Moveout approximation for P waves in a homogeneous VTI medium

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Introduction

Unconverted P-wave reflection in a homogeneous VTI layer

$$T^2(x) = (4H^2 + x^2)/v^2(\mathbf{n})$$

$T(x)$ - traveltime at the offset x

$v(\mathbf{n})$ - ray velocity

$\mathbf{n}$ - unit vector in the direction of the slowness vector $\mathbf{p}$

$\mathbf{N}$ - unit vector in the direction of the ray-velocity vector ($\mathbf{v}$)

H - depth of the reflecting interface

$\Rightarrow$ 2D problem in the (x_1, x_3) plane

Introduction

Normalized moveout formula

$$\bar{x} = x/2H \ , \qquad T_0 = 2H/\alpha$$

$$T^2(x) = \alpha^2 T_0^2 \, (1 + \bar{x}^2)/v^2(\mathbf{n})$$

T_0 - two-way zero-offset traveltime

$\bar{x}$ - normalized offset

α - vertical P-wave velocity, $\alpha^2 = A_{33}$

$A_{\alpha\beta}$ - density-normalized elastic moduli in the Voigt notation

Introduction

Specification of the vector N

$$N_1 = \bar{x}/\sqrt{1+\bar{x}^2}\,, \qquad N_3 = 1/\sqrt{1+\bar{x}^2}$$

Weak-anisotropy approximation

- first-order approximation of c^2 (c - phase velocity)

$$c^2(\mathbf{n}) = \alpha^2[1 + 2(\delta - \epsilon)n_1^2 n_3^2 + 2\epsilon n_1^2]$$

$$\epsilon = (A_{11} - \alpha^2)/2\alpha^2\,, \quad \delta = (A_{13} + 2A_{55} - \alpha^2)/\alpha^2\,, \quad \alpha^2 = A_{33}$$

Introduction

Relation between $\mathbf{N}$ and $\mathbf{n}$

$$\mathbf{N}(\mathbf{n}) = \mathbf{n} + 2c^{-2}(\mathbf{n})B(\mathbf{n})\mathbf{e}(\mathbf{n})$$

$$B(\mathbf{n}) = \alpha^2 n_1 n_3[\delta - 2(\delta - \epsilon)n_1^2]$$

$\mathbf{e}$ - unit vector, $\mathbf{e} \perp \mathbf{n}$ in the (x_1, x_3) plane

Relations between $v^2(\mathbf{n})$, $c^2(\mathbf{n})$ and $c^2(\mathbf{N})$

$$c^2(\mathbf{n}) = c^2(\mathbf{N}) - 8c^{-2}(\mathbf{N})B^2(\mathbf{N})$$

$$v^2(\mathbf{n}) = c^2(\mathbf{n}) + 4c^{-2}(\mathbf{n})B^2(\mathbf{n})$$

First-order traveltime approximations

a) Assumption $\mathbf{n} = \mathbf{N}$

$$T^2(\bar{x}) = T_0^2(1+\bar{x}^2)^3/P(\bar{x})$$

$$v^2(\mathbf{n}) = c^2(\mathbf{N})$$

b) Assumption $\mathbf{n} \neq \mathbf{N}$

$$T^2(\bar{x}) = T_0^2(1+\bar{x}^2)^3/[P(\bar{x}) - 4Q(\bar{x})P^{-1}(\bar{x})]$$

$$v^2(\mathbf{n}) = c^2(\mathbf{N}) - 4c^{-2}(\mathbf{N})B^2(\mathbf{n})$$

$$P(\bar{x}) = (1+\bar{x}^2)^2 + 2\delta\bar{x}^2 + 2\epsilon\bar{x}^4 \;, \qquad Q(\bar{x}) = \bar{x}^2[2\epsilon\bar{x}^2 + \delta(1-\bar{x}^2)]^2$$

Second-order traveltime approximation

$$T^2(\bar{x}) = T_0^2(1+\bar{x}^2)^3/[P(\bar{x}) - aQ(\bar{x})P^{-1}(\bar{x})]$$

$$P(\bar{x}) = (1+\bar{x}^2)^2 + 2\delta\bar{x}^2 + 2\epsilon\bar{x}^4 \quad , \qquad Q(\bar{x}) = \bar{x}^2[2\epsilon\bar{x}^2 + \delta(1-\bar{x}^2)]^2$$

$$a = (4r^2-3)/(r^2-1) \; , \qquad r = \beta/\alpha$$

β - vertical S-wave velocity, $\beta^2 = A_{55}$

$$v^2(\mathbf{n}) = c^2(\mathbf{N}) + c^{-2}(\mathbf{N})B^2(\mathbf{N})[(1-r^2)^{-1} - 4]$$

