Second-order traveltimes and first-order spreading of reflected/transmitted P waves in inhomogeneous, weakly anisotropic media

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Conclusions

Introduction

Available: P and coupled S waves (FORT, FODRT)

in smoothly varying, weakly anisotropic media

Generalization: layered, weakly anisotropic media

Two-step generalization:

- transformation of slowness vectors across an interface
- transformation of amplitudes across an interface

This presentation: P-wave traveltimes and spreading

in layered media

FORT and FODRT in smooth media

Decisive role in RT and DRT played by eigenvalue $G(\mathbf{x}, \mathbf{p})$

of the Christoffel matrix $\boldsymbol{\Gamma}(\mathbf{x}, \mathbf{p})$

$$G(\mathbf{x}, \mathbf{p}) = a_{ijkl} p_j p_l g_i g_k$$

a_{ijkl} - density-normalized elastic moduli

$\mathbf{x}$ - Cartesian coordinates

$\mathbf{p}$ - slowness vector

$\mathbf{g}$ - polarization vector

FORT and FODRT in smooth media

Basic idea: replace exact $G(\mathbf{x},\mathbf{p})$ by $G^{(1)}(\mathbf{x},\mathbf{p})$ everywhere!

Perturbation of P-wave eigenvalue $G(\mathbf{x},\mathbf{p})$

$$G(\mathbf{x},\mathbf{p}) \sim G^{(1)}(\mathbf{x},\mathbf{p}) = G^{(0)}(\mathbf{x},\mathbf{p}) + \Delta G(\mathbf{x},\mathbf{p})$$

$G^{(0)}(\mathbf{x},\mathbf{p})$ - eigenvalue of $\mathbf{\Gamma}$ in reference isotropic medium

$\Delta G(\mathbf{x},\mathbf{p})$ - first-order perturbation of $G^{(0)}(\mathbf{x},\mathbf{p})$

$G^{(1)}(\mathbf{x},\mathbf{p})$ - first-order approximation of $G(\mathbf{x},\mathbf{p})$ independent of the choice of reference

FORT and FODRT in smooth media

$G(\mathbf{x}, \mathbf{p})$ - first-order approximation of P-wave eigenvalue

Eikonal equation: $G(\mathbf{x}, \mathbf{p}) = 1$

Ray-tracing equations (FORT):

$$dx_i/d\tau = \frac{1}{2}\partial G/\partial p_i \, , \qquad dp_i/d\tau = -\frac{1}{2}\partial G/\partial x_i$$

x_i - ray coordinates of the first-order ray

p_i - components of the first-order slowness vector $\mathbf{p}$

τ - first-order traveltime

FORT and FODRT in smooth media

$\mathcal{L} = |\mathbf{X}^{(1)} \times \mathbf{X}^{(2)}|^{1/2}$ - first-order geometrical spreading

Dynamic ray-tracing equations (FODRT):

$$dX_i^{(I)}/d\tau = \frac{1}{2}\left(\frac{\partial^2 G(\mathbf{x},\mathbf{p})}{\partial p_i \partial x_j} X_j^{(I)} + \frac{\partial^2 G(\mathbf{x},\mathbf{p})}{\partial p_i \partial p_j} Y_j^{(I)}\right)$$

$$dY_i^{(I)}/d\tau = -\frac{1}{2}\left(\frac{\partial^2 G(\mathbf{x},\mathbf{p})}{\partial x_i \partial x_j} X_j^{(I)} + \frac{\partial^2 G(\mathbf{x},\mathbf{p})}{\partial x_i \partial p_j} Y_j^{(I)}\right)$$

$$X_i^{(I)} = [\partial x_i/\partial \gamma^{(I)}]_{\tau=const} \, , \qquad Y_i^{(I)} = [\partial p_i/\partial \gamma^{(I)}]_{\tau=const}$$

$\gamma^{(I)}$ - ray parameters (e.g., take-off angles)

FORT and FODRT in smooth media

Second-order traveltime correction

$$\Delta\tau = -\frac{1}{2}\int_\Omega [c(\mathbf{x},\mathbf{n})]^{-2}[B_{13}^2(\mathbf{x},\mathbf{n}) + B_{23}^2(\mathbf{x},\mathbf{n})]/(V_P^2 - V_S^2)d\tau$$

