Smoothing the 1-D velocity model of Dobrá Voda for ray tracing

Petr Bulant
Department of Geophysics, Faculty of Mathematics and Physics, Charles University, Ke Karlovu 3, 121 16 Praha 2, Czech Republic, http://sw3d.cz/staff/bulant.htm

Summary
The algorithm of constructing smooth velocity models suitable for ray tracing by inversion of given data with minimization of the Sobolev norm composed of the second derivatives of velocity is applied to the 1-D P-wave velocity model of the Dobrá Voda locality. The data available for the Dobrá Voda locality are strongly heterogeneous, and the application of the smoothing algorithm to the 1-D model requires proper selection of the amount of smoothing and appropriate parameterization of the model. Numerical tests performed for individual vertical profiles indicate that the data do not allow to apply the smoothing algorithm with uniform smoothing to the 3-D velocity model of the locality.

Keywords
Ray tracing, velocity model, smoothing, Sobolev norm, model parametrization.

1. Introduction

The construction of the velocity model of a geological structure is the first step in any calculation based on the application of ray methods. If the discrete values of velocity are given, we need to fit the data by a velocity model. In order to successfully perform ray tracing, proper smoothing of the velocity model is a key issue. The recommended method is the construction of a velocity model by data fitting with minimization of the Sobolev norm of the model composed of second velocity derivatives. Several papers on this topic were published. The first paper on this topic by Klimeš (2000a) is devoted to the detailed theoretical description of the smoothing algorithm. The papers by Bulant (2002) and Žáček (2002) contain a shortened overview of the theory and show numerical application to 2-D and 3-D models; a preliminary versions of the papers were published by Bulant (2000), Žáček (2000), and Bulant (2001). The paper by Klimeš (2000b) contains the description of application of the smoothing algorithm to the Marmousi model, and contains detailed description of the history file used to smooth the model using the SW3D software.

Recently, the ray method was often applied to calculate travel times for hypocenter locations, or to calculate Green functions for moment tensor inversion in (micro)seismic monitoring of induced or natural earthquakes. In these applications, the knowledge about the velocity structure is usually very limited, and the data are usually sufficient for creation of 1-D models only. Thus the most frequent task in these applications is to create a smooth 1-D model, which fits the data as closely as possible, but is smooth enough for ray tracing.

In this paper, we show an example of such calculation, using the data which are available for the P-wave velocity in locality Dobrá Voda.

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2. Input information for velocity model smoothing

There are three kinds of input information for the process of smoothing the velocity model. These are the available velocity data, the manually selected parametrization of the model, and the manually selected parameters controlling smoothing of the model.

2.1. Data for the P-wave velocity

The P-wave velocity data for the locality are available in the form of a very sparsely sampled 3-D model consisting of $7 \times 8 \times 8$ discrete values of P-wave velocity. The data grid is rectangular but irregular, namely in the vertical direction where 6 grid points are available for depths from 0 km to 4.8 km, with two remaining grid points in depths 25 km and 50 km. The values at depths 0 to 4.8 km display lateral variation of the velocity, whereas the velocity at the remaining two depth levels is laterally invariant, see Figure 1. At depths 0 to 4.8 km the data display strong velocity gradient, whereas the values at depths 25 km and 50 km indicate much lower velocity gradient. Vertically the data thus consist of two very different parts, the upper densely sampled part with a strong velocity gradient, and the lower sparsely sampled part with a weak gradient.

2.2. Model parametrization, amount of smoothing

The slowness in the velocity model is interpolated by B-splines. As an input for velocity smoothing, the parametrization of the model, i.e. the number and the depths of the spline points must be manually specified. The values of velocity in the prescribed spline points are calculated during smoothing, based on the velocities at the data points and on the applied amount of smoothing. The amount of smoothing is controlled by numerical parameter which we shall call SOBMUL. This parameter is also prescribed manually.

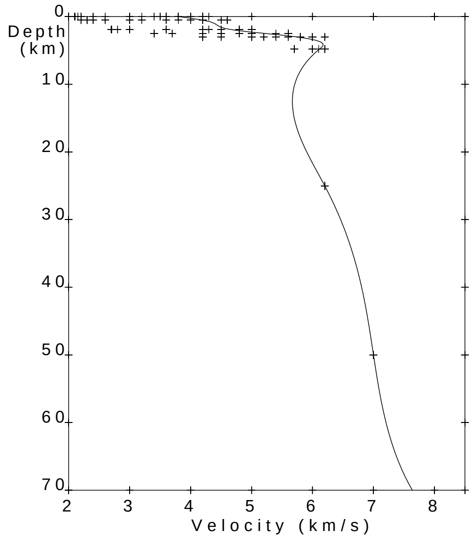
3. Model with 8 spline points at the depths of data points

In 1-D, the exact solution of simultaneous least-square fitting of discrete data and minimizing the square of the Sobolev norm composed of second derivatives yields natural cubic splines with the spline grid equal to the data grid. For the first test, we thus choose the 1-D model parametrized by 8 spline points located at the same depths as the depths of the data points, i.e. in the depths of 0.0, 0.5, 1.9, 2.5, 3.0, 4.8, 25.0 and 50.0 km.

