# LOCAL DETERMINATION OF WEAK ANISOTROPY PARAMETERS FROM WALKAWAY VSP qP-WAVE DATA IN THE JAVA SEA REGION

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*Received: 11 January 2002; Revised: 1 September 2003; Accepted: 8 October 2003*

ABSTRACT

*We apply the inversion scheme of Zheng and Pšenčík (2002) to the walkaway VSP data of Horne and Leaney (2000) collected in the Java Sea region. The goal is a local determination of parameters of the medium surrounding the borehole receiver array. The inversion scheme is based on linearized equations expressing qP-wave slowness and polarization vectors in terms of weak anisotropy (WA) parameters. It thus represents an alternative approach to Horne and Leaney (2000), who based their procedure on inversion of the Christoffel equation using a global optimization method. The presented inversion scheme is independent of structural complexities in the overburden and of the orientation of the borehole. The inverson formula is local, and has therefore potential to separate effects of anisotropy from effects of inhomogeneity. The data used are components of the slowness vector along the receiver array and polarization vectors. The inversion is performed without any assumptions concerning the remaining components of the slowness vector. The inversion is made (a) assuming arbitrary anisotropy, i.e., without any assumptions about symmetry of the medium, (b) assuming transverse isotropy with a vertical axis of symmetry and (c) assuming isotropy of the medium. Inverted are the raw data as well as data, in which weighting is used to reduce the effect of outliers. It is found that the WA parameters $\epsilon_z$, $\epsilon_{15}$ and $\epsilon_{35}$ are considerably more stable than the parameters $\epsilon_x$ and $\delta_x$. The latter two parameters are also found to be strongly correlated. Weaker correlation is also found between the mentioned two parameters and $\epsilon_z$. The results of inversion show clearly that the studied medium is not isotropic. They also seem to indicate that the studied medium does not possess the VTI symmetry.*

Keywords: Weak anisotropy, weak anisotropy (WA) parameters, $qP$ waves, slowness vector, polarization vector, local inversion, walkaway VSP

## 1. INTRODUCTION

We apply the inversion scheme of *Zheng and Pšenčík (2002)*, based on approximate formulae obtained from the first-order perturbation theory, to real data. The goal is the determination of parameters of the medium at a given receiver. The scheme was succesfully tested on synthetic data from a multi-azimuthal multiple-source offset VSP experiment. Here, the algorithm is used to invert the walkaway VSP data from an experiment in the Java Sea region, see *Leaney (1994)*, *Horne and Leaney (2000)*. Since the geology of the region is essentially horizontal, *Horne and Leaney (2000)* inverted the data assuming local homogeneity and VTI symmetry of the medium in a vicinity of the receiver. They based their inversion on successive tests of the Christoffel equation for varying values of the parameters of the medium. The basic part of their algorithm was the determination of the wave normal corresponding to a given, i.e., observed, polarization of transmitted and reflected $qP$ waves and of transmitted and reflected, converted $qS$ waves. This was done by solving iteratively the Christoffel equation. For the obtained wave normal the component of the slowness vector along the receiver array was calculated and compared with the vertical component of the observed slowness vector. The parameters of the medium which minimized the misfit of the vertical components of the observed and calculated slowness vectors were then accepted as the sought parameters. Adaptive simulated annealing was used for the minimization.

In this study, the parameters of the medium are determined from the linearized equations expressing $qP$-wave slowness and polarization vectors in terms of parameters of a reference isotropic medium and weak anisotropy (WA) parameters, see, e.g. *Pšenčík and Gajewski (1998)*, *Pšenčík and Vavryčuk (2002)*. For each source-receiver pair we have three equations. One equation relates the WA parameters to the slowness and two equations relate the WA parameters to the polarization vector. The equations require knowledge of slowness and polarization vectors at a receiver. With three-component receivers, the determination of the complete polarization vector should be, generally, possible. As to the slowness vector, only the component along the receiver array is usually available from the measurements in a borehole. In order to determine the unknown horizontal components, either cross-hole measurements are required or various assumptions about the structure of the overburden are often made. More details about this problem and its history can be found, together with references, in *Zheng and Pšenčík (2002)*. If we wish to avoid artificial assumptions about the structure, we must eliminate the two unknown horizontal components from the above-mentioned three linearized equations, reducing them into a single equation. The resulting equation is independent of structural complexities in the overburden, does not depend on the orientation of the borehole, and requires no assumptions about local homogeneity around the receiver. This removes the problems encountered by *Dewagan and Grechka (2003)* in their scenario 3. In this contribution, we use this equation and specify it for the case of a walkaway profile. For a given receiver and a system of sources we can thus form a system of linear equations, from which the WA parameters can be determined. The WA parameters are determined by minimizing the misfit in the mentioned equations using the SVD algorithm (*Press et al., 1986*). We invert the data (a) without making any assumption about the anisotropic symmetry of the medium (ANI experiment), (b) assuming that the medium is transversely isotropic with a vertical axis of symmetry (VTI experiment) and (c) assuming that the medium is isotropic (ISO experiment). Because the data are collected along a single profile, only 5 $qP$-wave WA parameters from 15, which describe $qP$ waves in a general weakly anisotropic medium (*Pšenčík and Gajewski, 1998*) can be determined in the ANI experiment, 3 parameters in the VTI experiment and a single parameter ($P$-wave velocity) in the ISO experiment.

The above three experiments are performed with unweighted (i.e. raw) and weighted data. The weighting is used to reduce the effects of outliers in the data. In the former case, the data are treated without any additional assumption. In the latter case, the effect of the data points which differ considerably (outliers) from a general trend between vertical components of the slowness vector and the polarization angles is reduced. This leads to a considerable improvement of the misfit. We must keep in mind, however, that the results obtained with weighting are obtained with the above subjective assumption. Another factor affecting the results is the choice of the reference isotropic medium. Its effects are also investigated. Throughout the paper we assume that the noise of the data is Gaussian, i.e., it is additive, uncorrelated and has zero average.

