

# Moveout approximation for P waves in a homogeneous VTI medium

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## SUMMARY

We propose an alternative expression for the P-wave moveout approximation in a VTI medium based on the weak-anisotropy approximation. The proposed formulae are relatively simple, they yield highly accurate results for zero and far offsets. First-order formulae depend on four parameters, two-way zero-offset traveltime  $T_0$  related to the vertical velocity  $\alpha$ , the depth  $H$  of the reflector and two weak-anisotropy (WA) parameters  $\epsilon$  and  $\delta$ . Very accurate second-order formula is slightly more complicated and, in addition to the mentioned parameters, depends on an additional parameter,  $r$ , the ratio of the S- and P-wave velocities. Since the dependence of the moveout on  $r$  is very weak,  $r$  can be specified as a typical S- to P-wave velocity ratio and the number of parameters necessary for the specification of moveout is four too.

**Keywords:** moveout approximation, VTI medium, weak-anisotropy approximation

## 1 INTRODUCTION

Reflection traveltime (moveout) approximations find applications in several branches of processing of reflection data. There is quite an extensive literature devoted to these approximations in anisotropic media (Aleixo and Schleicher, 2010; Stovas, 2010; Tsvankin, 2001). In most cases, they are based on the Taylor expansion of the square of reflection traveltime  $T$  in terms of the square of the source-receiver offset  $x$ . If only the first and second terms of the expansion are kept, we speak about normal (hyperbolic) moveout, broadly used in reflection data processing in isotropic media. If the medium is anisotropic, the approximation based on the two terms becomes very inaccurate, especially for increasing offset. In fact, the moveout is generally non-hyperbolic in anisotropic media. In order to accommodate anisotropy, many researchers consider the next term in the Taylor ex-

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pansion (Tsvankin, 2001) or use various multiparametric approximations based partially on physics and partially on intuition. Lists of many such approximations with illustrations of their accuracy can be found, for example, in Fowler (2003) or Aleixo and Schleicher (2010). Quite accurate, but also quite complicated, formula for a homogeneous VTI medium was proposed recently by Stovas (2010).

In this paper, we propose alternative reflection traveltime formulae for homogeneous transversely isotropic media with vertical axis of symmetry (VTI media) based on weak-anisotropy approximation. They differ from approximations known from literature by not representing a Taylor expansion in offset. The formulae are specified by four parameters: two-way zero-offset traveltime  $T_0$ , related to the vertical velocity  $\alpha$ , the depth  $H$  of the reflector and two weak anisotropy (WA) parameters, linearized versions of Thomsen's (1986) parameters. If we do not take into account the depth  $H$  of the reflector, the proposed formulae require the same number of parameters as the shifted hyperbola approximation, by one parameter less than the rational approximation and by two parameters less than generalized moveout formula, see Stovas (2010). The proposed formulae are relatively simple, for example, they do not contain square roots. Their complexity increases with their higher accuracy. In most cases, the number of considered parameters remains four (including  $H$ ). Only in case of the second-order formula, an additional parameter  $r$ , the ratio of S- and P-wave velocities, must be considered. However, because of the weak dependence of the moveout formula on  $r$ ,  $r$  can be specified as a typical S- to P-wave velocity ratio and the number of parameters specifying the moveout formula remains four. The formulae work well close to the zero-offset and for wide angle reflections. They are less accurate for intermediate offsets (see the numerical examples), especially for offsets, for which vectors of ray-velocity and phase-velocity deviate significantly. For zero-offset and for great offsets the formulae work well because in these directions ray-velocity and phase velocity vectors tend to be equal because vertical and horizontal propagation in a VTI medium represents propagation in a longitudinal direction (Pšenčík and Gajewski 1998, Farra and Pšenčík 2003), in which both velocities coincide.

The lower-case indices  $i, j, k, l, \dots$  take the values of 1,2,3, the upper-case indices  $I, J, K, L, \dots$  take the values of 1,2. The Einstein summation convention over repeated indices is used.

