (to avoid complex-valued rays).

Symmetric with respect to its first and second pairs of indices (to simplify the transport equation for amplitude):

$$\chi^{\alpha\beta\gamma\delta} = \chi^{\gamma\delta\alpha\beta} \quad .$$

Time-independent (to apply the ray-theory approximation in the frequency domain):

$$\chi^{\alpha\beta\gamma\delta} = \chi^{\alpha\beta\gamma\delta}(x^m) \quad .$$

Gabor representation of medium perturbations

We consider infinitesimally small perturbations $\delta\chi^{\alpha\beta\gamma\delta}$ of the constitutive tensor $\chi^{\alpha\beta\gamma\delta}$ in the electromagnetic wave equation.

We decompose the perturbations of the constitutive tensor into Gabor functions $g^{\Omega}(x^m)$ indexed here by Ω:

$$\delta\chi^{\alpha\beta\gamma\delta}(x^m) = \sum_{\Omega} \chi_{\Omega}^{\alpha\beta\gamma\delta}\, g^{\Omega}(x^m) \quad ,$$

$$g^{\Omega}(x^m) = \exp\left[\mathrm{i}\, k_i^{\Omega}\,(x^i - x_{\Omega}^i) - \tfrac{1}{2}(x^i - x_{\Omega}^i)\, K_{ij}^{\Omega}\,(x^j - x_{\Omega}^j)\right] \quad .$$

Gabor functions $g^{\Omega}(x^m)$ are centred at various spatial positions x_{Ω}^i and have various structural wavenumber vectors k_i^{Ω}.

The wavefield scattered by the perturbations is then composed of waves $A_{\alpha}^{\Omega}(x^{\mu})$ scattered by the individual Gabor functions:

$$\delta A_{\alpha}(x^{\mu}) = \sum_{\Omega} A_{\alpha}^{\Omega}(x^{\mu}) \quad .$$

Applied approximations

Short-duration broad-band wavefield with a smooth frequency spectrum incident at the Gabor function, expressed in terms of the amplitude and travel time.

First-order Born approximation of each wave $A_{\alpha}^{\Omega}(x^{\mu})$ scattered by one Gabor function.

Ray-theory approximation of the Green tensor in the Born approximation.

High-frequency approximation of spatial derivatives of both the incident wave and the Green tensor. In this high-frequency approximation, we neglect the derivatives of the amplitude, which are of order $1/frequency$ with respect to the derivatives of the travel time.

Paraxial ray approximation of the incident wave in the vicinity of central point x_{Ω}^{i} of the Gabor function.

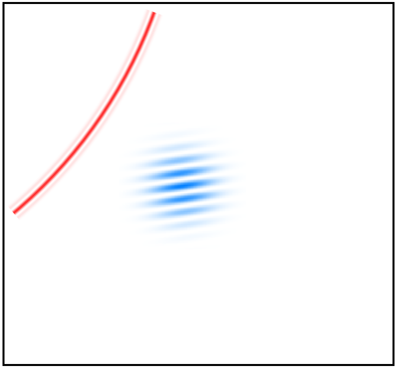
Two-point paraxial ray approximation of the Green tensor at point x_{Ω}^{i} and at the receiver. The paraxial ray approximation consists in a constant amplitude and in the second-order Taylor expansion of the travel time.

Sensitivity Gaussian packets

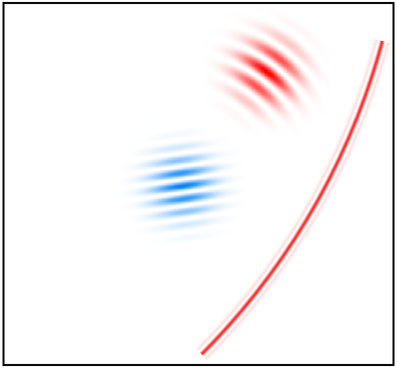
The mentioned approximations enable us to calculate the waves scattered by Gabor functions analytically.