In Sec. 2, basic equations used for the inversion are specified and briefly discussed. The observed data and their treatment are described in Sec. 3. The inversion procedure is outlined and results of the three inversion experiments ANI, VTI and ISO are discussed in Sec. 4.

## 2. DESCRIPTION OF THE ALGORITHM

Let us consider a Cartesian coordinate system $(x, y, z)$, with positive $z$-axis pointing down along the borehole, see Figure 1. For local determination of weak anisotropy (WA) parameters in a vicinity of a receiver situated in a borehole, we use modified form of Eq.(22) of *Zheng and Pšenčík (2002)*:

$$(\alpha^2 - \beta^2)^{-1} B_{13} D - \frac{1}{2}\alpha^{-1} B_{33}\eta = D e_i^{(1)} g_i + \alpha \Delta\eta \ , \tag{1}$$

where

$$D = (n_1^2 + n_2^2)^{1/2}, \quad \eta = \alpha^{-1} n_3 \ . \tag{2}$$

Eq. (1) is a result of elimination of two horizontal components of a slowness vector from three linearized equations for $qP$-wave slowness and polarization vectors. The

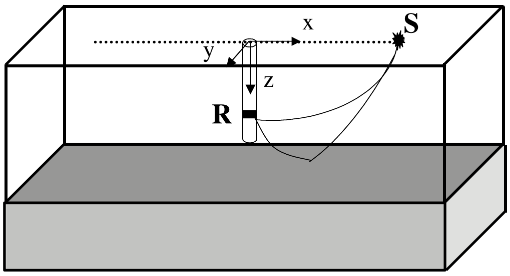


**Fig. 1.** Configuration of the walkaway experiment: single radial profile along the $x$-axis with 117 shots to the left and 111 shots to the right side of the borehole, with approximately 0.025 km separation; a three-component receiver at a depth of 1.63 km. S - a shot point, R - the receiver.

symbols $\alpha$ and $\beta$ denote $P$- and $S$-wave velocities of the isotropic reference medium. Observed data appear in Eq. (1) through the observed polarization vector $g_i$, and as the quantity $\Delta\eta = p_z^{obs} - \eta$, which represents the vertical component of the vectorial difference of the observed slowness vector from the slowness vector in the reference medium. The vector $n_i$ in (2) denotes the unit wave normal in the reference isotropic medium. The vector $e_i^{(1)}$ in (1) is a unit vector perpendicular to $n_i$. Both vectors belong to the vectorial frame $e_i^{(1)}$, $e_i^{(2)}$ and $e_i^{(3)} = n_i$ in the reference medium, which is chosen as follows

$$\vec{e}^{(1)} = D^{-1}(n_1 n_3, n_2 n_3, n_3^2 - 1), \quad \vec{e}^{(2)} = D^{-1}(-n_2, n_1, 0), \quad \vec{e}^{(3)} = (n_1, n_2, n_3) \; . \quad (3)$$

The symbols $B_{mn}$ in Eq. (1) denote the elements of the *weak anisotropy* matrix,

$$B_{mn} = a_{ijkl} e_i^{(m)} e_j^{(3)} e_l^{(3)} e_k^{(n)} - c_0^2 \delta_{mn} \; . \quad (4)$$

Here $a_{ijkl}$ is a tensor of density-normalized elastic parameters, $c_0$ stands for $\alpha$ when $m = n = 3$, or $\beta$ when $m = n = 1, 2$. The elements of the WA matrix depend on the sought WA parameters. Eq. (1) is independent of any structural complexity in the overburden and holds for an arbitrarily oriented borehole. Eq. (1) is local. This means that variation of observed quantities appearing in it is considered to be solely an effect of anisotropy.

Let us now consider a walkaway experiment. Let us identify the walkaway profile with the $x$-axis of the Cartesian coordinate system and let us consider the wave normal $n_i$ in the reference medium to be situated in the $(x, z)$ plane. In such a case, we have $n_2 = 0$ and Eq. (1) can be rewritten into the following form

$$(\alpha^2 - \beta^2)^{-1} B_{13} |n_1| - \frac{1}{2} \alpha^{-2} B_{33} n_3 = |n_1| e_i^{(1)} g_i + \alpha \Delta\eta \; . \quad (5)$$

All quantities appearing in Eq. (5) have the same meaning as in Eq. (1). The elements of the WA matrix $B_{mn}$ used in (5) (specified for $n_2 = 0$) for an anisotropic medium of an arbitrary symmetry, for a VTI symmetry and for an isotropic medium are shown in the Appendix A, in Eqs. (A1), (A3) and (A4), respectively. Eq. (A2) gives the five WA parameters, which are the only WA parameters which can be recovered along a single profile over a medium with anisotropy of arbitrary symmetry. All quantities in Eq. (5) are considered at the receiver.

There are several ways to choose the wave normal $n_i$ in the reference isotropic medium. The most straightforward is to determine $n_i$ in the direction of the straight line connecting the source and the receiver. *Zheng and Pšenčík (2002)* constructed a reference isotropic model in the overburden through which they traced rays in order to determine $n_i$. Here we choose a more effective way. We choose the wave normal $n_i$ to be parallel with the projection of the observed polarization vector $g_i$ onto the plane $(x, z)$. This has two important consequences. First, there is no need to perform the time-consuming ray tracing in the reference medium. Second, for this choice, the first term on the RHS of Eq. (5) disappears (information about the polarization vector is now contained in the orientation of the vector $n_i$). The term on the RHS disappears always, no matter whether or not the polarization vector $g_i$ is situated in the plane $(x, z)$. Synthetic tests based on the use of the above three choices of the wave normal $n_i$ showed that for weak anisotropy all above approaches lead to effectively equivalent results.

Solving the system of equations (5) at a given receiver for a system of sources along the walkaway profile, we can find WA parameters at the receiver. Different types of waves can be used in the process.