## 2 TRAVELTIME APPROXIMATION

Exact expression for the square of the traveltime of an unconverted reflected wave propagating from the source  $S$  to the receiver  $R$ , both situated at the same horizontal level in a homogeneous VTI medium, has the form:

$$T^2(x) = \frac{4H^2 + x^2}{v^2(\mathbf{n})} . \quad (1)$$

Here  $x$  is the offset (distance between  $S$  and  $R$ ) and  $H$  is the depth to the reflector.  $T = T(x)$  denotes the traveltime of the considered reflected wave. It is function of the offset. The symbol  $v = v(\mathbf{n})$  denotes the ray velocity, which is a function of the direction  $\mathbf{n}$  of the slowness vector  $\mathbf{p}$ .

We can rewrite eq. (1) using the notation common in moveout analysis:

$$\bar{x} = \frac{x}{2H}, \quad T_0 = \frac{2H}{\alpha}. \quad (2)$$

Here  $\alpha$  is the vertical velocity and  $T_0$  is the two-way zero-offset travelttime. Eq. (1) then reads:

$$T^2(\bar{x}) = \alpha^2 T_0^2 \frac{1 + \bar{x}^2}{v^2(\mathbf{n})}. \quad (3)$$

Equation (3) can be used for unconverted reflected P or S waves. In the following, we are going to deal with P waves only.

In order to evaluate  $T^2$  from eq.(3), it is necessary to know the direction  $\mathbf{n}$  of the slowness vector. It may differ considerably from the direction  $\mathbf{N}$  of the ray velocity, which specifies the ray along which (1) was evaluated. The vector  $\mathbf{N}$  can be determined from the geometry, which leads to eq.(1). Because we consider a homogeneous VTI medium, it is not important if  $\mathbf{N}$  specifies the direction of the downgoing or upgoing part of the ray of a reflected wave. Let us consider the downgoing part, for which  $N_1$  and  $N_3$  components of the vector  $\mathbf{N}$  are positive. We assume that the axis of symmetry, which is parallel to the  $x_3$ -axis, and the vector  $\mathbf{N}$  are situated in the coordinate plane  $(x_1, x_3)$ . They can then be expressed in terms of the normalized offset  $\bar{x}$  as:

$$N_1 = \frac{\bar{x}}{\sqrt{1 + \bar{x}^2}}, \quad N_3 = \frac{1}{\sqrt{1 + \bar{x}^2}}. \quad (4)$$

It is relatively simple to determine the ray-velocity direction  $\mathbf{N}$  for a given  $\mathbf{n}$ . It is, however, quite complicated to determine  $\mathbf{n}$  for  $\mathbf{N}$  given. In fact, this problem is commonly solved in the two-point ray tracing in anisotropic media: in order to construct a ray between two specified points, one needs to find the slowness vector at one of the points corresponding to the ray connecting them. The problem simplifies if anisotropy of the studied medium is weak. Backus (1965) showed that in such a medium, for a given  $\mathbf{n}$ , the ray velocity  $v(\mathbf{n})$  is equal to the phase velocity  $c(\mathbf{n})$  in the first-order approximation with respect to deviations of anisotropy from isotropy. In other words, the difference of  $v(\mathbf{n})$  and  $c(\mathbf{n})$  is of the second order. Pšenčík and Vavryčuk (2002) and Farra (2004) confirmed Backus' (1965) observation and, in addition, they showed that the difference between directions of ray-velocity and phase-velocity vectors,  $\mathbf{N}$  and  $\mathbf{n}$ , is of the first order. Neglecting this difference may thus have more important consequences than neglecting the difference of  $v(\mathbf{n})$  and  $c(\mathbf{n})$ . With respect to this, we consider the following two ways of evaluation of equation (3):

- 1) We neglect the difference between vectors  $\mathbf{n}$  and  $\mathbf{N}$ ;
- 2) We take the difference between vectors  $\mathbf{n}$  and  $\mathbf{N}$  into account.