Wave $A^{\Omega}_{\alpha}(x^{\mu})$ scattered by one Gabor function is composed of a few (i.e., 0 to 3 as a rule) Gaussian packets. Each of these “sensitivity” Gaussian packets has a specific frequency and propagates from point x^{i}_{Ω} in a specific direction:

A single Gabor function $g^{\Omega}(x^{i}_{\Omega})$ centred at point x^{i}_{Ω}.



Broad-band wave incident at the Gabor function.



Scattered wave $A^{\Omega}_{\alpha}(x^{\mu})$ composed of one sensitivity Gaussian packet.

Each of these sensitivity Gaussian packets scattered by Gabor function $g^{\Omega}(x^m)$ is sensitive to just a single linear combination

$$R^{\Omega} = \frac{\chi_{\Omega}^{\alpha\beta\gamma\delta} \, E_{\alpha} \, P_{\beta} \, e_{\gamma} \, p_{\delta}}{-2 \, \chi^{\alpha\beta\gamma\delta}(x_{\Omega}^{n}) \, e_{\alpha} \, P_{\beta} \, e_{\gamma} \, p_{\delta}}$$

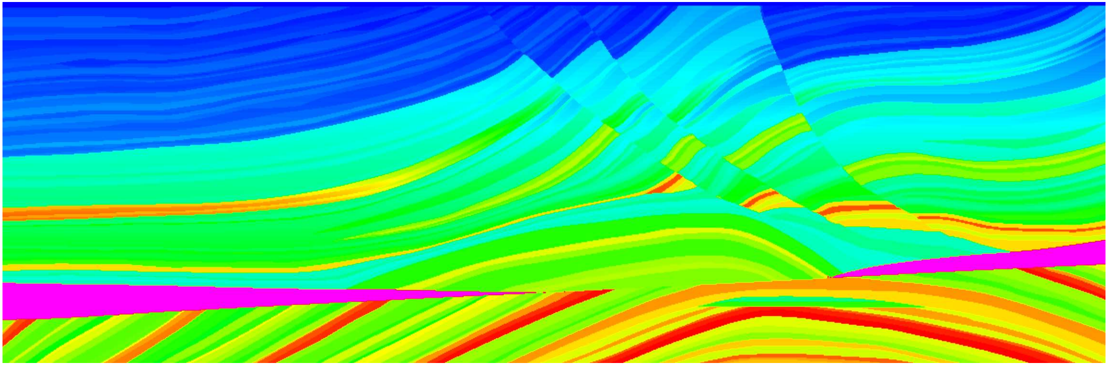
of coefficients $\chi_{\Omega}^{\alpha\beta\gamma\delta}$ corresponding to the Gabor function.

P_i ... slowness vector of the incident wave; $P_4 = -1$
E_{α} ... polarization vector of the incident wave
p_i ... slowness vector of the sensitivity Gaussian packet; $p_4 = -1$
e_{α} ... polarization vector of the sensitivity Gaussian packet

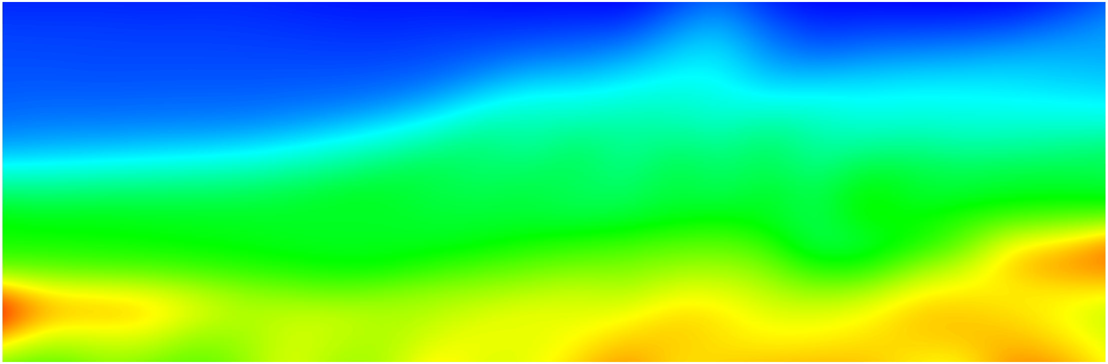
Coefficient R^{Ω} represents the weak-contrast reflection-transmission coefficient at the interface at which the constitutive tensor changes by $\chi_{\Omega}^{\alpha\beta\gamma\delta}$.