$c \quad (c^{-2} = p_i p_i)$ - first-order phase velocity

V_P, V_S - P- and S-wave reference velocities along ray Ω

efficient choice: $V_P^2 = B_{33}$, $V_S^2 = 0.5(B_{11} + B_{22})$

$$B_{mn}(\mathbf{x},\mathbf{n}) = a_{ijkl}(\mathbf{x}) n_i n_l e_j^{[m]} e_k^{[n]}$$

a_{ijkl} - density-normalized elastic moduli

$\mathbf{e}^{[m]}$ - triplet of orthogonal unit vectors, $\mathbf{e}^{[3]} = \mathbf{n} = c\mathbf{p}$

Transformation of slowness vector

$$p_i^G - (p_k^G N_k) N_i = p_i - (p_k N_k) N_i$$ (Snell law)

$\mathbf{N}$ - unit normal to the interface

$\mathbf{p}$ - first-order slowness vector of incident wave

$\mathbf{p^G}$ - first-order slowness vector of generated wave

$$p_i^G = b_i + \xi^G N_i \qquad b_i = p_i - (p_k N_k) N_i$$

ξ^G - sought quantity

$G(b_i + \xi^G N_i) = 1$ - polynomial equation of 4th degree

G corresponds to generated wave

Transformation of slowness vector

$G(b_i + \xi^G N_i) = 1$ - polynomial equation of 4th degree

a) numerical solution

b) iterative solution (arbitrary incidence, arbitrary contrast)

Iterative solution

commonly ≤ 2 iterations to reach $\xi^{G(j)} - \xi^{G(j-1)} < 10^{-5}$

$$\mathbf{p}^{G\{j\}} = \mathbf{b} + \xi^{G\{j\}}\mathbf{N}$$

$$\xi^{G\{j\}} = \xi^{G\{j-1\}} - \frac{G(\mathbf{p}^{G\{j-1\}}) - 1}{N_k \partial G / \partial p_k(\mathbf{p}^{G\{j-1\}})}$$

$\xi^{G\{0\}}$ - reference isotropic medium

Transformation of slowness vector

MODEL: ISO/HTI

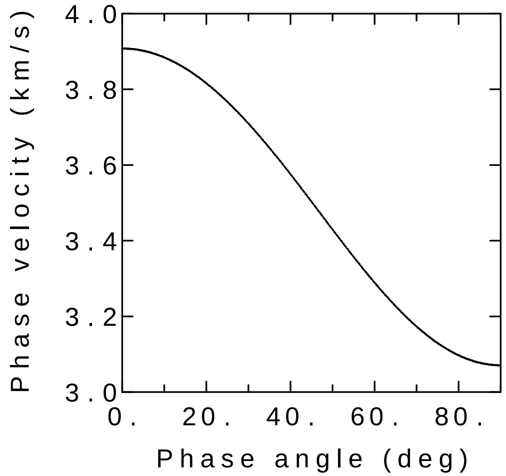
ISO A: α=4.0 km/s, β=2.31 km/s

ISO B: α=3.0 km/s, β=1.73 km/s

0^0 - vertical propagation

90^0 - propagation along x-axis (axis of symmetry)

HTI:



Anisotropy: $\sim$ 24%

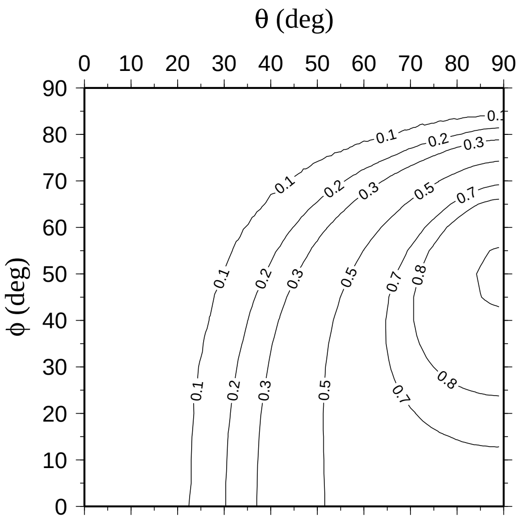
Contrast (normal $\rightarrow$ tangent. inc.): A: 2% $\rightarrow$ 26%, B: 26% $\rightarrow$ 2%