Before we start to treat the mentioned two cases, let us introduce some relations, which will be useful in the following considerations. First, let us introduce the equation for the square of the first-order phase velocity in a general weakly anisotropic medium (see, e.g., Pšenčík and Gajewski, 1998):

$$c^2(\mathbf{n}) = B_{33}(\mathbf{n}) = a_{ijkl} n_j n_l n_i n_k. \quad (5)$$

The symbol  $B_{33}$  denotes an element of the first-order symmetric matrix  $\mathbf{B}(\mathbf{n})$ , given by the formula

$$B_{mn}(\mathbf{n}) = a_{ijkl}n_jn_l e_{mi}e_{nk} . \quad (6)$$

Here  $a_{ijkl}$  are density-normalized elastic moduli and  $n_i$  is the  $i$ -th component of the vector  $\mathbf{n}$ . The symbol  $e_{ij}$  denotes  $j$ -th component of the vector  $\mathbf{e}_i$ . The vectors  $\mathbf{e}_i$  form an orthonormal triplet, in which  $\mathbf{e}_3 = \mathbf{n}$ . Vectors  $\mathbf{e}_K$  can be chosen arbitrarily in the plane perpendicular to  $\mathbf{n}$ . In the VTI medium, eq.(5) reduces to (see, e.g., Pšencík and Farra, 2005):

$$c^2(\mathbf{n}) = \alpha^2[1 + 2(\delta - \epsilon)n_1^2n_3^2 + 2\epsilon n_1^2] . \quad (7)$$

The parameters  $\epsilon = (A_{11} - \alpha^2)/2\alpha^2$  and  $\delta = (A_{13} + 2A_{55} - \alpha^2)/\alpha^2$  are the weak anisotropy (WA) parameters, which represent linearized Thomsen (1986) parameters. In accord with (3), we use  $\alpha^2 = A_{33}$ . The symbols  $A_{\beta\gamma}$  with  $\beta, \gamma = 1, 2, \dots, 6$  denote density-normalized elastic moduli in the Voigt notation. We can see from eq.(7) that the square of the first-order P-wave phase velocity  $c$  depends on three parameters of the medium  $\alpha$  ( $\alpha^2 = A_{33}$ ),  $\epsilon$  and  $\delta$ , and on the direction of the slowness vector  $\mathbf{n}$ .

In the following, we shall need an estimate of the difference between the vectors  $\mathbf{n}$  and  $\mathbf{N}$ , and an estimate of the change of the square of the phase velocity due to the replacement of  $c^2(\mathbf{n})$  by  $c^2(\mathbf{N})$ . These problems were studied by Pšencík and Vavryčuk (2002) and Farra (2004). They found that the unit vector  $\mathbf{N}$  in the direction of the ray velocity can be expressed in terms of  $\mathbf{n}$  as follows

$$\mathbf{N}(\mathbf{n}) = \mathbf{n} + \Delta\mathbf{N} , \quad (8)$$

where

$$\Delta\mathbf{N}(\mathbf{n}) = 2c^{-2}(\mathbf{n})B_{I3}(\mathbf{n})e_I(\mathbf{n}) . \quad (9)$$

The fact that  $B_{I3}$  in eq.(9) is the first-order quantity implies that the difference between vectors  $\mathbf{n}$  and  $\mathbf{N}$  is also of the first order. The components of the two vectors can be therefore interchanged within the first-order approximation everywhere, where they are multiplied by some first-order quantity. Thus, within the first-order approximation, we have  $\mathbf{B}(\mathbf{n}) = \mathbf{B}(\mathbf{N})$ .