This information about the Gabor function is lost if the sensitivity Gaussian packet does not fall into the aperture covered by the receivers and into the legible frequency band.

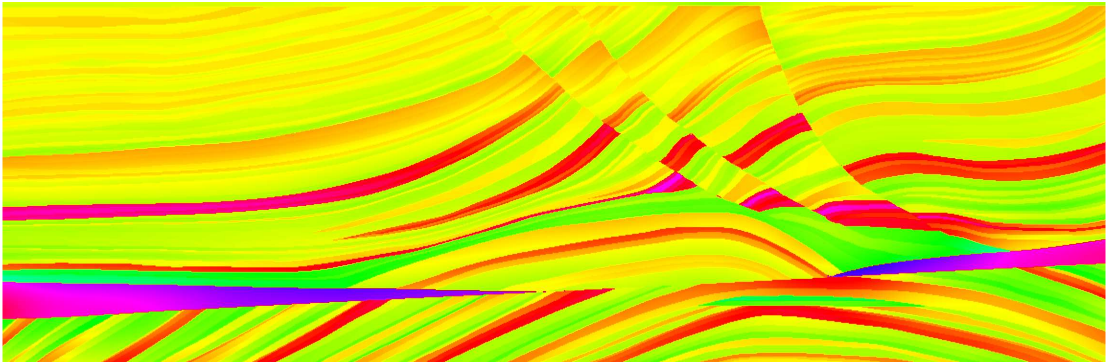
Example



Wave–propagation velocity in the Marmousi structure.

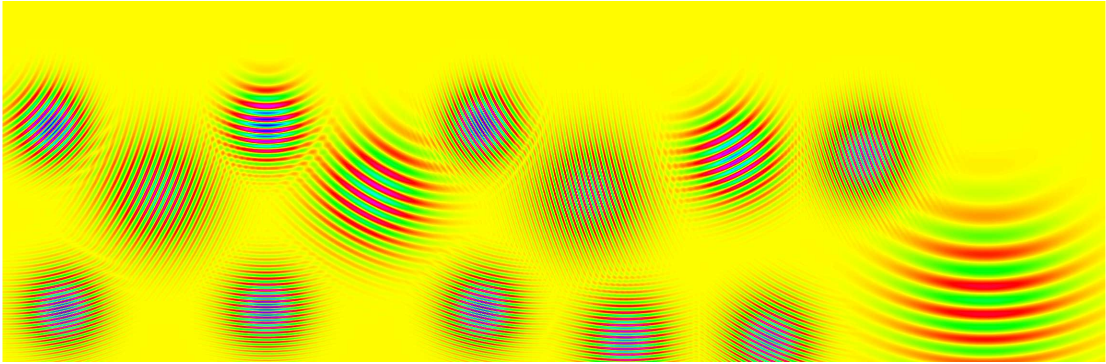


Wave–propagation velocity in the velocity model for ray tracing.



Velocity difference
between the Marmousi structure and the velocity model.