Long-spread moveout approximation

Tsvankin and Thomsen (1994); Tsvankin (2001)

$$T^2(\bar{x}) = T_0^2[1 + R\bar{x}^2 - 2(\epsilon_T - \delta_T)R^3\bar{x}^4/(1 + S\bar{x}^2)]$$

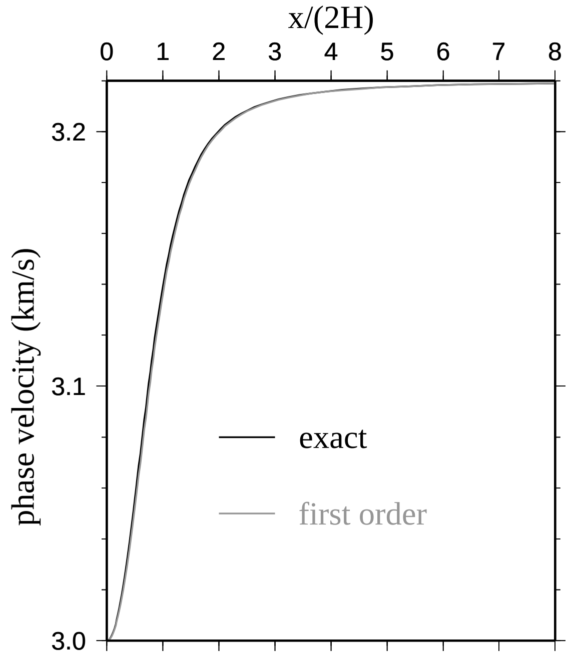
$$R = (1 + 2\delta_T)^{-1} \quad , \qquad S = R^2(1 + 2\epsilon_T)$$

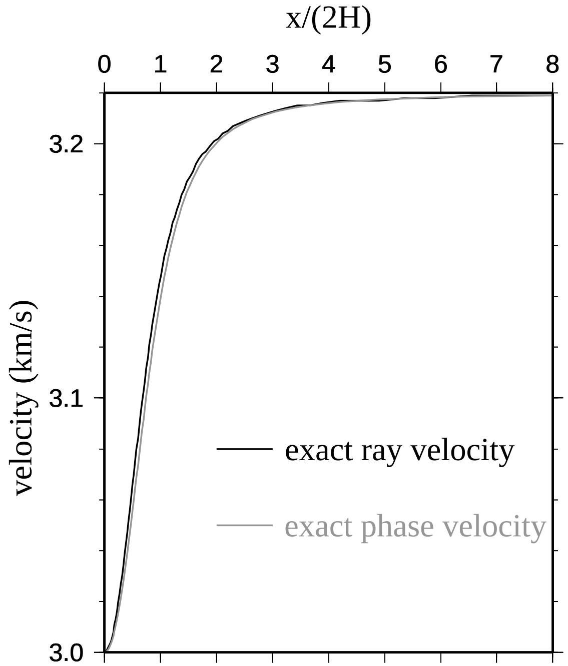
$\epsilon_T = \epsilon$, δ_T - Thomsen's parameters

Tests of the formulae

Limestone (anisotropy $\sim 8\%$)

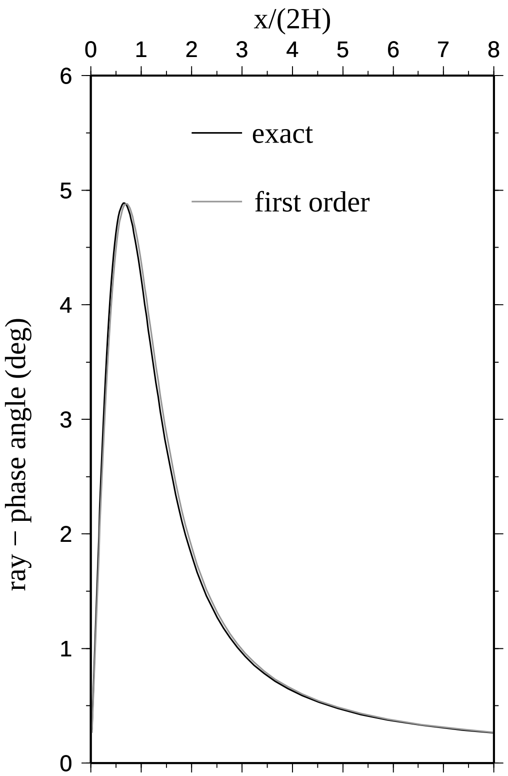
$$\alpha = 3.0km/s, \quad \beta = 1.707km/s, \quad \epsilon = 0.076, \quad \delta = -0.133$$