## 3. DATA

We apply the inversion scheme based on a system of equations (5) to walkaway VSP data of *Leaney (1994)* collected in the Java Sea region. For a description of local geology see *Leaney (1994)*. A schematic configuration of the experiment is shown in Figure 1. The data consist of vertical components, $p_z$ and $g_z$, of the slowness and polarization vectors of transmitted and reflected $qP$ and converted $qS$ waves. A detailed description of how these data were obtained from the observations can be found in *Horne and Leaney (2000)*. Decomposition into $qP$ and $qS$ waves was achieved using a parametric inversion of *Esmersoy (1990)*. The parametric inversion included both upgoing and downgoing waves simultaneously (*Leaney, 1990*). The components of the slowness and polarization vectors were treated independently. Each shot provided the vertical component of the slowness vector and the polarization vector of all considered waves. In this study, we concentrate on the information related to the $qP$ waves only. We have available vertical components of observed slowness and polarization vectors of $qP$ waves generated by 228 sources distributed irregularly in the range from -2.5 to 2.5 km with approximately 0.025 km spacing along the walkaway profile. The considered receiver is at a depth of 1.63 km in the borehole situated in the middle of the profile.

Since only vertical components of observed slowness and polarization vectors are available, we can make various assumptions about the remaining components of these vectors. Here we assume that the observed slowness and polarization vectors are arbitrarily oriented in space. Such an assumption leads to Eq. (5). For the wave normal $n_i$ chosen as a unit vector parallel to the projection of the polarization vector $g_i$ into the plane $(x, z)$ (see Sec. 2), it does not matter whether or not the polarization vector $g_i$ is confined to the plane $(x, z)$. (The assumption that the slowness vector is confined to the plane $(x, z)$ would imply that its two components, $p_2 = 0$ and $p_3$ are known. Such a situation is described by two equations (20) of *Zheng and Pšenčík (2002)*. For the case of the walkaway VSP, one of these equations reduces to Eq. (5) and the other yields $\epsilon_{16} = \epsilon_{34} = \chi_x = \chi_z = 0$. Thus, the results for arbitrarily oriented slowness and polarization vectors can be easily transformed to the results for the case that the mentioned two vectors are confined in the plane $(x, z)$.)

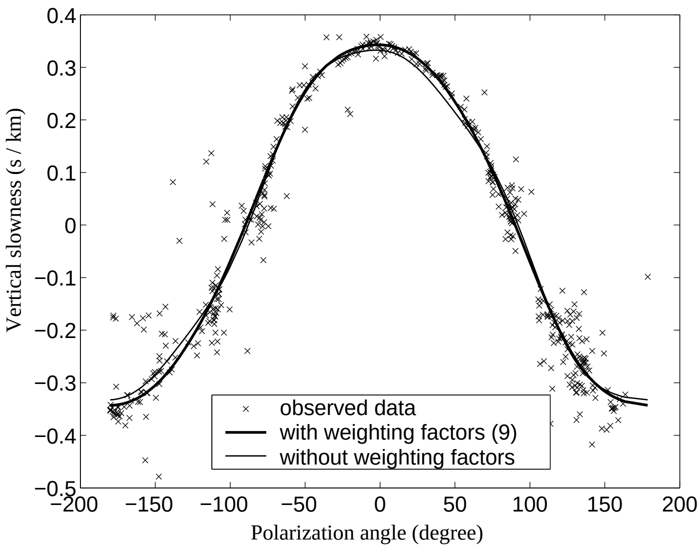


**Fig. 2.** Vertical components of the slowness vector with respect to the polarization angles. Positive and negative vertical components of the slowness vector correspond to down- and upgoing waves, respectively. The observed data shown by ×. Bold and thin lines denote results of the ANI experiment obtained with and without weighting factors (9), respectively. Reference velocity $\alpha = 2.83$ km/sec.

The observed data from all sources are shown by symbols × in Figure 2. This figure has the form used by *Horne and Leaney (2000)*. It shows vertical components of the slowness vector (in s/km) versus polarization angles (angles between the polarization vector and the vertical; in degrees).

## 4. INVERSION

There are various ways to determine the $P$-wave velocity $\alpha$ of the reference isotropic medium. We used the formula

$$\alpha = \frac{1}{N} \sum_{i=1}^{N} \frac{g_z^{(i)}}{p_z^{(i)}} \ . \tag{6}$$

In Eq. (6), $p_z^{(i)}$ and $g_z^{(i)}$ are the vertical components of the observed slowness and polarization vectors, respectively, $N$ is the number of data points for all $qP$ waves considered in the inversion. Using Eq. (6), the reference velocity was found to be $\alpha = 2.5$ km/s. The $S$-wave velocity $\beta$ was taken as $\beta = \alpha/\sqrt{3}$.

We performed three experiments. In the experiment ANI, we made no a priori assumptions about the symmetry of the medium. For data from a single profile, we could recover only five WA parameters: $\epsilon_x$, $\epsilon_z$, $\delta_x$, $\epsilon_{15}$ and $\epsilon_{35}$, see (A2). In the experiment VTI, we assumed that the studied medium was VTI and thus only three WA parameters could be recovered: $\epsilon_x$, $\epsilon_z$ and $\delta_x$. Let us note that the assumption of the VTI anisotropy of the medium has been used by *Horne and Leaney (2000)* in their inversion study. In the experiment ISO, we assumed that the studied medium was isotropic and was thus specified by a single parameter $\epsilon_z$, related to the $P$-wave velocity of the medium. With 228 sources we had 228 equations (Eq. (5)) if only the downgoing $qP$ waves were considered and 456 equations if both downgoing and up-going waves were considered. The studied inversion problems were thus strongly overdetermined.