Transformation of slowness vector

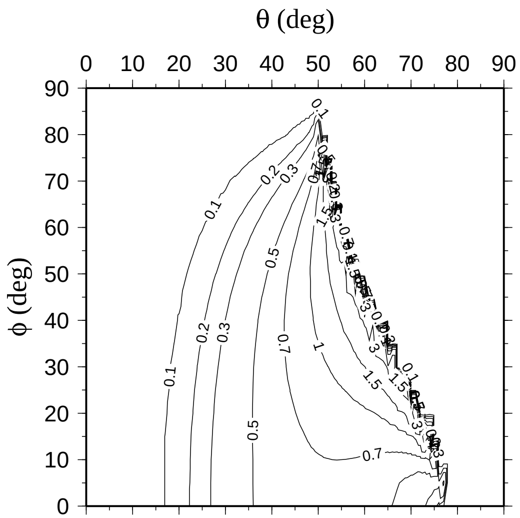
Angular deviations (deg) of approximate from exact slowness vectors of transmitted P waves

θ, ϕ specify the direction of slowness vector of incident P wave

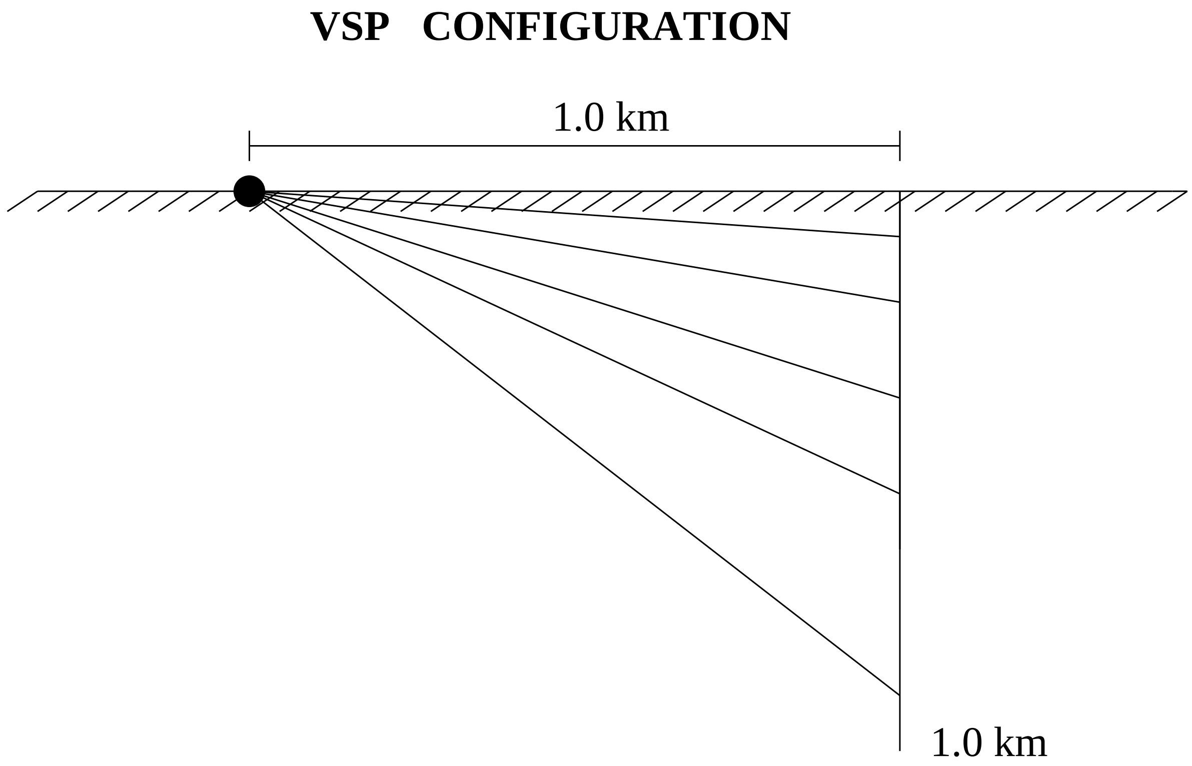
MODEL A



MODEL B



Traveltime and spreading across an interface



Traveltime and spreading across an interface

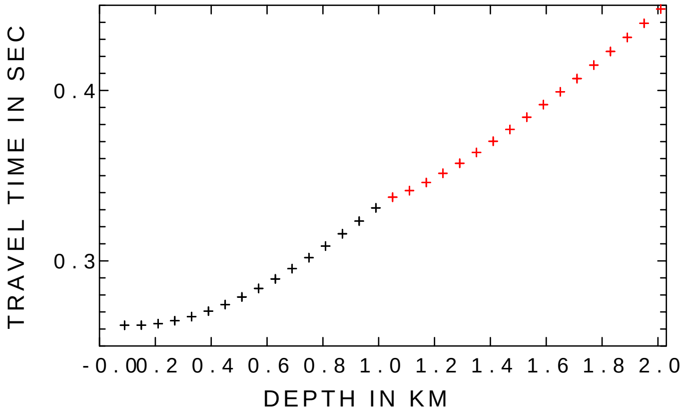