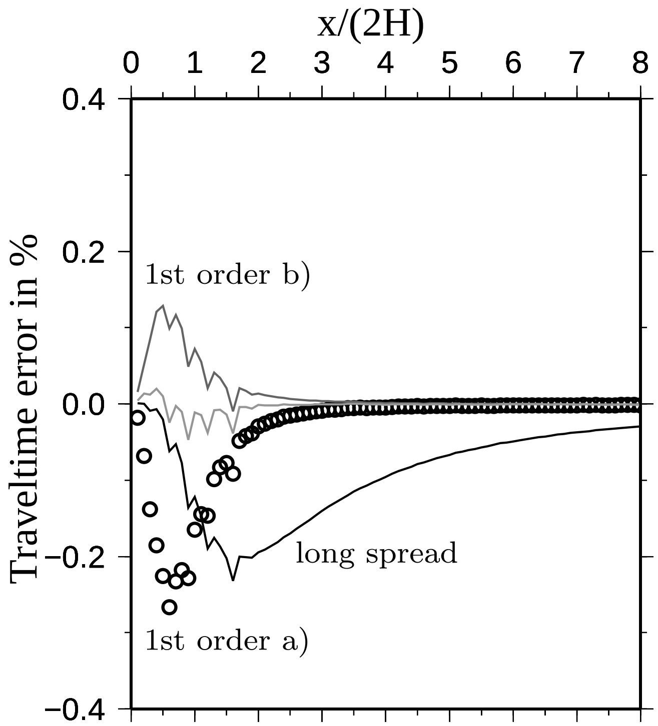




Tests of the formulae

Limestone (anisotropy $\sim$ 8%)

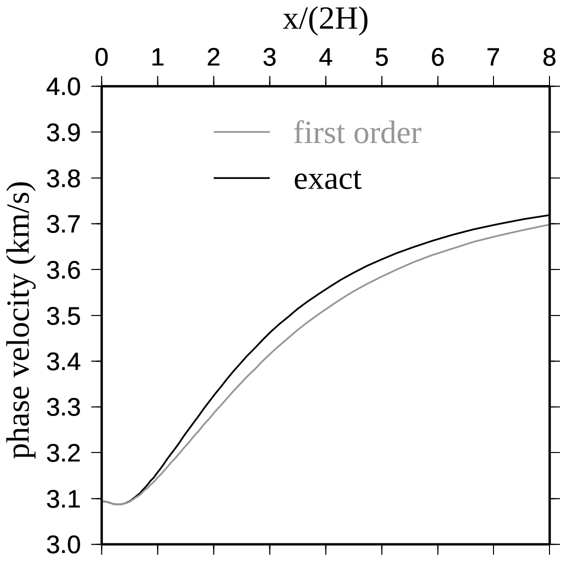


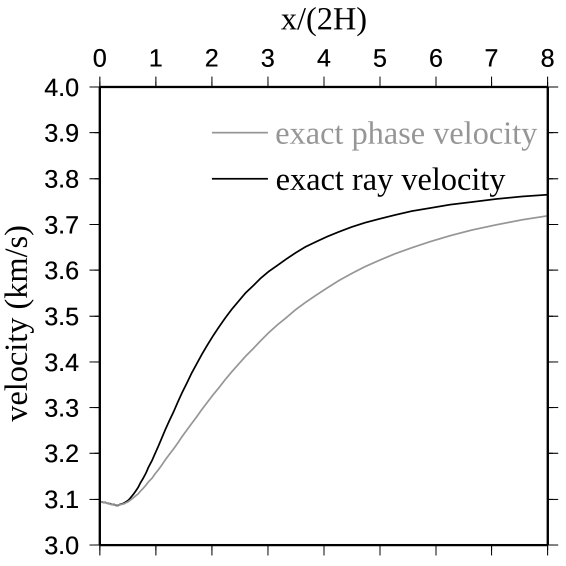


Tests of the formulae

Greenhorne shale (anisotropy $\sim$ 26%)

$$\alpha = 3.094km/s, \quad \beta = 1.51km/s, \quad \epsilon = 0.256, \quad \delta = -0.0523$$