The system of equations (5) was solved by minimizing the objective function $\Phi$

$$\Phi^2 = \sum_{i=1}^{N} w_i (y_i^{obs} - A_{ij} m_j)^2 \ . \tag{7}$$

The symbol $N$ denotes again the number of data points. The symbols $y_i^{obs}$ denote the RHS of Eq.(5), which are determined by values of observed parameters and of parameters of the reference model. The symbols $A_{ij}$ denote elements of the $N \times NPAR$ matrix $\mathbf{A}$, which depends on the parameters of the reference medium and the considered source. The symbols $m_j$ denote elements of a vector $\mathbf{m}$ of the sought WA parameters, whose dimension $NPAR$ is the number of the sought parameters. The WA parameters were estimated using weighted least squares, from

$$\mathbf{m} = \left( \mathbf{A}^T \mathbf{w} \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{w} \mathbf{y}^{obs} \ . \tag{8}$$

The vector $\mathbf{y}^{obs}$ is an $N$-dimensional vector with elements $y_i^{obs}$.

The symbol $w_i$ in Eq. (7) is a diagonal element of the $N \times N$-dimensional diagonal matrix $\mathbf{w}$, see (8), and it denotes a weight of the $i$-th data point. The weights were introduced to reduce effects of outliers in the observed data. In order to estimate the effect of weighting, we performed all the inversions in two modes. In the first mode, we made no weighting, i.e., we used Eq. (7) with $w_i = 1$ for $i = 1, ..., N$. In the second mode, we proceeded in the following way. The data were approximated by a polynomial whose coefficients were determined by least squares. A polynomial of the fourth degree was found as most convenient approximation. The deviations of the observed data from the polynomial of the fourth degree were then used for the determination of the weights of individual data points in the following way:

$$w_i = \exp\Big( - \frac{(|(p_z^{obs})_i| - |(p_z^{pol})_i|)^2}{2\sigma^2}\Big) \; . \tag{9}$$

The symbol $(p_z^{obs})_i$ denotes the $i$-th vertical component of the observed slowness vector. The symbol $(p_z^{pol})_i$ denotes the value of the vertical component of the slowness vector calculated from the fourth-degree polynomial for the same polarization angle as $(p_z^{obs})_i$. The symbol $\sigma$ denotes an averaged deviation of $|(p_z^{obs})_i|$ and $|(p_z^{pol})_i|$.

The SVD algorithm was used to minimize the objective function (7) in the three experiments, ANI, VTI and ISO with $w_i = 1$ and with $w_i$ determined from Eq. (9). No criterion to improve conditionality of the inversion matrix $\mathbf{A}^T\mathbf{w}\mathbf{A}$ was used, i.e., all singular values were kept.

In the following, we show results of inversion, in which data for down- and upgoing waves are used together. The observed data for upgoing waves have greater scatter as can be seen in Figure 2 for the polarization angles less than $-90^o$ and greater than $90^o$. Because of this, the inversion of data for upgoing waves alone yields unreliable results. The inversion based on the use of data for downgoing waves alone yields results comparable with results of the inversion, in which data for both waves are considered together. Use of data for both waves leads to a greater stability of inverted parameters than with downgoing waves alone.

The estimated values of WA parameters inverted with no weights ($w_i = 1$) and their standard deviations for the three experiments ANI, VTI and ISO are given in Table 1. The table also gives the values of the corresponding elastic parameters or their combinations, and the range, determined from the estimated standard deviations, within which they can vary. The best estimated values in Table 1 and the following tables are the central values. The misfits, i.e., the values of the minimized objective functions $\Phi$, were found to be 3.08, 3.15 and 3.24 for the ANI, VTI and ISO experiments, respectively. We can see that the values of density-normalized elastic parameters $A_{11}$ and $A_{33}$ differ by more than 25% from the value 6.25 km$^2$/sec$^2$ of the square of the reference velocity $\alpha$. The formulae, on which the inversion scheme is based, work best if the "distance" between actual anisotropic medium and the isotropic reference medium is small, i.e. if the difference between the velocities of the corresponding $qP$ and $P$ wave is small. This is obviously not the case in Table 1. Tests with synthetic data without noise have shown that best estimates of the

**Table 1.** Estimated values of inverted WA parameters (non-dimensional) with their standard deviations, and values of the corresponding density-normalized elastic parameters or their combinations (in km$^2$/sec$^2$) with their range of variation for ANI, VTI and ISO experiments. $A_{1355} = A_{13} + 2A_{55}$. Reference velocity $\alpha = 2.5$ km/sec. No weighting.

| | WA par. | Est. val. | St. dev. | El. par. | Variation |
|---|---|---|---|---|---|
| *ANI* | $\epsilon_x$ | 4.00E-01 | 1.80E-01 | $A_{11}$ | [8.99 13.52] |
| | $\epsilon_z$ | 1.70E-01 | 1.78E-02 | $A_{33}$ | [8.13 8.57] |
| | $\delta_x$ | 5.40E-01 | 2.70E-01 | $A_{1355}$ | [7.96 11.23] |
| | $\epsilon_{15}$ | -2.50E-02 | 9.41E-03 | $A_{15}$ | [-0.21 -0.09] |
| | $\epsilon_{35}$ | 5.70E-02 | 1.32E-02 | $A_{35}$ | [0.27 0.44] |
| *VTI* | $\epsilon_x$ | 5.40E-01 | 1.82E-01 | $A_{11}$ | [10.74 15.29] |
| | $\epsilon_z$ | 1.80E-01 | 1.80E-02 | $A_{33}$ | [8.22 8.67] |
| | $\delta_x$ | 7.20E-01 | 2.64E-01 | $A_{1355}$ | [9.06 12.36] |
| *ISO* | $\epsilon_z$ | 1.20E-01 | 1.09E-02 | $A_{33}$ | [7.62 7.89] |