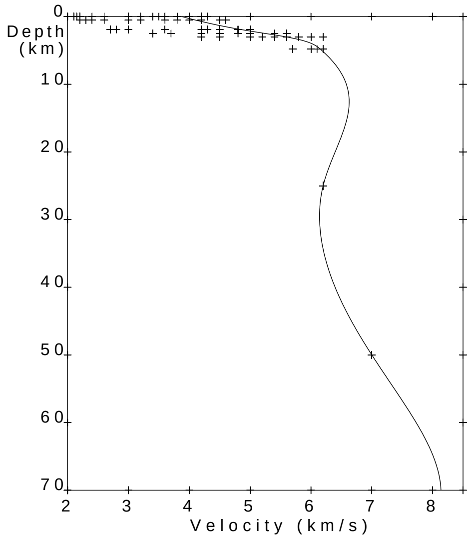
We then perform the inversion with different amounts of smoothing, starting from the minimum smoothing and gradually increasing the smoothing by increasing the parameter called here SOBMUL. The resulting models are shown in Figure 1.

Figure 1 (right page): The velocity models obtained for the model parametrized by 8 spline points in the depths corresponding to data points, and with the slowness interpolated by splines. The vertical axis is the depth, horizontal axis is the P-wave velocity. The crosses show the values of the velocity in the data points. The solid line shows the velocity in the 1-D models resulting from the inversion. The amount of smoothing applied ranges from SOBMUL=1 km s to SOBMUL=10 000 000 km s. In the depths 0 to 5 km, the model approximately fits the data for lower values of SOBMUL, and diverges considerably from the data for the values of SOBMUL larger than 100 000 km s. In the depths around 12 km, we can observe the development of low velocity channel for SOBMUL=1 km s, and the development of high velocity channel for SOBMUL from 1 000 km s to 100 000 km s. In the depths around 30 km, we can observe the development of low velocity channel for SOBMUL from 1 000 km s to 100 000 km s. The high and low velocity channels disappear for values of SOBMUL higher than 1 000 000 km s. From these observations we can conclude that the optimum value of smoothing needs to be searched for between SOBMUL=1 km s and SOBMUL=1 000 km s.

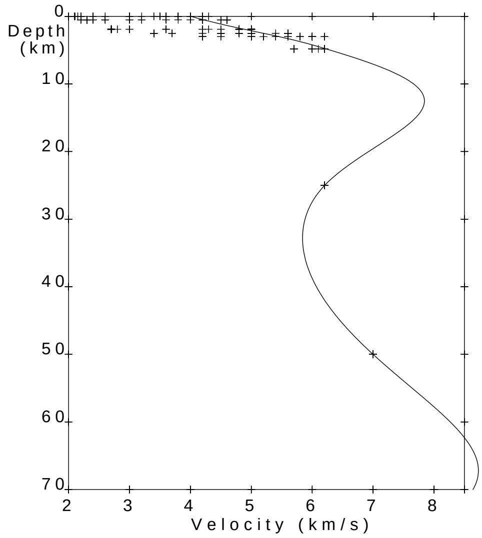
SOBMUL=1



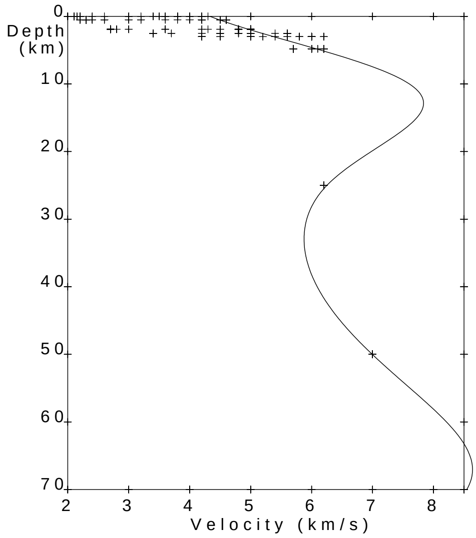
SOBMUL=1 000



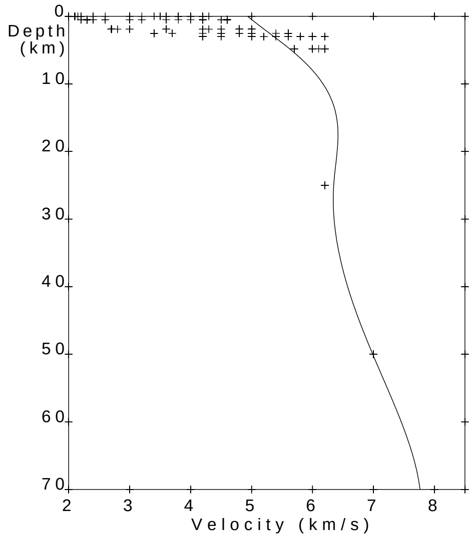
SOBMUL=10 000



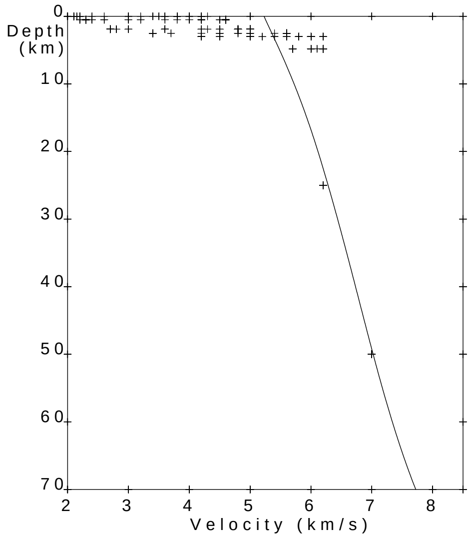
SOBMUL=100 000



SOBMUL=1 000 000



SOBMUL=10 000 000



We would like to obtain the velocity model which reasonably fits the data in the upper part of the model in depths 0 to 5 km, and whose lower part follows the low velocity gradient suggested by the two data points available at depths 25 km and 50 km. From Figure 1, we can see that, with increasing SOBMUL, the model rapidly changes from the model with low velocity channel in the depths around 12 km to the models with high velocity channel in these depths, and that the smooth models obtained with high SOBMUL do not fit the data sufficiently. We can thus try to find the optimum value of smoothing between SOBMUL=1 km s and SOBMUL=1 000 km s.