Tests of the formulae

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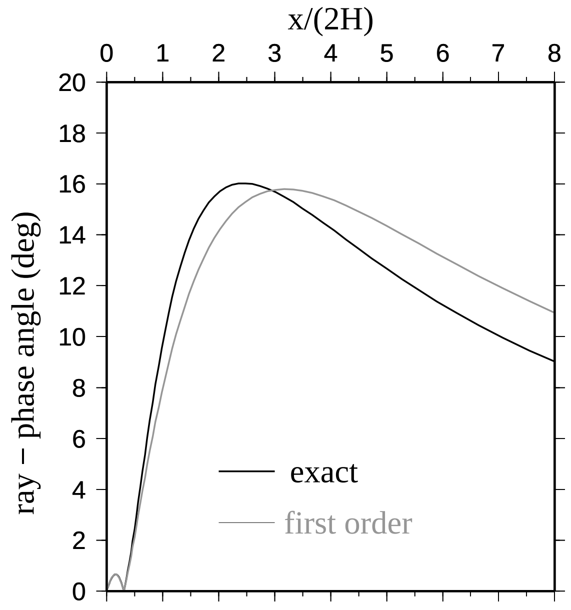
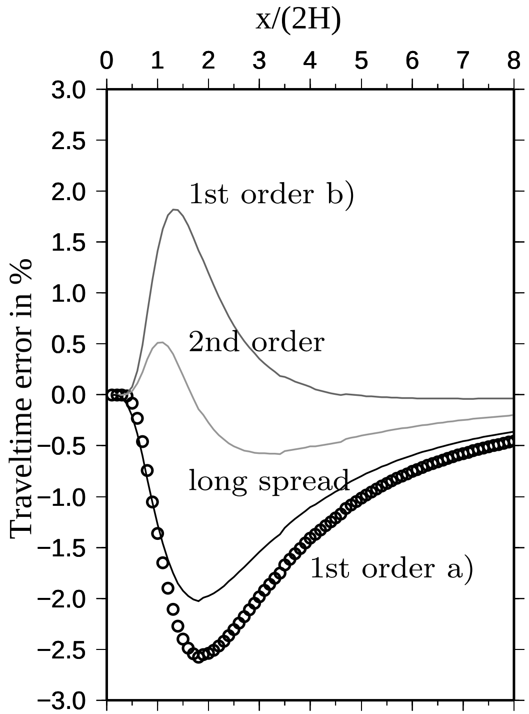


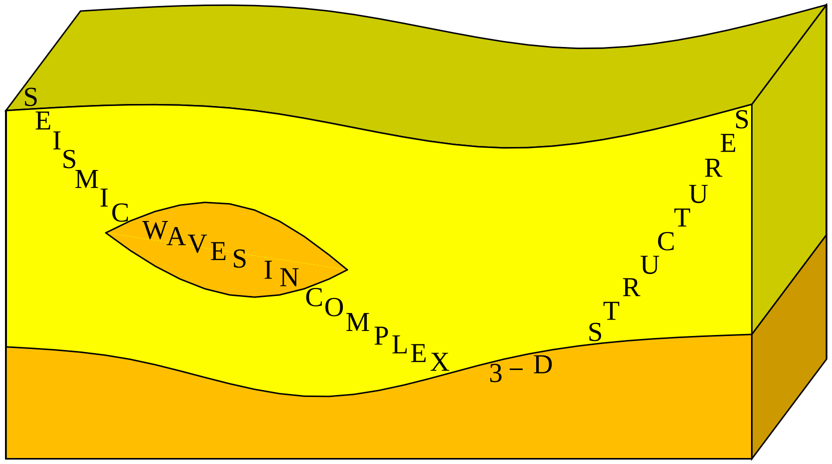


Conclusions

- based on WA approximation
- no non-physical assumptions
- relatively simple formulae
- inaccuracies for large deviations of $\mathbf{n}$ and $\mathbf{N}$
- for small and large offsets accurate
- second-order formula very accurate
- dependence on H, α, ϵ, δ, (second-order formula weakly on r)
- straightforward generalization for SV waves
- generalization for TTI or lower symmetry, converted waves

  not so straightforward

Acknowledgements



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