Two-layer HTI model

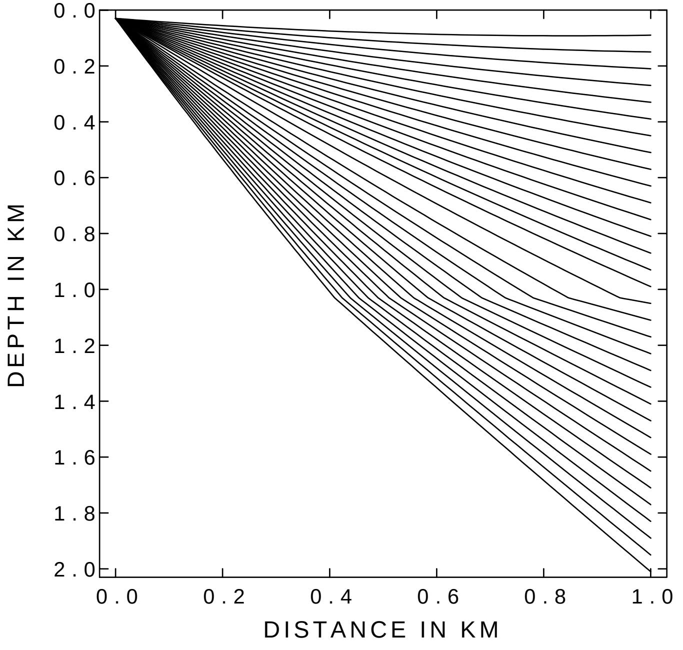
Layer 1: 0-1 km; symmetry axis rotates $45^0 \rightarrow 0^0$
vertical variation of WA parameters

Layer 2: 1-3 km; symmetry axis rotates $0^0 \rightarrow 45^0$
no variation of WA parameters

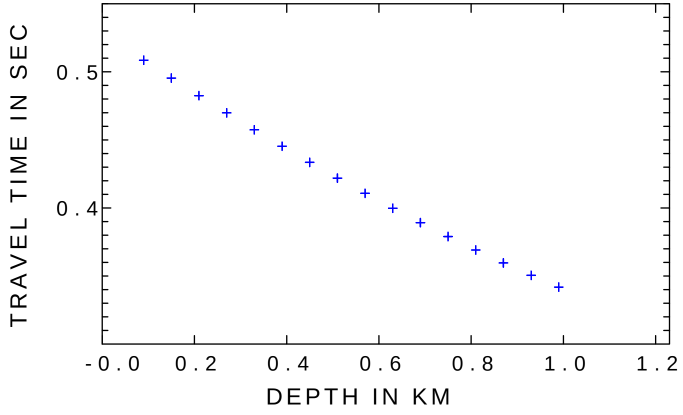
Anisotropy: $\sim$ 8%; contrast: $\sim$ 23%

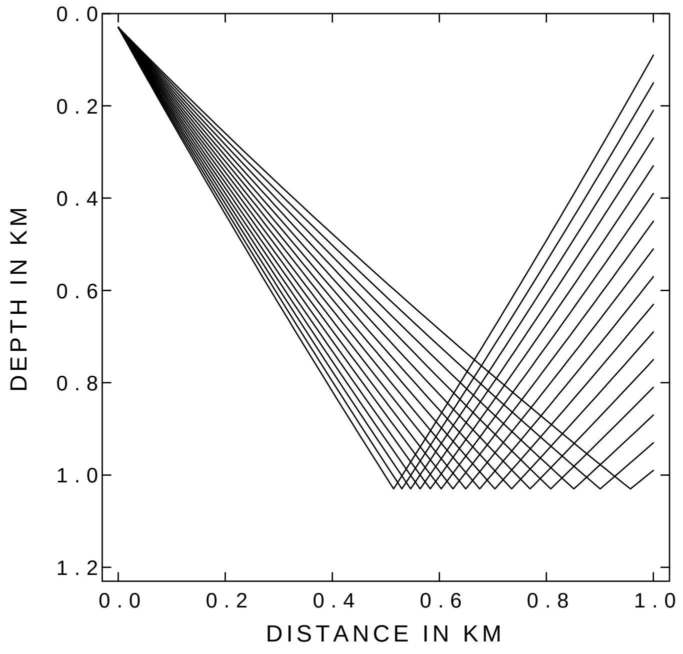
Traveltime and spreading across an interface
transmitted wave