All quantities appearing on the right-hand side of eq.(9) have been defined above. Eqs (8) and (9) simply follow from eq.(22) of Farra (2004) if we take into account that, to the first order,  $v(\mathbf{n}) = c(\mathbf{n})$ . With (8) and (9) we can seek the relation between  $c^2(\mathbf{n})$  and  $c^2(\mathbf{N})$ . As shown in (7), we have available simple expression for  $c^2(\mathbf{n})$ , but we do not know  $\mathbf{n}$ . From the configuration leading to eq.(1) we know  $\mathbf{N}$ , but not  $\mathbf{n}$ . From (5) and (8), we simply get

$$c^2(\mathbf{n}) = c^2(\mathbf{N}) - \Delta c^2(\mathbf{N}) , \quad (10)$$

where

$$\Delta c^2(\mathbf{N}) = 4a_{ijkl}\Delta N_j N_l N_i N_k . \quad (11)$$

Inserting  $\Delta\mathbf{N}$  from (9) to (11), we get

$$\Delta c^2(\mathbf{N}) = 8c^{-2}(\mathbf{N})B_{I3}(\mathbf{N})a_{ijkl}e_{Ij}N_l N_i N_k = 8c^{-2}(\mathbf{N})[B_{13}^2(\mathbf{N}) + B_{23}^2(\mathbf{N})] . \quad (12)$$

We can thus write, see (10)

$$c^2(\mathbf{n}) = c^2(\mathbf{N}) - 8c^{-2}(\mathbf{N})[B_{13}^2(\mathbf{N}) + B_{23}^2(\mathbf{N})] . \quad (13)$$

We end this section by specifying the elements  $B_{I3}$  of the first-order matrix  $\mathbf{B}$  for the VTI medium. As Pšenčík and Gajewski (1998), we consider the vectors  $\mathbf{e}_1$  and  $\mathbf{e}_2$  chosen so that  $\mathbf{e}_2$  is perpendicular to the axis of symmetry (vertical) and the vectors  $\mathbf{e}_i$  form a right-handed orthonormal triplet. Then

$$B_{13}(\mathbf{n}) = \alpha^2 n_1 n_3 [\delta - 2(\delta - \epsilon)n_1^2] , \quad B_{23} = 0 . \quad (14)$$

Let us note that, as shown by Farra and Pšenčík (2003), the term  $B_{13}^2(\mathbf{N}) + B_{23}^2(\mathbf{N})$  appearing in (13) does not depend on the choice of the vectors  $\mathbf{e}_I$ .

## 2.1 Case 1

If we neglect the difference between the vectors  $\mathbf{n}$  and  $\mathbf{N}$ , we have, in the first-order approximation,  $v^2(\mathbf{n}) = c^2(\mathbf{N})$ , and equation (3) can be rewritten as follows:

$$T^2(\bar{x}) = \alpha^2 T_0^2 \frac{1 + \bar{x}^2}{c^2(\mathbf{N})} . \quad (15)$$

From (15), (7) and (4), we get, after some algebra, the first-order expression for  $T^2$ :

$$T^2(\bar{x}) = T_0^2 \frac{(1 + \bar{x}^2)^3}{P(\bar{x})} , \quad (16)$$

where

$$P(\bar{x}) = (1 + \bar{x}^2)^2 + 2\delta\bar{x}^2 + 2\epsilon\bar{x}^4 \quad (17)$$

is the polynomial containing terms of zero and first order in WA parameters. Separating the terms emphasizing small and large offsets, this can be rewritten in the following form:

$$T^2(\bar{x}) = T_0^2 \left[ 1 + (1 - 2\delta)\bar{x}^2 + 2\bar{x}^4 \frac{\delta - \epsilon + 2\delta^2 + (\delta - \epsilon + 2\delta\epsilon)\bar{x}^2}{P(\bar{x})} \right] . \quad (18)$$

We can see that eq.(18) contains also some second-order WA terms. Linearization of eq.(18) leads to a simplified expression, which is, however, very inaccurate at large offsets.