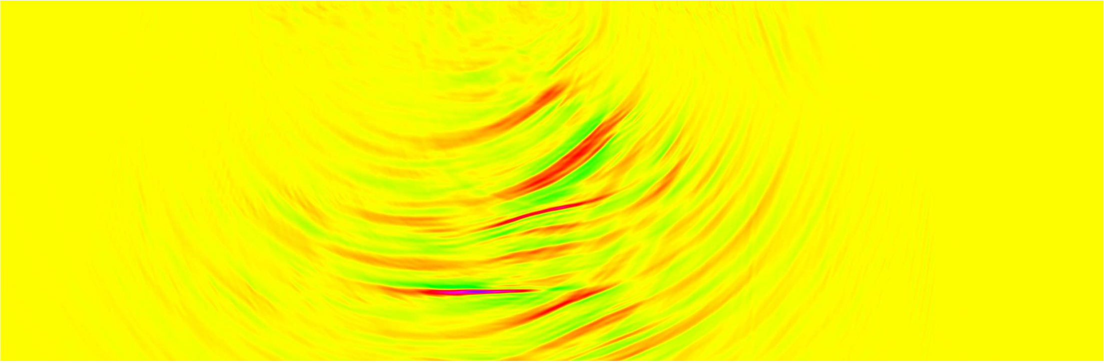
For the decomposition of the velocity difference, we generate the set of Gabor functions $g^{\Omega}(x^m)$ with optimized matrices K_{ij}^{Ω}.
We obtain 67014 Gabor functions within the selected wavenumber domain.



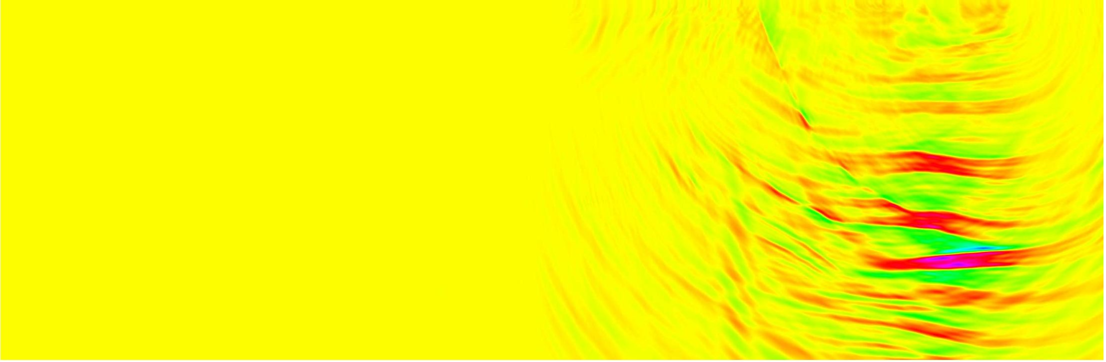
Example showing 14 ones of 67014 optimized Gabor functions used to decompose the velocity difference.

We decompose the velocity difference into the sum of Gabor functions. For each wave source, we calculate the quantities describing the paraxial approximation of the incident wave at all central points of Gabor functions. For each wave source and each Gabor function, we calculate the initial conditions for the corresponding sensitivity Gaussian packets which form the scattered wave. We consider Gaussian packets corresponding to the given frequency band only. We then trace the central ray of each sensitivity Gaussian packet. If a sensitivity Gaussian packet arrives to the receiver array within the registration time, the recorded wavefield contains information on the corresponding Gabor function.

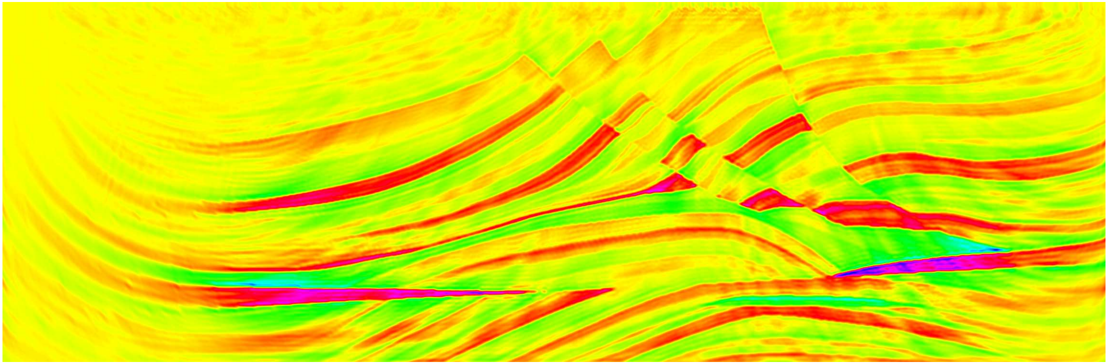
The velocity difference can thus be decomposed into the part to which the recorded waves are sensitive and into the part to which the recorded waves are not sensitive.