**Table 2.** The same as in Table 1. Reference velocity $\alpha = 2.83$ km/sec. No weighting.

| | WA par. | Est. val. | St. dev. | El. par. | Variation |
|---|---|---|---|---|---|
| *ANI* | $\epsilon_x$ | 3.20E-01 | 2.00E-01 | $A_{11}$ | [9.89 16.48] |
| | $\epsilon_z$ | 5.88E-02 | 2.01E-02 | $A_{33}$ | [8.63 9.27] |
| | $\delta_x$ | 3.40E-01 | 3.00E-01 | $A_{1355}$ | [8.39 13.15] |
| | $\epsilon_{15}$ | -2.82E-02 | 1.07E-02 | $A_{15}$ | [-0.31 -0.14] |
| | $\epsilon_{35}$ | 6.41E-02 | 1.50E-02 | $A_{35}$ | [0.39 0.63] |
| *VTI* | $\epsilon_x$ | 4.80E-01 | 2.10E-01 | $A_{11}$ | [12.43 19.05] |
| | $\epsilon_z$ | 6.71E-02 | 2.04E-02 | $A_{33}$ | [8.76 9.41] |
| | $\delta_x$ | 5.50E-01 | 3.00E-01 | $A_{1355}$ | [9.99 14.79] |
| *ISO* | $\epsilon_z$ | 4.52E-03 | 1.24E-02 | $A_{33}$ | [7.88 8.28] |

sought parameters and minimum misfit calculated from Eq.(7) are obtained for the reference medium whose "distance" from the sought medium is minimum. Similar tests with noisy data, i.e., with data like our data set, have shown, however, that best estimates of the sought parameters are found for a reference medium whose "distance" from the sought medium is minimum but the corresponding misfit might not be minimum. Therefore, we decided to substitute the reference velocity of 2.5 km/sec by a value whose square is closer to estimated values of the parameter $A_{33}$ in the experiment ANI. Since estimated value of $A_{33}$ is slightly greater than 8 km$^2$/sec$^2$, we took its square root, 2.83, as the reference velocity, and used it in all following inversions.

The results in Table 2 were thus obtained with the reference velocity of 2.83 km/sec and with no weighting. From the sizes of the WA parameter $\epsilon_z$ we can see that the reference velocity is now close to the vertical velocity of the studied medium. The misfits were found to be 3.5, 3.6 and 3.7 for the ANI, VTI and ISO experiments, respectively. They are thus slightly greater than the misfits for the reference velocity 2.5 km/sec. In agreement with results of synthetic tests described in the previous paragraph, we expect that the inversion with the reference velocity of 2.83 km/sec yields better estimates than with the reference velocity of 2.5 km/sec. Table 2 shows standard deviations of individual estimated parameters. For the determination of the standard deviations it is necessary to know standard deviations of the observed data. Since they are not known, we estimated them from the RHS of Eq.(7) divided by the number of data points $N$, assuming that the distribution of errors in the data is Gaussian. We can see that the parameters with the smallest values of standard deviation are parameters $\epsilon_z$ (in both ANI and VTI experiments), $\epsilon_{15}$ and $\epsilon_{35}$. This is in agreement with results of synthetic tests, see, e.g., *Zheng and Pšenčík (2002)*. The mentioned parameters were also found to be most stable ones, i.e., to have least sensitivity to perturbations of observed data. The standard deviations of the remaining parameters are about an order of magnitude greater. Anisotropy calculated from estimated values of elastic parameters as $((A_{11} - A_{33}/(A_{11} + A_{33}) \times 100\%)$ is about 19% for ANI and 27% for VTI experiment. These values must be, however, taken with caution because possible variations of the parameter $A_{11}$ are rather large, see Table 2. We can also see in Table 2 that the parameters $\epsilon_{15}$ and $\epsilon_{35}$ in the experiment ANI are nonzero which seems to indicate that the inverted medium is not of VTI symmetry. Vertical components of the slowness vector calculated from the estimated values of the WA parameters for the ANI experiment in Table 2 as functions of the polarization angle are shown by thin line in Figure 2.

**Table 3.** The same as in Table 1. Reference velocity $\alpha = 2.83$ km/sec. Weighting (9).

| | WA par. | Est. val. | St. dev. | El. par. | Variation |
|---|---|---|---|---|---|
| *ANI* | $\epsilon_x$ | 3.70E-01 | 8.16E-02 | $A_{11}$ | [12.56 15.17] |
| | $\epsilon_z$ | 2.92E-02 | 7.86E-03 | $A_{33}$ | [8.35 8.60] |
| | $\delta_x$ | 4.00E-01 | 1.20E-01 | $A_{1355}$ | [10.27 12.17] |
| | $\epsilon_{15}$ | -2.84E-03 | 4.33E-03 | $A_{15}$ | [-0.06 0.01] |
| | $\epsilon_{35}$ | 2.79E-02 | 5.92E-03 | $A_{35}$ | [0.17 0.27] |
| *VTI* | $\epsilon_x$ | 4.20E-01 | 8.26E-02 | $A_{11}$ | [13.40 16.04] |
| | $\epsilon_z$ | 3.12E-02 | 8.00E-03 | $A_{33}$ | [8.39 8.65] |
| | $\delta_x$ | 4.70E-01 | 1.20E-01 | $A_{1355}$ | [10.81 12.73] |
| *ISO* | $\epsilon_z$ | -2.01E-02 | 5.25E-03 | $A_{33}$ | [7.60 7.77] |

In Table 3, we show the results of inversion with weighting based on Eq. (9). This weighting reduces effects of outliers. It leads to reduced values of misfit: 1.14, 1.17 and 1.31 for the ANI, VTI and ISO experiment, respectively. The misfit is thus reduced more than three times in comparison with unweighted case. We must keep in mind, however, that by using weights we made an assumption that the vertical component of the slowness vector varies smoothly with varying polarization angle, see the bold line in Figure 2. Weighting leads to a significant reduction of the standard deviations. It also leads to lower estimates of the parameters $\epsilon_z$ and $\epsilon_{35}$. Consequently, anisotropy in the ANI experiment increases to about 24% while in the VTI experiment remains nearly the same, 26%. Again, these values must be taken with caution. We can see that the parameter $\epsilon_{15}$ in the experiment ANI is now effectively zero but $\epsilon_{35}$ remains nonzero indicating, as in the case of unweighted data, that the inverted medium does probably not possess the VTI symmetry. Vertical components of the slowness vector calculated from the estimated values of the WA parameters for the ANI experiment in Table 3 as functions of the polarization angle are shown by bold line in Figure 2. We do not show the curve for the VTI experiment since it practically coincides with the ANI curve in this display.