## 2.2 Case 2

We take now into account the difference between the vectors  $\mathbf{n}$  and  $\mathbf{N}$ . We use again the first-order equation (22) of Farra (2004) relating the ray-velocity and phase-velocity vectors for a given  $\mathbf{n}$ :

$$\mathbf{v}(\mathbf{n}) = c(\mathbf{n})\mathbf{n} + 2c^{-1}(\mathbf{n})[B_{13}(\mathbf{n})\mathbf{e}_{[1]} + B_{23}(\mathbf{n})\mathbf{e}_{[2]}] . \quad (19)$$

From (19) we simply obtain the expression for the square of the first-order ray velocity:

$$v^2(\mathbf{n}) = c^2(\mathbf{n}) + 4c^{-2}(\mathbf{n})[B_{13}^2(\mathbf{n}) + B_{23}^2(\mathbf{n})] . \quad (20)$$

Substitution of  $c^2(\mathbf{n})$  from (13) to (20) and taking into account (14) yields

$$v^2(\mathbf{n}) = c^2(\mathbf{N}) - 4c^{-2}(\mathbf{N})B_{13}^2(\mathbf{N}) . \quad (21)$$

Inserting (21) to (3) and taking into account (7) and (4), we get, after some algebra, more accurate first-order expression for  $T^2$ :

$$T^2(\bar{x}) = T_0^2 \frac{(1 + \bar{x}^2)^3}{P(\bar{x}) - 4Q(\bar{x})P^{-1}(\bar{x})} . \quad (22)$$

In (22), the polynomial  $P(\bar{x})$  (containing terms of zero and first order in WA parameters) is given in (17) and  $Q(\bar{x})$ , which is of the second order in WA parameters, is given by:

$$Q(\bar{x}) = \bar{x}^2 [2\epsilon\bar{x}^2 + \delta(1 - \bar{x}^2)]^2 . \quad (23)$$

Although (22) is the first-order expression as (16), it is more accurate because it takes into account different directions of  $\mathbf{n}$  and  $\mathbf{N}$ . Let us note that eq.(16) can be obtained from (22) by neglecting the second-order term  $Q(\bar{x})$ .

The accuracy of (22) can be further enhanced by replacing the first-order expression for the phase velocity squared in (21) by its second-order expression (Farra, 2001). Equation (21) then reads:

$$v^2(\mathbf{n}) = c^2(\mathbf{N}) + c^{-2}(\mathbf{N})B_{13}^2(\mathbf{N})[(1 - r^2)^{-1} - 4] . \quad (24)$$

In (24),  $r = \beta/\alpha$ ,  $\alpha^2 = A_{33}$  and  $\beta^2 = A_{55}$ . Inserting (24) to (3) and taking into account (7) and (4), we get, after some algebra, the second-order expression for  $T^2$ :

$$T^2(\bar{x}) = T_0^2 \frac{(1 + \bar{x}^2)^3}{P(\bar{x}) + aQ(\bar{x})P^{-1}(\bar{x})} . \quad (25)$$

Here  $a = (4r^2 - 3)/(1 - r^2)$ .

The approximation (22) depends on four parameters: two-way zero-offset traveltime  $T_0$ , related to  $\alpha$ , the depth  $H$  of the reflector and two WA parameters  $\epsilon$  and  $\delta$ . In addition to these parameters, the approximation (25) depends on an additional parameter  $r$ , the ratio of S- and P-wave velocities.

For estimation of accuracy of the above formulae for  $T^2$ , we compare their results with results obtained from the long-spread moveout equation (Tsvankin, 2001). When we use the notation (2), the equation reads

$$T^2(\bar{x}) = T_0^2 \left[ 1 + R\bar{x}^2 - \frac{2(\epsilon_T - \delta_T)R^3\bar{x}^4}{1 + S\bar{x}^2} \right] . \quad (26)$$

Here

$$R = (1 + 2\delta_T)^{-1} , \quad S = \frac{2R^2(\epsilon_T - \delta_T)}{1 - V_{nmo}^2 V_{hor}^{-2}} , \quad (27)$$

where  $\epsilon_T$  and  $\delta_T$  are Thomsen's (1986) parameters (nonlinearized),  $V_{nmo}$  is the normal moveout velocity [ $V_{nmo}^2 = \alpha^2(1 + 2\delta_T)$ ] and  $V_{hor}$  is the horizontal velocity [ $V_{hor}^2 = \alpha^2(1 + 2\epsilon_T)$ ].