Note that this behaviour differs from our previous experiences with smoothing, for example from the experiences with models Marmousi, Hess or SEG/EAGE Salt Model. In those examples the models were too rough for lower values of SOBMUL, reasonably smooth and reasonably fitting the data for optimal SOBMUL, and even smoother and slowly diverging from the data for higher values of SOBMUL. In the described model of Dobrá Voda, the range of optimum values of SOBMUL is quite narrow, see Figure 2.

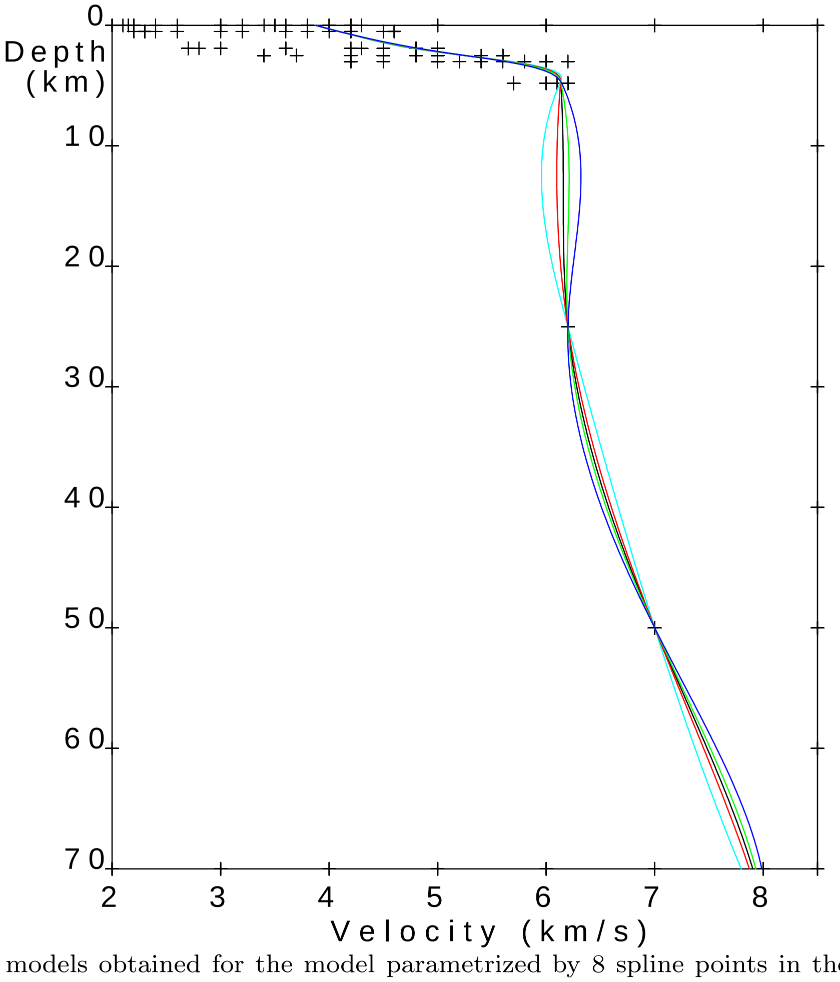


Figure 2: The models obtained for the model parametrized by 8 spline points in the depths corresponding to data points for the values of SOBMUL=200 (cyan), SOBMUL=300 (red), SOBMUL=350 (black), SOBMUL=400 (green) and SOBMUL=600 (blue). In the upper part of the model in the depths 0 to 5 km the velocity remains almost unchanged. As the optimum value of SOBMUL we consider the value of SOBMUL=350 km s, because the available data do not indicate neither low nor high velocity channel to be present in the depths between 5 to 25 km. In the lower part of the model in the depths around 12 km, we can see the quick change from a low velocity channel for SOBMUL less than 350 km s to the high velocity channel for SOBMUL higher than 350 km s.

The second velocity derivatives are minimized in order to limit a ray chaos. The chaotic behaviour of rays in the velocity model is characterized by the Lyapunov exponents, which describe exponential spreading of ray tubes, and by the rotation numbers, which describe the frequency of caustics along rays (Klimeš, 2002). In a vertically heterogeneous 1-D velocity model, the Lyapunov exponents and rotation numbers become largest for horizontal propagation. We thus calculated the depth-dependent Lyapunov exponents and rotation numbers for horizontal propagation in the velocity models shown in Figure 2. The Lyapunov exponents and rotation numbers are defined with respect to the horizontal distance. The Lyapunov exponents and rotation numbers in the selected model obtained for SOBMUL=350 km s are displayed in Figure 3. The Lyapunov exponents and rotational numbers in other four velocity models are similar. The Lyapunov exponents and rotation numbers in the upper part of the velocity model are very large, but their influence on rays considerably depends on the actual values of the velocity gradient in the respective depth, which determines the curvature of rays and consequently the lengths of ray segments situated in the corresponding depth. We thus cannot estimate the behaviour of rays from the Lyapunov exponents and rotation numbers in this case, and have to calculate rays.