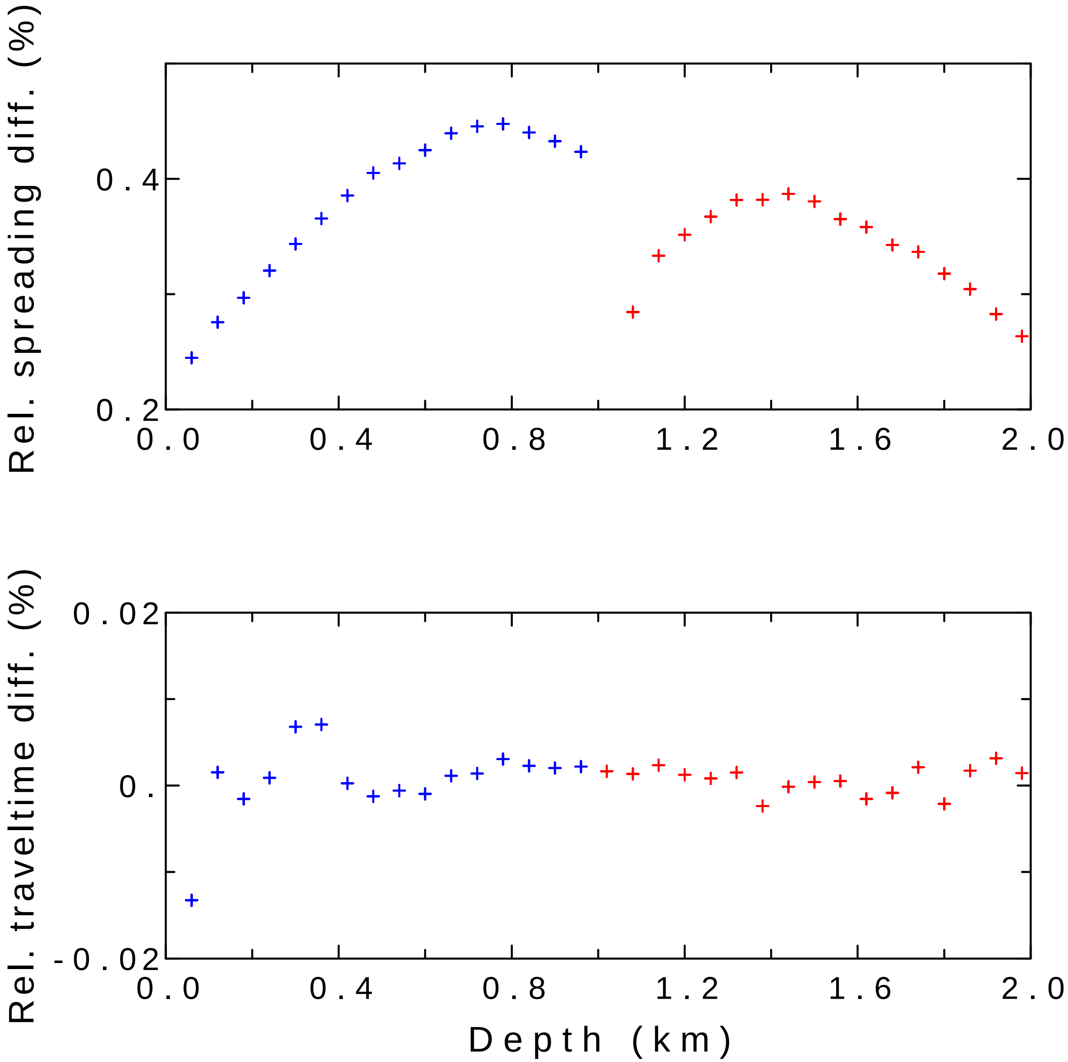


Traveltime and spreading across an interface reflected wave

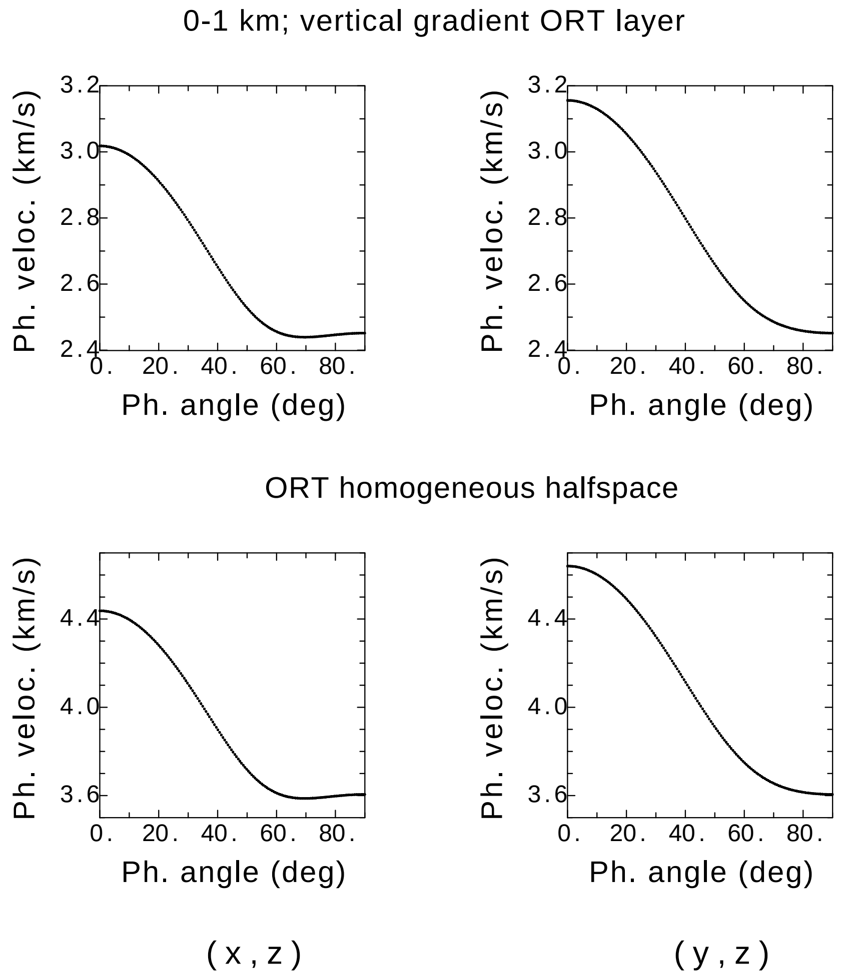




Traveltime and spreading across an interface



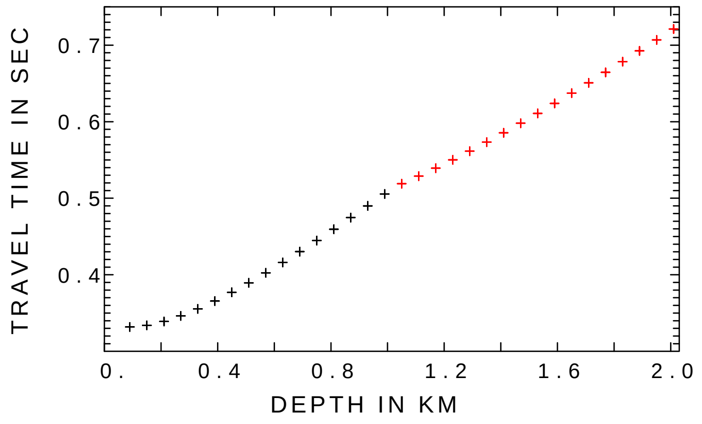
Traveltime and spreading across an interface
ORT model