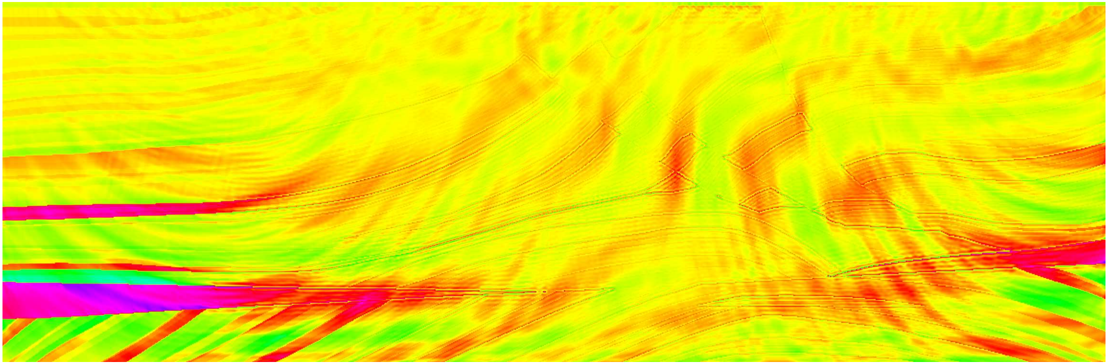
Sum of the Gabor functions
to which the wavefield recorded for wave source 70 is sensitive.



Sum of the Gabor functions
to which the wavefield recorded for wave source 220 is sensitive.



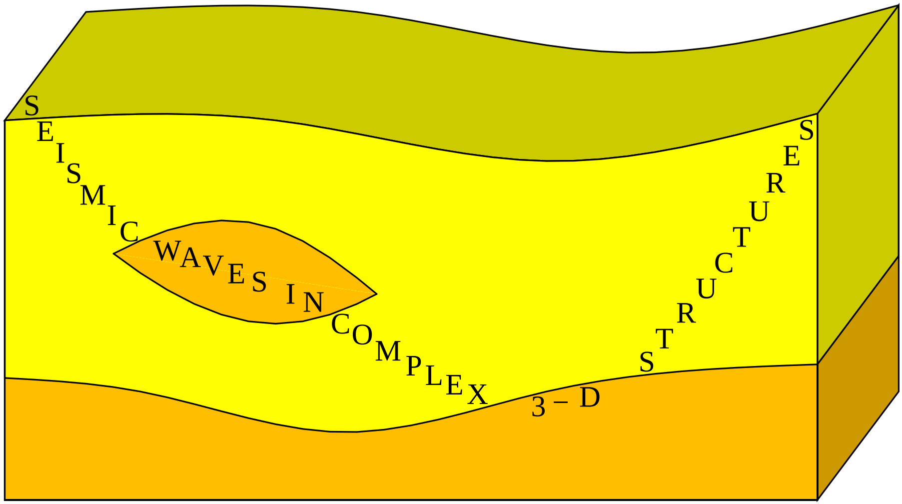
Sum of the Gabor functions
to which the wavefields collected from all wave sources are sensitive.



Part of the velocity difference
influencing no recorded wavefield.

Acknowledgements

Ray theory for wave propagation in heterogeneous anisotropic media
is being developed within consortium research project
"Seismic Waves in Complex 3–D Structures"



http://sw3d.cz