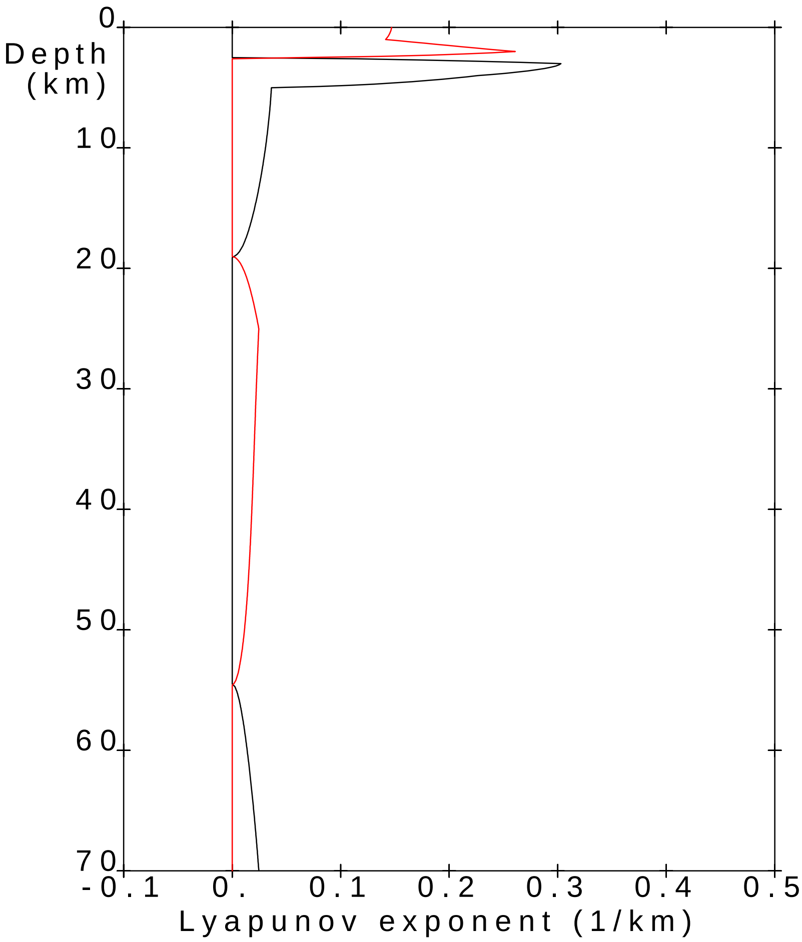


Figure 3: Horizontal Lyapunov exponents (black line) and rotational numbers (red line) for model obtained with SOBMUL=350 km s.

Figure 4 shows the examples of the initial-value ray tracing in the models displayed in Figure 2. The examples suggest that the selected model obtained for SOBMUL=350 km s is suitable for two-point ray tracing, see the discussion in the caption of Figure 4.