### 3 TESTS OF ACCURACY

Here we test the above formulae, specifically (16), (22), (25) and, for comparison, (26) by determining their relative errors  $(T - T_{ex})/T_{ex} \times 100\%$ . By  $T_{ex}$  we denote the traveltimes calculated using the package ANRAY (Gajewski and Pšenčík, 1990), which we consider as an exact reference. We test the above formulae on two models used also by Stovas (2010). One is the weakly anisotropic (anisotropy  $\sim 8\%$ ) Limestone model (Figs 1-4), the other is the strongly anisotropic (anisotropy  $\sim 26\%$ ) Greenhorn shale model (Figs 5-8).

The Limestone model is specified by P- and S-wave velocities  $\alpha = 3\text{km/s}$ ,  $\beta = 1.707\text{km/s}$ ,  $\epsilon = 0.076$  and  $\delta = 0.133$ . The parameters necessary for the evaluation of eq.(26) are  $V_{nmo} = 3.41\text{ km/s}$ ,  $V_{hor} = 3.22\text{ km/s}$ ,  $\epsilon_T = 0.076$  and  $\delta_T = 0.146$ .

The Greenhorn shale model is specified by  $\alpha = 3.094\text{ km/s}$ ,  $\beta = 1.51\text{ km/s}$ ,  $\epsilon = 0.256$  and  $\delta = -0.0523$ . The parameters necessary for the evaluation of eq.(26) are  $V_{nmo} = 2.934\text{ km/s}$ ,  $V_{hor} = 3.805\text{km/s}$ ,  $\epsilon_T = 0.256$  and  $\delta_T = -0.0505$ .

In Figure 1, we show the exact (black) and first-order (grey) phase velocity in the Limestone model as a function of the increasing normalized offset  $\bar{x}$ . We can see that the difference between both velocities is negligible, both curves effectively coincide. By a close inspection, we could see that the first-order phase velocity is always less than the exact one. This confirms observation made on the basis of exact and first-order formulae by, for example, Farra and Pšenčík (2003).

The curves of exact ray and phase velocities in Figure 2 behave in a similar way, the small difference between them being observable. The phase velocity is less than the ray velocity. Both velocities are equal for the zero offset. This direction represents the longitudinal direction. Similarly, both curves converge to each other with increasing offset because horizontal direction represents another longitudinal direction, in which values and directions of both velocities coincide.

In Figure 3, we can see the variation of the absolute value of the angle between the ray-velocity and phase-velocity vectors, i.e., between vectors  $\mathbf{N}$  and  $\mathbf{n}$  as a function of an increasing normalized offset  $\bar{x}$ . The black curve shows the angle between exact vectors, the grey curve between first-order approximations of the vectors. We can again see that the vectors  $\mathbf{N}$  and  $\mathbf{n}$  coincide for the zero offset and approach each other for increasing offset. Most important feature of this figure is the magnitude of the difference between the two angles for the intermediate offsets. Although anisotropy is weak, the difference makes nearly  $5^\circ$ . This difference is responsible for the worse performance of all approximate moveout formulae. If neglected, as it is the case of eq.(16), it can lead to further decrease of accuracy of approximate formulae for the corresponding offsets. This can be seen in Figure 4.

