SP39

Seismogram Improvement Using the Born Aproximation

L. Sachl* (Charles University)

SUMMARY

FORTRAN 77 program for computation of the Born aproximation of the first order in inhomogenous isotropic medium without attenuation and dissipation has been coded. Several numerical examples were computed. Here is presented just one of them. The unperturbed model is a homogenous model without interface. The perturbed model has one horizontal interface, it is a model with two homogenous layers. There is only a direct wave in the unperturbed medium, but there is a direct as well as a reflected wave in the perturbed medium. The aim is to regain this lacking information lost by the use of the unperturbed medium. The reference solution is the seismogram computed in the perturbed model using the ray theory.

Introduction

When computing seismograms in a complex seismic structure, we can meet a situation for which the method we are using is not suitable or even applicable. A good example is the ray theory, which has certain advantages when compared with other methods, but it is not applicable, if the medium is not "smooth enough". A possible solution is to use a model, which is "close" to the original one and satisfies the theory requirements. Let us call this model unperturbed and the original one perturbed. The results obtained in the unperturbed model contain, of course, some deviations when compared with the accurate solution (e.g. the ray theory vs. finite difference results). The method how to improve the solution is using the Born aproximation, which requires quantities computed in the unperturbed medium and differences between the perturbed and unperturbed model called perturbations. The result is a seismogram with corrected travel times and amplitudes.

FORTRAN 77 program for computation of the Born aproximation of the first order in inhomogenous isotropic medium without attenuation and dissipation has been coded. It is designed to cooperate with SW3D software packages (see *http://sw3d.cz*). Several numerical examples were computed. Here is presented just one of them.

Theory

We introduce a perturbation parameter α. The unperturbed medium is given by $\alpha = 0$, the perturbed medium is given by $\alpha = 1$. The wavefield in now denoted by $u_i(\mathbf{x}, \omega, \alpha)$ in the frequency domain, where $\mathbf{x}$ denotes radius vector and ω angular frequency. The wavefield in the perturbed medium can be related to the wavefield in the unperturbed medium using Taylor (perturbation) expansion

$$u_i(\mathbf{x}, \omega, 1) = u_i(\mathbf{x}, \omega, 0) + u_{i,\alpha}(\mathbf{x}, \omega, 0) + \ldots,$$

where $u_{i,\alpha}$ means derivative of the wavefield with respect to the perturbation parameter α. Let us have a source at point $\mathbf{x}^s$ and a receiver at point $\mathbf{x}$. If we decompose the wavefield and the Green function into the amplitudes a_i, A_{ij} and the phase terms $\exp(i\omega\tau)$, $\exp(i\omega T)$

$$u_i = a_i \exp(i\omega\tau),$$

$$G_{ij} = A_{ij} \exp(i\omega T),$$

where τ and T are travel times of the wavefield and the Green function respectively, Born aproximation in the isotropic medium with applied high frequency aproximation of the spatial derivatives can be written in the form

$$u_{i,\alpha}(\mathbf{x}, \omega, 0) = \omega^2 \int_{\Omega} \exp[i\omega(\tau + T)][\rho_{,\alpha} A_{ji} a_j + \lambda_{,\alpha} A_{ji} P_j a_k p_k + \mu_{,\alpha} A_{mi} P_j (a_m p_j + a_j p_m)] d^3\mathbf{x}',$$

where p_i or P_i denotes derivative of the travel times τ or T with respect to spatial coordinates, $\rho_{,\alpha}$ and $\lambda_{,\alpha}$ and $\mu_{,\alpha}$ are perturbation derivatives of density ρ and elastic moduli λ, μ. In the previous formula the wavefield from the source is incident at the integration point $\mathbf{x}'$, the Green function from the receiver is incident at the same integration point. Derivatives $\rho_{,\alpha}$ and $\mu_{,\alpha}$, $\lambda_{,\alpha}$ are equal to perturbations of density ρ and elastic moduli λ, μ, because we have chosen linear dependence of model parameters on α and because of our convention $\alpha = 0 \Leftrightarrow$ unperturbed medium, $\alpha = 1 \Leftrightarrow$ perturbed medium. Ω is the volume where these perturbations are nonzero.