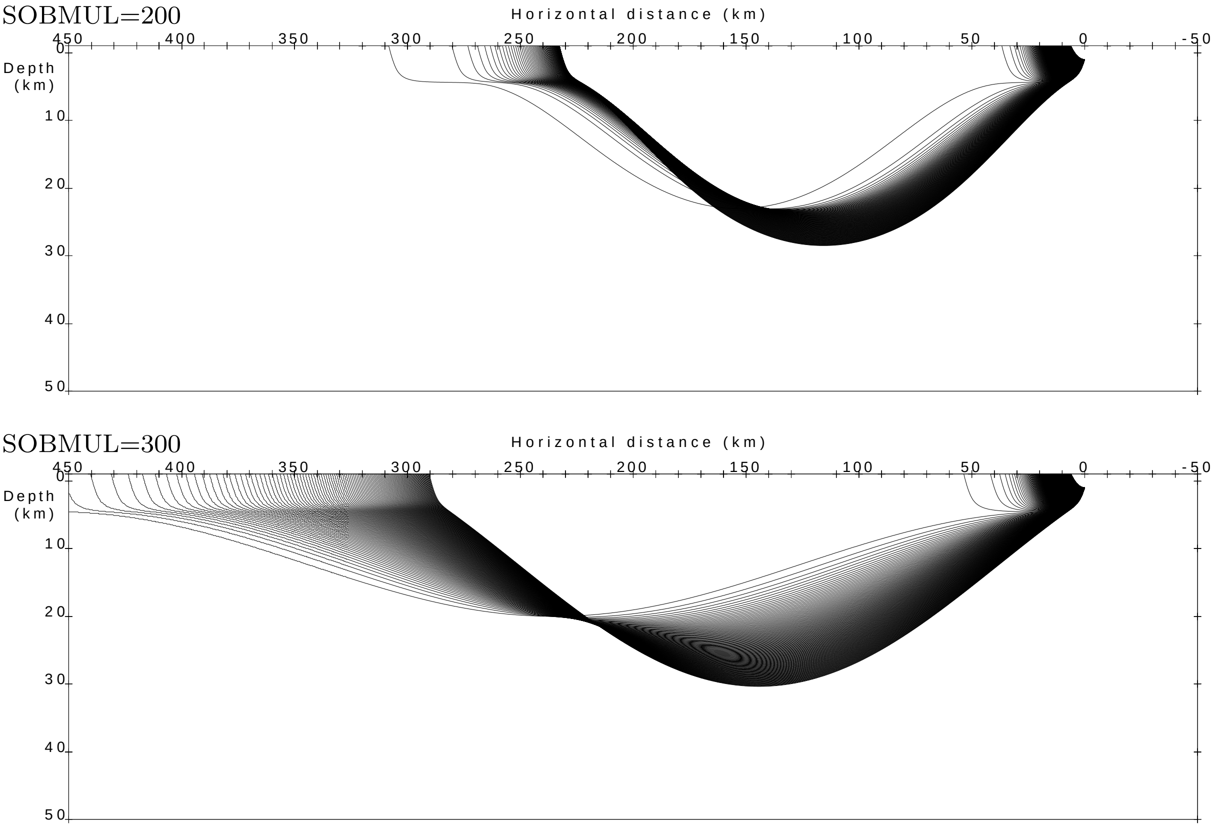


Figure 4: Rays calculated by initial-value ray tracing in the models from Figure 2. The first ray is shot from the source in a horizontal direction, and then the next rays are traced with a constant step of 0.0002 rad in the vertical shooting angle, the last ray is shot under the angle of 0.82 rad from the horizontal direction. The ratio between horizontal and vertical scale of the figure is 1:3. We can see that the low velocity channel in the depths under 5 km in the two models with SOBMUL lower than 350 km s causes an unacceptably large geometrical spreading between the rays emerging on the surface in distances longer than 50 km from the source. For model with SOBMUL=350 km s the velocity under 5 km is almost constant, and the rays are emerging at the reference surface quite regularly with maximum mutual distance between neighbouring rays about 20 km for the rays emerging at the surface in the distances up to 250 km from the source. Thus, if the distance of the rays traced with the step of 0.0002 rad is shorter than 20 km, and if we consider that we are able to trace the rays with a step of 0.000001 rad, we can estimate that it will be always possible to find two-point rays emerging less than 50 m from the receivers. The model is thus suitable for two-point ray tracing, although it is on the edge of acceptable models. The high velocity channel under 5 km in the two models with SOBMUL higher than 350 km s causes large geometrical spreading for rays emerging on the surface in distances longer than 250 km.

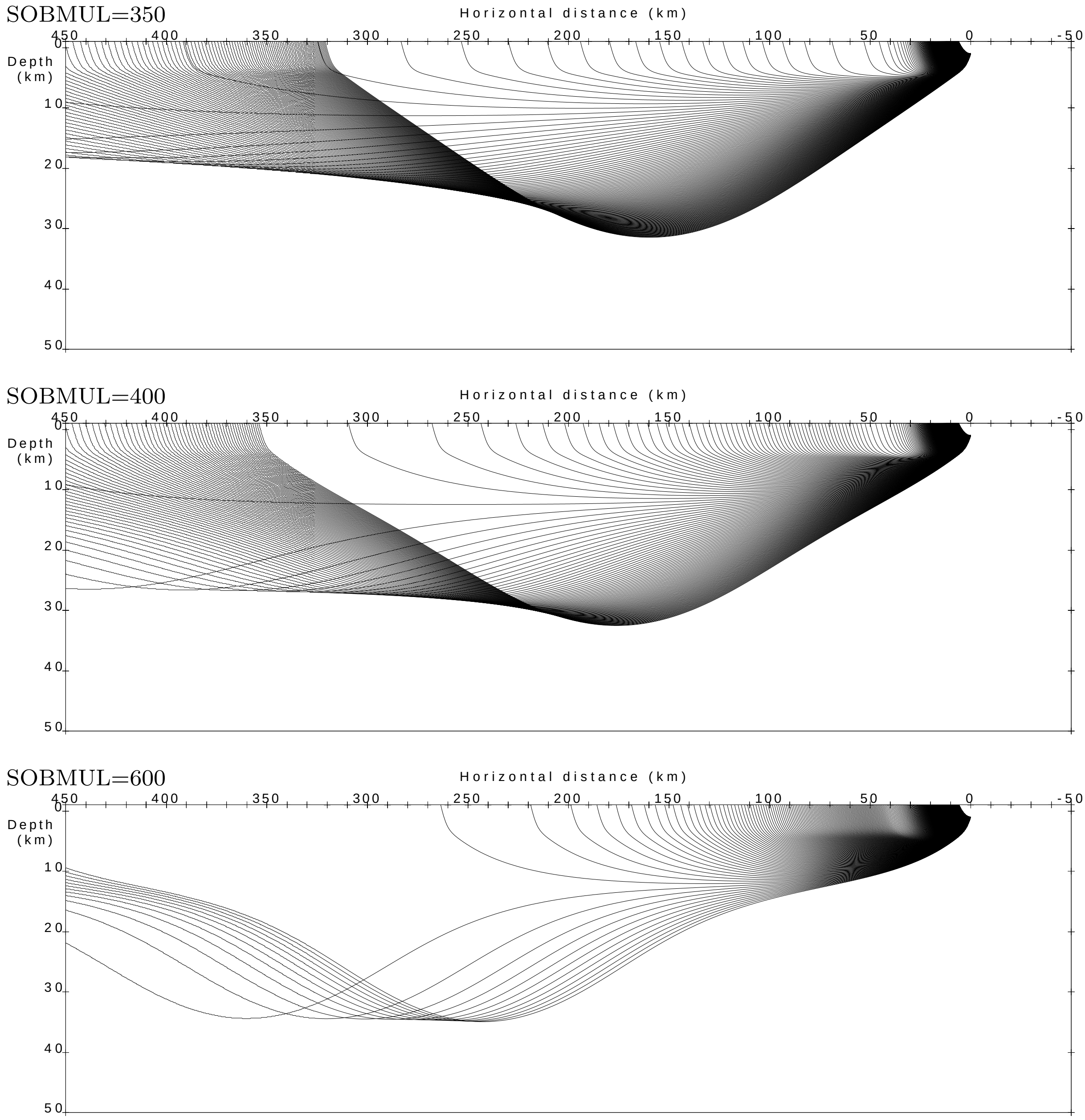


Figure 4 (continued)