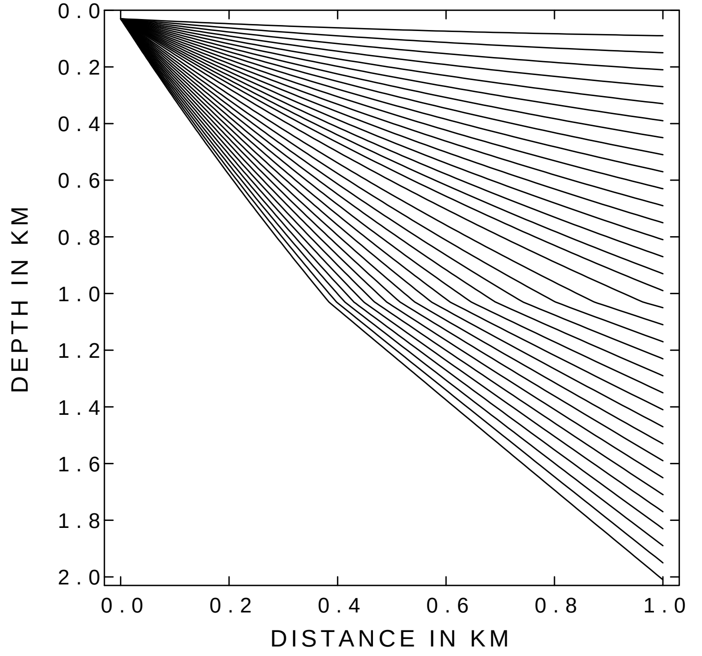


Anisotropy: $\sim$ 20%; contrast (normal $\rightarrow$ tangent. inc.): $\sim$ 22% $\rightarrow$ 39%

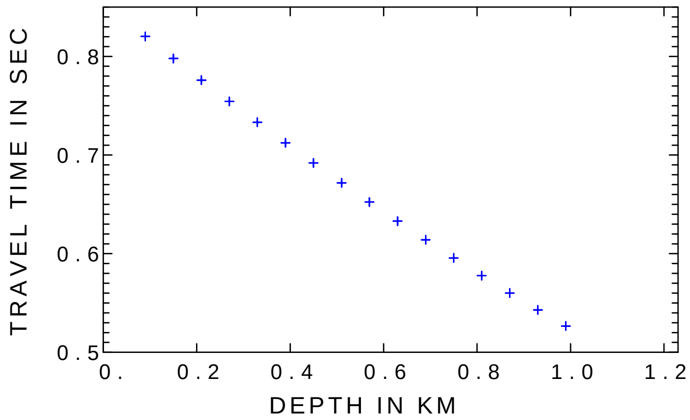
Traveltime and spreading across an interface