Numerical example

We take into account one source and one receiver. Only P waves are considered. The unperturbed model is a homogenous model without interface. The perturbed model has one horizontal interface. Density and elastic moduli in the upper part of the model are the same as in the unperturbed model, in the lower

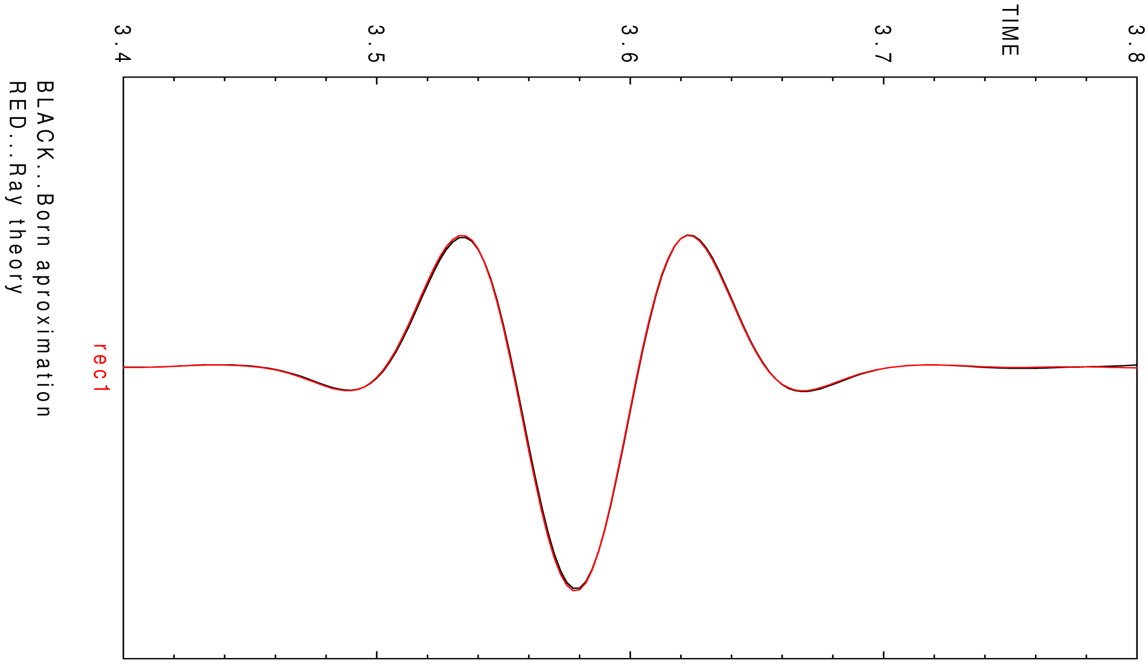

Figure 1 *Comparison of the seismogram computed using the Born approximation and using the ray theory in the perturbed model*

part they are slightly higher. Perturbations are small enough to use the first order Born aproximation only, higher orders are not necessary.

There is only a direct wave in the unperturbed medium, but there is a direct as well as a reflected wave in the perturbed medium. The aim is to regain this lacking information lost by the use of the unperturbed medium.

The ray tracing is performed from the source and from the receiver. The results of the ray tracing are used to interpolate travel times and other quantities to the regular rectangular grid of points. Both the wavefield incident from the point source and the Green function from the receiver are aproximated by the ray theory. Finally seismograms are computed using the Born approximation.

For the result see fig. 1. Grid was composed of 100x100x400 points and was situated in the lower part of the model. The reference solution is the seismogram computed in the perturbed model using the ray theory.

Conclusions

Program for computation of the Born aproximation has been coded and used to improve seismograms computed in the unperturbed model. Seismogram computed by the Born aproximation shows a good agreement with seismogram computed directly in the perturbed medium. On the other hand there are some problems. Possible solutions will be shown in the poster. It is planned to test the program in a more complex model.

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