4. Alternative parametrizations of the model

As explained in the previous chapter, for the 1-D model we know that the parametrization with the same number and same depths of spline points as the data points is sufficient to describe the model. To check this theoretical knowledge numerically, we perform the inversion for the model with doubled number of splines, where we added always a new spline point in between the two already used points. The spline points are thus in the depths of 0.0, 0.25, 0.5, 1.2, 1.9, 2.2, 2.5, 2.75, 3.0, 3.9, 4.8, 15.0, 25.0, 37.5 and 50.0 km. As expected, the resulting model coincides with the model with spline points at data points, see Figure 5 (a).

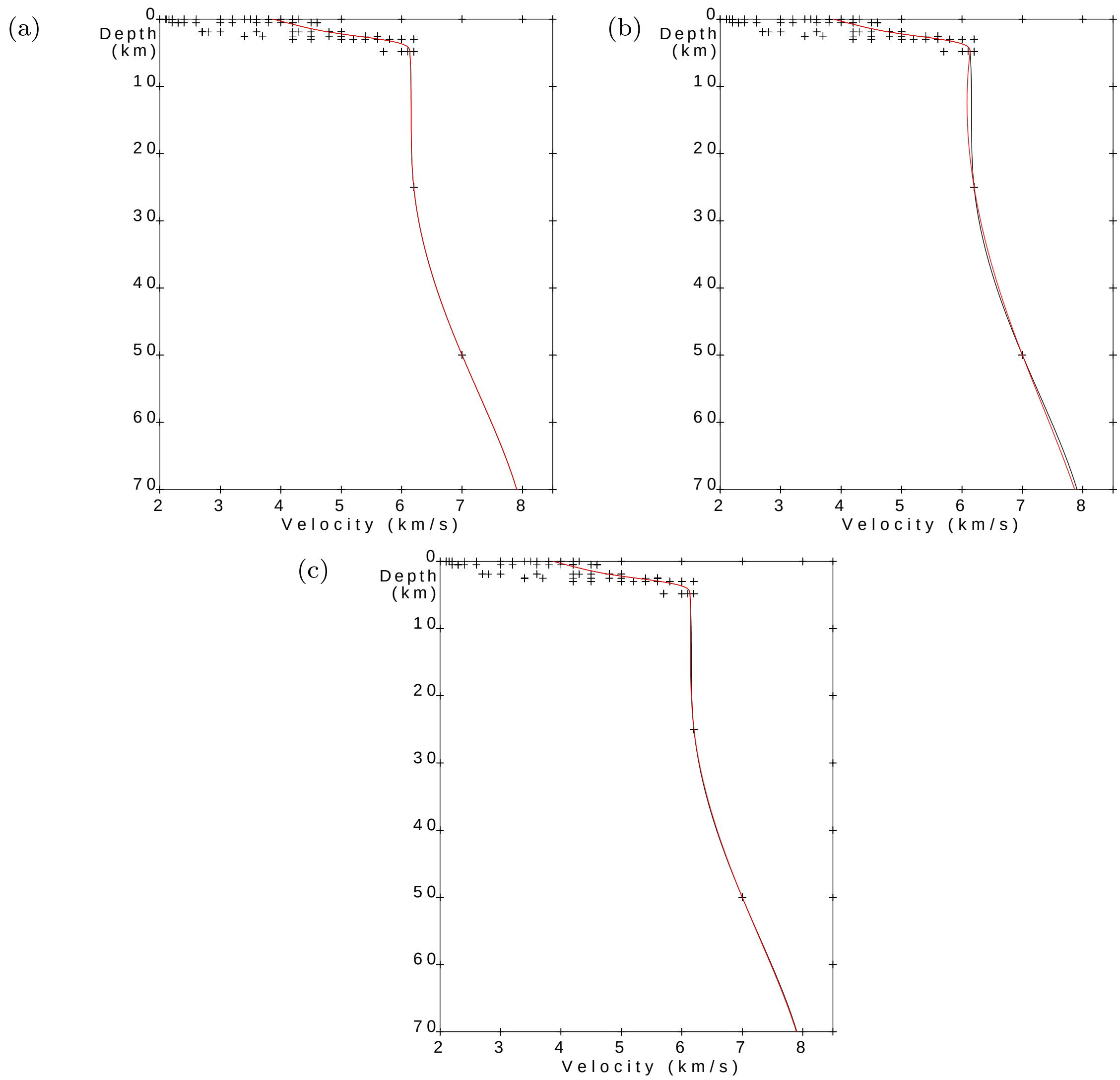
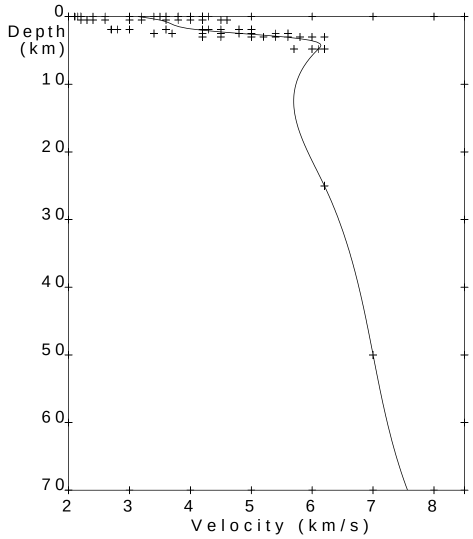