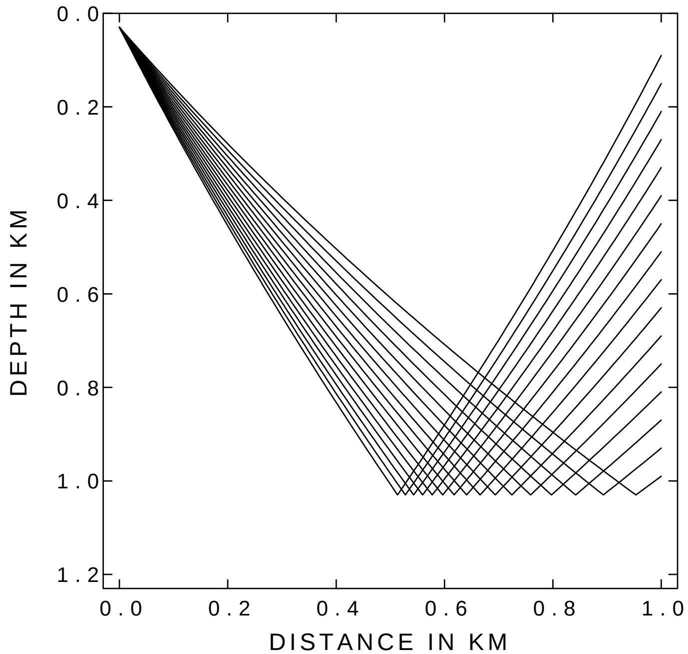
transmitted wave



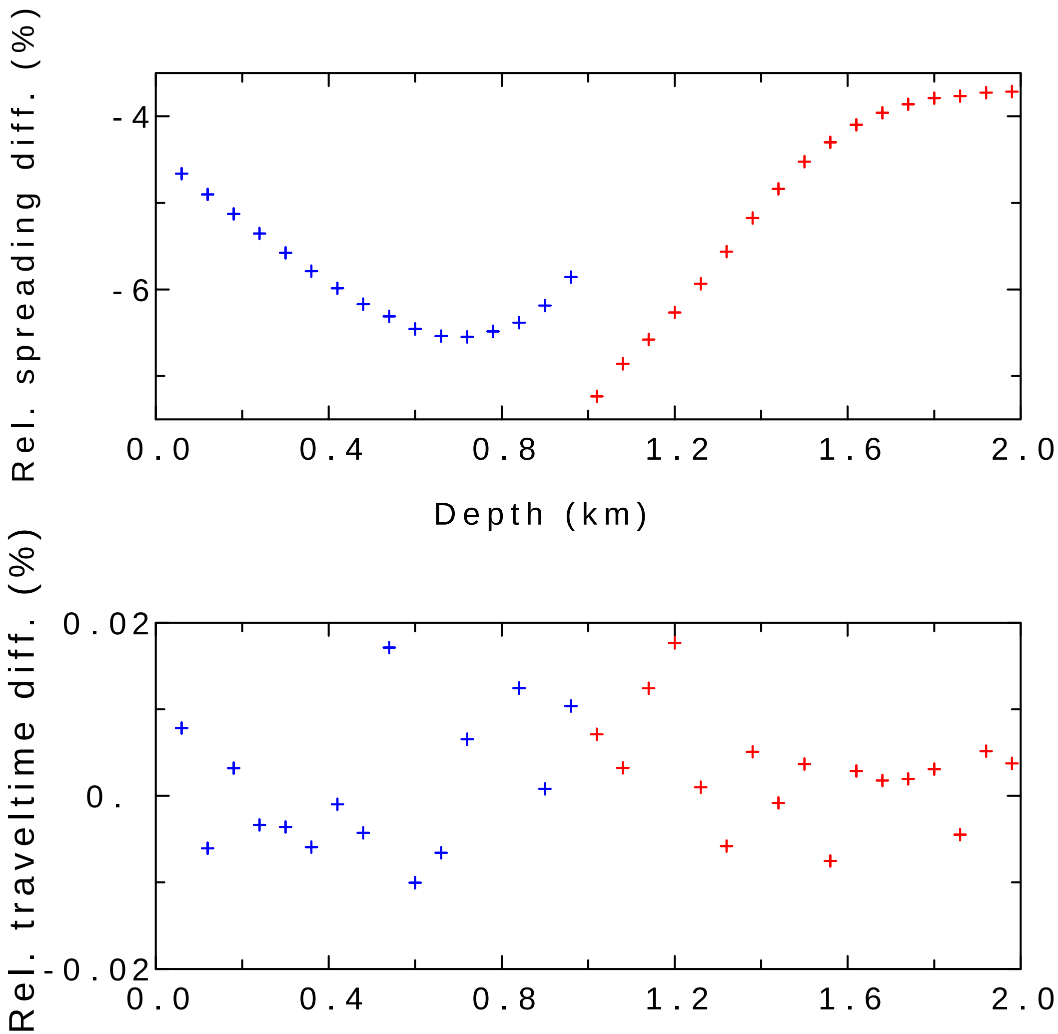


Traveltime and spreading across an interface reflected wave





Traveltime and spreading across an interface



Conclusions

P-wave separated from S wave

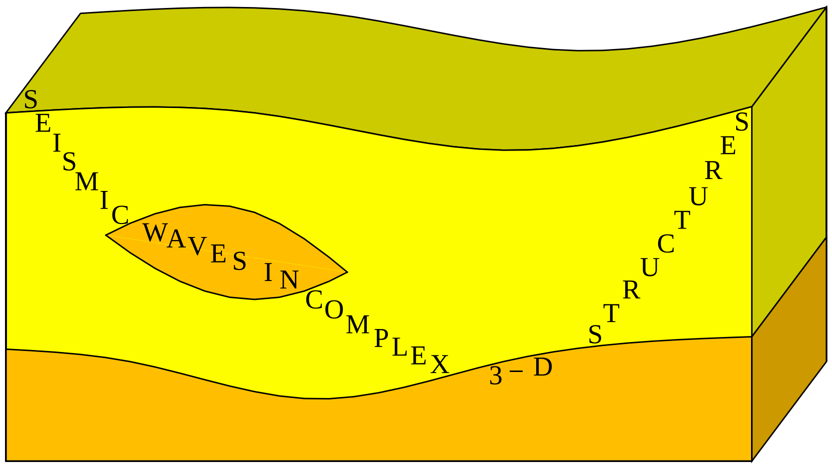
Second-order traveltime, first-order spreading

Accuracy of approximate traveltime and spreading in layered media comparable with accuracy in smooth media

Efficient iterative determination of slowness vectors of generated waves, arbitrary contrast, arbitrary angle of incidence

P-wave WA R/T coefficients available (Farra & Pšenčík, 2010) $\Rightarrow$ next step: seismograms of R/T P waves

Acknowledgements



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