The results of Table 3 are illustrated in Figure 3. Assuming that the distribution of noise is Gaussian, we studied the confidence regions (see, e.g., *Menke, 1984*) of the estimated WA parameters from Table 3. Comparisons of selected projections of the confidence regions with 99% probability for the ANI and VTI experiments are shown in the upper and the middle rows of Figure 3. Narrow ellipses indicate a correlation between the WA parameter $\epsilon_z$ and the parameters $\epsilon_x$ and $\delta_x$. Note substantially smaller variation of the parameter $\epsilon_z$ than of the other two parameters, which indicates high stability of the parameter $\epsilon_z$. We can very clearly see that the confidence regions corresponding to the ANI experiment are smaller than in the VTI experiment. The $F$-test (*Beck and Arnold, 1976*) applied to the results of these two experiments also indicates that, with 95% confidence, only the ANI model satisfies the observed data. Projections of the confidence region of 99% probability of the WA parameters $\epsilon_{15}$ and $\epsilon_{35}$ in relation with $\epsilon_z$ estimated in the experiment ANI are shown in the bottom row of Figure 3. The ellipses in the bottom row indicate little correlation of the all involved WA parameters. Small variations of the involved parameters indicate their high stability. The results of the correlation analysis are shown quantitatively in Tables 4 and 5 for the ANI and VTI experiments, respectively. The parameters $\epsilon_{15}$ and $\epsilon_{35}$ show negligible mutual correlation and correlation with remaining WA parameters. We can see, however, very strong correlation between the WA parameters $\epsilon_x$ and $\delta_x$ and the above mentioned correlations of these two parameters with $\epsilon_z$. Note that an indication of the relation of these parameters can be already seen in formulae (A1) and (A3) in the Appendix. The substitution of the WA parameters $\epsilon_x$, $\epsilon_z$ and $\delta_x$ by $\epsilon_x$, $\epsilon_z$ and $\delta_x - \epsilon_x - \epsilon_z$ leads to reduction of standard deviations and also to the reduction of the size of the parameter $\epsilon_x$. This in turn leads to a substantial reduction of estimated anisotropy $((A_{11} - A_{33}/(A_{11} + A_{33}) \times 100\%)$. It is now approximately 6% for the ANI and 9% for the VTI experiment without weighting. The above-mentioned improvement of

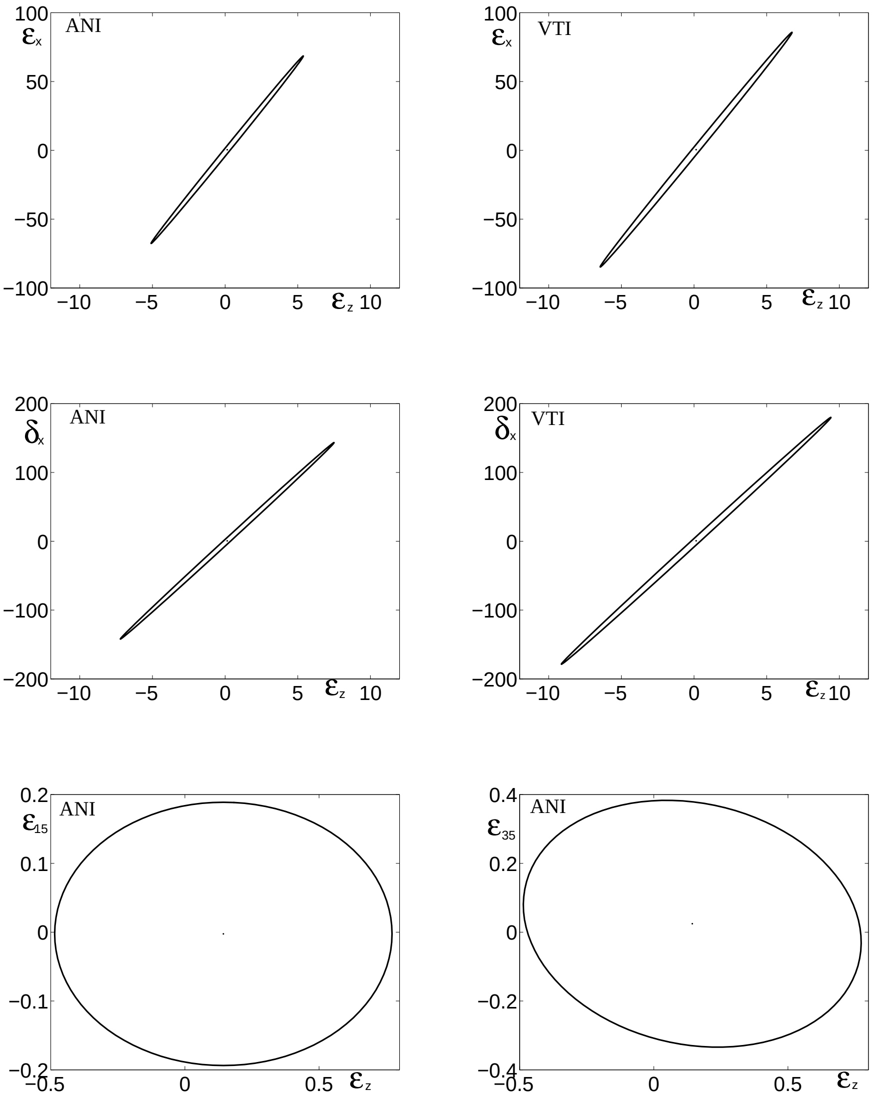


**Fig. 3.** Projections of 99% confidence region into the $(\epsilon_z, \epsilon_x)$ plane (top) and $(\epsilon_z, \delta_x)$ plane (middle), for the ANI (left) and VTI (right) experiment. Note different sizes of the confidence regions for ANI and VTI. Projections of 99% confidence region into the $(\epsilon_z, \epsilon_{15})$ and $(\epsilon_z, \epsilon_{35})$ planes (bottom) for the ANI experiment. Results obtained with weighting factors (9). Reference velocity $\alpha = 2.83$ km/sec.