Figure 5: Alternative parametrizations of the model. In all of the three figures, the velocity in the model with 8 spline points at the depths of data points is plotted by black line. In figure (a) it is overlaid by red line showing the velocity in the model parametrized more densely by adding a new spline point in the middle of each two spline points. From this figure we can see that the parametrization by 8 spline points at the depths of data points is sufficient. In figure (b) the red line shows the velocity in the model parametrized by the same number of 8 splines, but distributed more regularly (see the text of the paper for details). We can see that this parametrization is not sufficient. In figure (c) the red line shows the velocity in the model parametrized by 13 more regularly distributed splines, and we can see that this parametrization is again sufficient to describe the velocity. All the models were obtained by inversion with SOBMUL=350 km s.

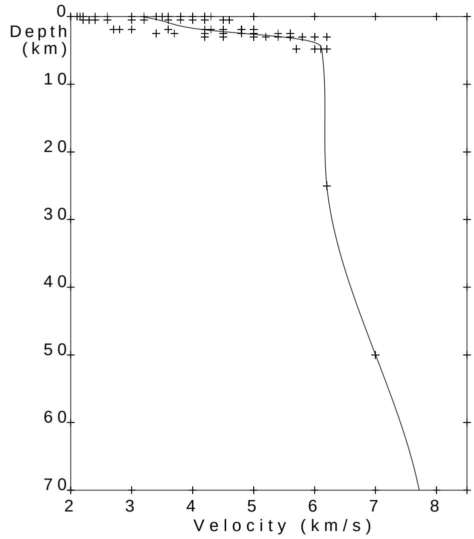
We also tried to use a model with slightly more regular parametrization. We used again 8 spline points with 6 points in the depths 0 to 5 km, but this time the spline points are distributed regularly in the depths of 0., 1., 2., 3., 4., 5., 25. and 50. km. As can be seen from Figure 5 (b), this parametrization is slightly insufficient and causes a small but not negligible change in the behaviour of the velocity. When we try regular

Figure 6 (right page): The models obtained for the model parametrized by 8 spline points in the depths corresponding to data points similarly as in Figure 1, but in this test the quantity interpolated by splines is the velocity. The overall behaviour of the smoothing algorithm is very similar to the case when the slowness was interpolated (Figure 1).

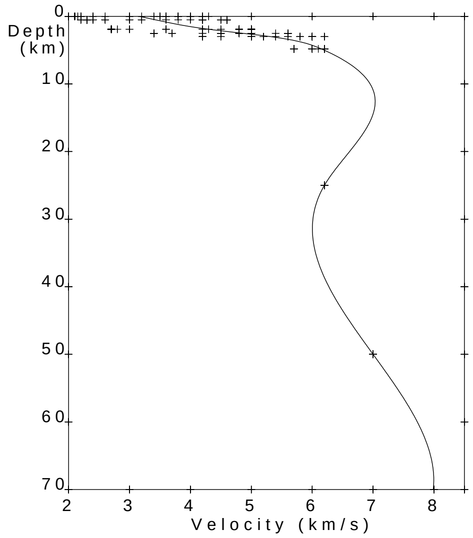
SOBMUL=0



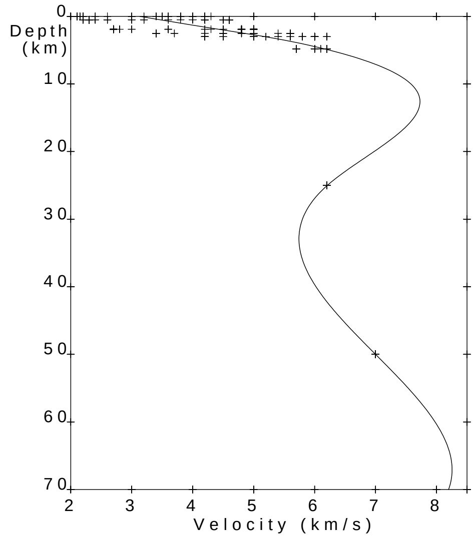
SOBMUL=0.15



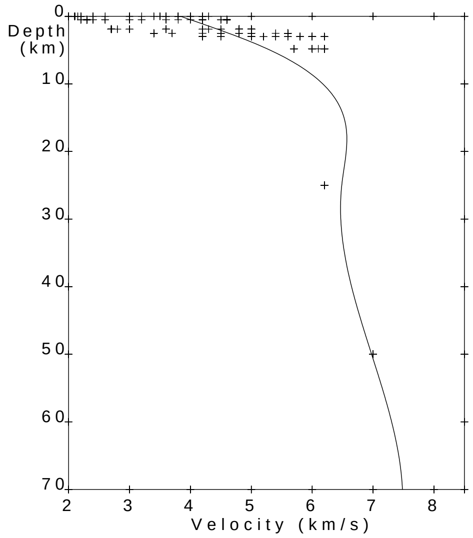
SOBMUL=1



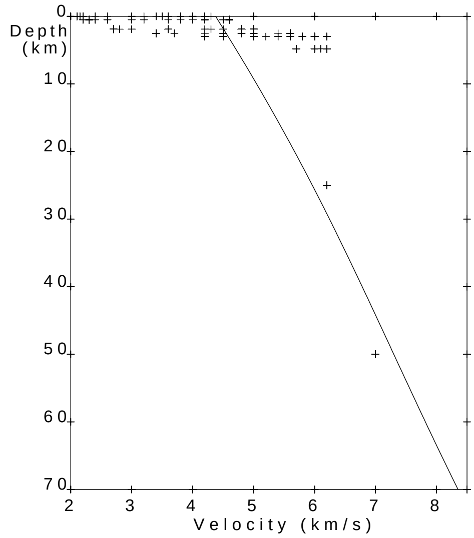
SOBMUL=10



SOBMUL=1 000



SOBMUL=100 000



but denser parametrization of the model with 13 spline points in the depths of 0., 0.7, 1.4, 2.1, 2.8, 3.5, 4.2, 4.9, 5.6, 10., 25., 37.5 and 50. km, we obtain again almost the same velocity as in the model with 8 spline points at the depths of data points, see Figure 5 (c).

As already mentioned in Section 2.2, the velocity model is paramterized in such a way that the quantity interpolated by splines is the slowness. In order to further test the behaviour of the smoothing algorithm, we performed a test similar to that shown in Figure 1, but this time for the model where the quantity interpolated by splines is the velocity, see Figure 6. The overall behaviour of the smoothing algorithm is very similar, yielding models with low velocity channel in depths around 12 km for low values of SOBMUL, then rapid change (with increasing SOBMUL) towards models with high velocity channel around 12 km, and finally yielding models not fitting the data sufficiently for high values of SOBMUL.

5. Comparison of local 1-D models

As already mentioned above, the data for the velocity model are available on a 3-D grid of 7 × 8 × 8 points. We thus can look at the data as a set of 56 vertical profiles, each of them consisting of 8 data points, and we can try to construct 1-D velocity models for the individual profiles. Figure 7 shows the constructed models for two of the 56 profiles. We can see that the value of SOBMUL, i.e. the amount of smoothing which needs to be applied during the inversion, differs considerably in the two cases. It is thus obvious that it will not be possible to find a single value of SOBMUL which would enable to successfully smooth a 3-D model.

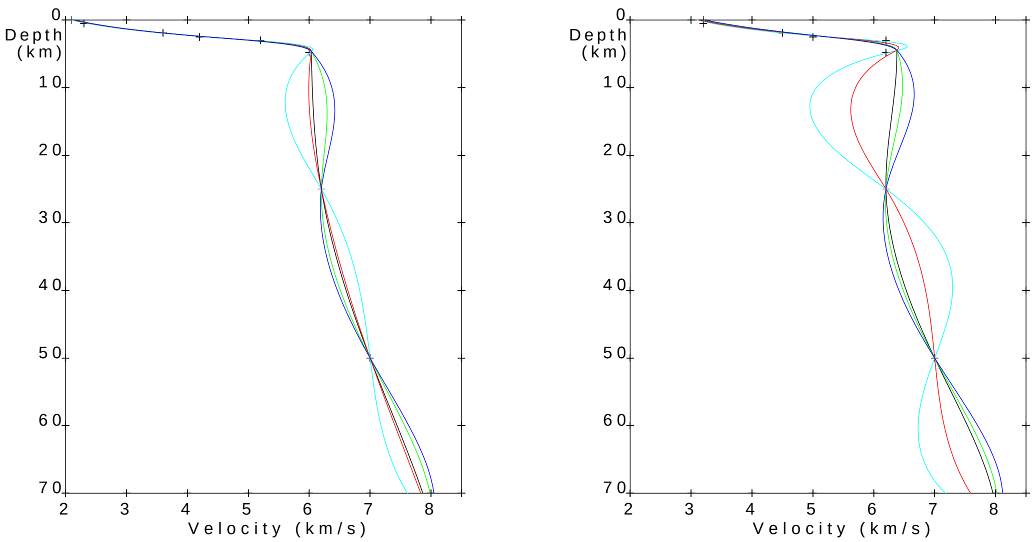


Figure 7: Velocity models obtained by inversion of individual vertical profiles consisting of 8 data points, two of the 56 profiles are shown. In the profile shown in the left figure, the five velocity models correspond to the values of SOBMUL=100 (cyan), SOBMUL=200 (red), SOBMUL=220 (black), SOBMUL=300 (green) and SOBMUL=350 (blue), the optimum value is SOBMUL=220 km s. In the right figure, the velocity models correspond to the values of SOBMUL=350 (cyan), SOBMUL=1000 (red), SOBMUL=2200 (black), SOBMUL=2500 (green) and SOBMUL=3000 (blue), the optimum value is SOBMUL=2200 km s. The optimum value of SOBMUL is thus 10 times higher for the right profile than for the left profile. Such differences between individual profiles do not allow to find a single value of SOBMUL suitable for creation of a smooth 3-D model.

Conclusions

The P-wave velocity data available for the Dobrá Voda locality are strongly heterogeneous. The upper part of the structure is relatively densely sampled with a strong velocity gradient and relatively high lateral variation of the data, while its lower part is sampled sparsely with a weak vertical gradient and no lateral variations. The 1-D smooth P-wave velocity model of the locality was successfully constructed using the algorithm of the least square inversion of given data with minimization of Sobolev norm composed of second derivatives of the velocity in the model. Application of the algorithm requires proper selection of the amount of smoothing and appropriate parametrization of the model. Numerical tests performed for individual vertical data profiles indicate that the data do not allow us to apply the smoothing algorithm with constant smoothing weight to create a smooth 3-D model of the locality.

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