**Table 4.** Correlation coefficients of the inverted WA parameters for the results of the experiment ANI shown in Table 3.

| | $\epsilon_x$ | $\epsilon_z$ | $\delta_x$ | $\epsilon_{15}$ | $\epsilon_{35}$ |
|---|---|---|---|---|---|
| $\epsilon_x$ | 1.000 | 0.796 | 0.993 | 0.005 | -0.137 |
| $\epsilon_z$ | 0.796 | 1.00 | 0.774 | 0. | 0.074 |
| $\delta_x$ | 0.993 | 0.774 | 1.00 | -0.017 | -0.120 |
| $\epsilon_{15}$ | 0.005 | 0. | -0.017 | 1.00 | -0.208 |
| $\epsilon_{35}$ | -0.137 | 0.074 | -0.120 | -0.208 | 1.00 |

**Table 5.** Correlation coefficients of the inverted WA parameters for the results of the experiment VTI shown in Table 3.

| | $\epsilon_x$ | $\epsilon_z$ | $\delta_x$ |
|---|---|---|---|
| $\epsilon_x$ | 1.000 | 0.796 | 0.994 |
| $\epsilon_z$ | 0.796 | 1.00 | 0.773 |
| $\delta_x$ | 0.994 | 0.773 | 1.00 |

the inverted results was, however, not observed in a series of synthetic tests, and thus it does not seem to be a general rule.

Figure 4 shows a comparison of observed and calculated data in a different display than Figure 2, in which the curves corresponding to individual experiments are separated. The horizontal axis is specified again by the polarization angle in degrees. The vertical axis shows, however, the ratio $\alpha p_z/g_z$, i.e., approximately the ratio of the vertical components of the wave vector and of the polarization vector. The symbols $\times$ denote again the observed data. The stars correspond to the experiment ANI, the triangles to the experiment VTI and open circles to the experiment ISO. The ratio $\alpha p_z/g_z$ was calculated from Eq. (5), in which the corresponding estimated values of the WA parameters found for reference velocity 2.83 km/sec and with weighting (see Table 3) were inserted. Most of the data corresponding to the direct wave have polarization angles in the interval $(-90^0, 90^0)$. The data corresponding to the reflected wave are mostly outside this interval. We can see again greater scatter of the observed data corresponding to the upgoing wave (some of the observed data are out of the chosen frame) indicating that the picking errors of the data related to upgoing waves are higher, see *Horne and Leaney (2000).* We can see clearly that results of the inversion under the assumption of isotropy do not fit the observed data at all. The results under the assumption of VTI symmetry approximate the data rather well for the polarization angles in the interval, approximately, $(-70^0, 70^0)$. The VTI results have sinusoidal character with always finite values of the ratio of the vertical components of the normalized slowness vector and of the polarization vector. This is a consequence of the fact that in a VTI medium the ratio $\alpha p_z/g_z$ varies symmetrically with respect to the axis of symmetry (polarization angle $0^0$). For $\pm 90^0$, both vertical components are zero. This symmetry is violated not only for lower symmetry anisotropy but also for TI anisotropy with an inclined axis of

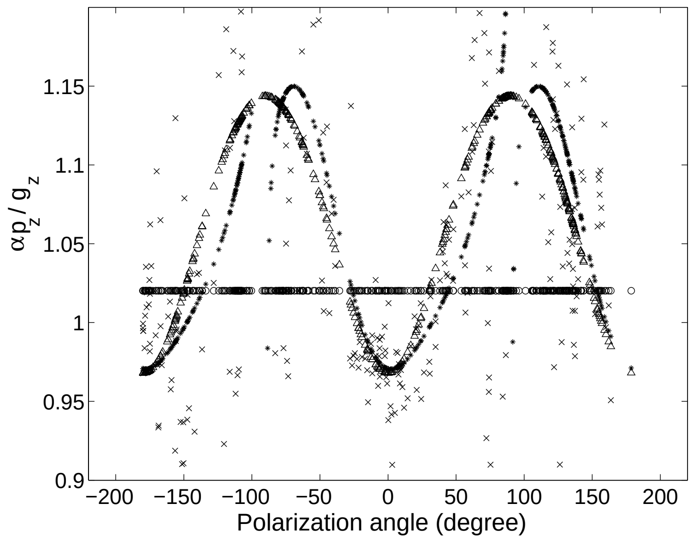


**Fig. 4.** Comparison of observed and calculated values of the ratios $\alpha p_z/g_z$ wrt polarization angles. The observed data shown by $\times$, the results of ANI, VTI and ISO experiments obtained with weighting factors (9) denoted by stars, triangles and open circles, respectively. Reference velocity $\alpha = 2.83$ km/sec.

symmetry. The ANI results tend to infinity since $p_z$ is nonzero for $g_z = 0$. We can see again that the observed data are best approximated by the results of the experiment ANI. From the obtained results, it is, however, not possible to deduce if the anisotropy of the medium is a TI with an inclined axis of symmetry or a lower symmetry anisotropy.

*Horne and Leaney's (2000)* polarization inversion under the assumption of VTI symmetry of the medium yields for $A_{11}$, $A_{33}$ and $A_{13}+2A_{55}$ values 11.35, 8.56, 8.02 km$^2$/sec$^2$, respectively. Compared with the values of the corresponding parameters in Tables 2 and 3, the above values seem to be slightly underestimated. The most stable parameter $A_{33}$ compares best. Differences in $A_{11}$ and $A_{13}+2A_{55}$ are larger. This also leads to differences in estimated anisotropy. For the values of *Horne and Leaney (2000)*, the anisotropy is about 14%. The anisotropy of media in Tables 2 and 3 is considerably larger. When making the above comparisons, we must keep in mind that there are several factors which are responsible for the differences. One is the above-mentioned greater uncertainty of the parameters $A_{11}$ and $A_{13}+2A_{55}$. It is also important to remember that in contrast to *Horne and Leaney (2000)*, we did not use

$qS$-wave data in the inversion (this affects mostly the term $A_{13} + 2A_{55}$). *Horne and Leaney (2000)* seek the vertical component of the slowness vector corresponding to the observed polarization vector, and compare it with observed vertical component of the slowness vector. They do not constrain the size of the slowness vector as we do. The treatment of outliers in the observed data differs in both approaches. In contrast to *Horne and Leaney (2000)*, we use approximate formulae for the inversion. Tests with synthetic data indicate, however, that this has negligible effects if the reference medium is properly chosen.

## 5. CONCLUSIONS

We have shown that the approach for the local determination of WA parameters from multi-azimuthal multiple-source offset VSP experiments proposed by *Zheng and Pšenčík (2002)* can also be successfully applied to data collected along a single profile.

The used inversion scheme is independent of structural complexities in the overburden, the borehole need not be vertical. Since the inversion formula is local, the variation of observed quantities at a receiver is considered to be caused solely by anisotropy. Thus the inversion scheme has potential to separate anisotropy from inhomogeneity. When only vertical components of the observed slowness vector are available, as in the studied case, information about polarization is necessary. The approach is very effective; convergence is typically achieved in one interation.

Analysis of the inverted parameters revealed considerably greater stability of the WA parameters $\epsilon_z$, $\epsilon_{15}$ and $\epsilon_{35}$ than of the parameters $\epsilon_x$ and $\delta_x$. It also showed strong correlation between the latter two WA parameters, and non-negligible correlations of these two parameters with $\epsilon_z$.

Figure 4 clearly indicates that the studied medium is not isotropic. Tables 2 and 3 seem to indicate that the medium is not even VTI. This is confirmed by the results of the F-test. At this stage, however, it is difficult to estimate which type of the anisotropic symmetry the medium possess. Use of $qS$-wave data could shed light on this problem. It would also allow more accurate comparison with results of *Leaney (1994)* and *Horne and Leaney (2000)*.

*Acknowledgements*: We are grateful to P.T.Maxus Southeast Sumatra for the release of the used data. A preliminary version of this work appeared in *Zheng et al. (2001)*. The authors wish to thank Peter Bakker, Jesse Costa, Leo Eisner, Petr Jílek, Leonardo Martins, Bob Nowack, Sláva Růžek, João B.C.da Silva and an anonymous reviewer for their important and useful comments. The authors are also grateful to the consortium project "Seismic waves in complex 3-D structures (SW3D)" for support. The work was also supported by the Grant Agency of the Academy of Sciences of the Czech Republic, project No. A3012309. EG and XZ are grateful to the consortium CTPETRO-FINEP/PETROBRAS/FADESP/UFPA and Chinese Joint Foundation of Seismological Sciences under contract No.103021, respectively, for support.

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## APPENDIX A

We present formulae for the elements $B_{13}$ and $B_{33}$ of the weak anisotropy matrix $B_{mn}$ appearing in Eq.(5) for a walkaway experiment in an anisotropic medium of arbitrary symmetry. The wave normal $n_i$ in the reference isotropic medium is situated in the vertical plane containing the walkaway profile, i.e., $n_2 = 0$. The following formulae represent a specification of the formulae of *Farra and Pšenčík (2003)* for general anisotropy:

$$B_{13} = \alpha^2 |n_1|^{-1} [\epsilon_{35} n_1 n_3^4 + (\delta_x - \epsilon_x - \epsilon_z) n_1^2 n_3^3 + (4\epsilon_{15} - 3\epsilon_{35}) n_1^3 n_3^2 + (\epsilon_x + \epsilon_z - \delta_x) n_1^4 n_3 + (\epsilon_x - \epsilon_z) n_1^2 n_3 - \epsilon_{15} n_1^3]$$

and

$$B_{33} = 2\alpha^2 [2\epsilon_{35} n_1 n_3^3 + \Big( (\delta_x - \epsilon_x - \epsilon_z) n_1^2 + \epsilon_z \Big) n_3^2 + 2\epsilon_{15} n_1^3 n_3 + \epsilon_x n_1^2]. \qquad (A1)$$

The elements of the weak anisotropy matrix in Eqs.(A1) depend on 5 WA parameters:

$$\epsilon_x = \frac{A_{11} - \alpha^2}{2\alpha^2}, \quad \epsilon_z = \frac{A_{33} - \alpha^2}{2\alpha^2}, \quad \delta_x = \frac{A_{13} + 2A_{55} - \alpha^2}{\alpha^2},$$
$$\epsilon_{15} = \frac{A_{15}}{\alpha^2}, \quad \epsilon_{35} = \frac{A_{35}}{\alpha^2}. \qquad (A2)$$

For the VTI symmetry, for which $\epsilon_{15} = \epsilon_{35} = 0$, Eqs.(A1) reduce to

$$B_{13} = \alpha^2 |n_1|^{-1} [(\delta_x - \epsilon_x - \epsilon_z) n_1^2 n_3^3 + (\epsilon_x + \epsilon_z - \delta_x) n_1^4 n_3 + (\epsilon_x - \epsilon_z) n_1^2 n_3]$$

and

$$B_{33} = 2\alpha^2 [\Big( (\delta_x - \epsilon_x - \epsilon_z) n_1^2 + \epsilon_z \Big) n_3^2 + \epsilon_x n_1^2]. \qquad (A3)$$

Thus assumption of the VTI symmetry reduces the number of possibly inverted parameters to three.

For the isotropic medium, for which $\epsilon_x = \epsilon_z = \frac{1}{2}\delta_x$, Eqs.(A3) reduce to

$$B_{13} = 0, \qquad B_{33} = 2\alpha^2 \epsilon_z. \qquad (A4)$$