Figure 4 shows relative traveltimes errors of the first-order formula (16) shown by open circles. In (16), the difference between the directions of ray- and phase-velocity vectors  $\mathbf{N}$  and  $\mathbf{n}$  was neglected. The interval of offsets with increased errors closely correlates with the interval of offsets with increased deviations of vectors  $\mathbf{N}$  and  $\mathbf{n}$ . We can see that relative errors of the formula (16) are generally small, less than, approximately, 0.28%. They reach these values only in a narrow region of small non-zero offsets. Their

maximum is approximately comparable with errors of the long-spread moveout formula (26) shown by black curve. However, for normalized offsets  $\bar{x}$  larger than 1.5, eq.(16) yields much better results than (26). Relative travelttime errors of the first-order formula (22), which takes into account different directions of vectors  $\mathbf{N}$  and  $\mathbf{n}$  is shown by grey curve. Maximum errors are now considerably reduced, they are less than 0.15%. These errors are effectively removed if the second-order equation (25), shown by light grey, is used. In this case, maximum errors are less than 0.03%.

Let us now test the approximate formulae on the Greenhorn shale model whose anisotropy cannot be considered weak.

In Figure 5, we can again see the comparison of the exact (black) and first-order (grey) phase velocities as a function of the normalized offset  $\bar{x}$ . We can observe features similar to Fig.1, but differences of both velocities at intermediate offsets are now clearly visible. In Figure 6, we can observe substantially larger differences between ray and phase velocities. Otherwise, the main features of the curves are again preserved. Figure 7 shows that for anisotropy of approximately 26%, the deviation of vectors  $\mathbf{N}$  and  $\mathbf{n}$  may reach about  $15^\circ$ . For small offsets, we can observe an interesting effect: change of mutual positions of vectors  $\mathbf{N}$  and  $\mathbf{n}$  at about  $\bar{x} = 0.3$ . The absolute value of the angle between  $\mathbf{N}$  and  $\mathbf{n}$  is zero there. The deviations of the two vectors have again a strong effect on travelttime errors, especially for small non-zero offsets. We can see it in Figure 8. We can see that eq.(16), in which we neglected the deviation of  $\mathbf{N}$  and  $\mathbf{n}$ , gives errors, whose maximum values reach 2.5% (open circles). For  $\bar{x} > 1$ , eq. (16) yields worse results than the long-spread formula (26) shown by black curve. When deviations of  $\mathbf{N}$  and  $\mathbf{n}$  are taken into account, see the grey curve obtained from eq.(22), the maximum errors reduce under 2%. They further reduce, under about 0.5%, when we use the second-order formula, eq.(25). This is shown by light grey curve in Fig.8. Note that stronger anisotropy not only leads to greater maximum errors, but also to the extension of offsets with increased errors. Nevertheless, the maximum error of 0.5% for anisotropy of about 26% seems to be very good result.

## 4 CONCLUSIONS

We propose alternative approximate reflection moveout formulae for a P wave in a homogeneous VTI medium based on the weak-anisotropy approximation. For large offsets, the formulae automatically converge to correct values. Generally, they depend on four parameters of the medium, the vertical P-wave velocity  $\alpha$ , the depth  $H$  to the reflector and WA parameters  $\epsilon$  and  $\delta$ . Highly accurate second-order formula (25) depends also on the ratio  $r$  of the S- and P-wave velocities, but this dependence is very weak so that  $r$  can be estimated as a typical S- to P-wave velocity ratio, e.g., the Poisson ratio,  $\beta^2/\alpha^2 = 1/3$ .

As shown by numerical examples, the second-order formula yields highly accurate results even for strong anisotropy (26% in our case). Stovas' (2010) generalized moveout approximation formula yields even better results, but it is rather complicated when compared with the formula (25). The generalized moveout approximation depends on five parameters while even the second-order formula (25) depends effectively on four parameters only.



Accuracy of the proposed formulae strongly depends on the difference of directions  $\mathbf{N}$  and  $\mathbf{n}$  of the ray- and phase-velocity vectors. For offsets, for which the above two vectors are significantly different, the formulae may be less accurate.

Generalization for SV waves in homogeneous VTI media is straightforward. Not so straightforward is the generalization of the presented formulae for TTI media or media with anisotropy of lower symmetry, and for converted waves. Nevertheless, such generalizations are possible. We must only expect results of lower accuracy.

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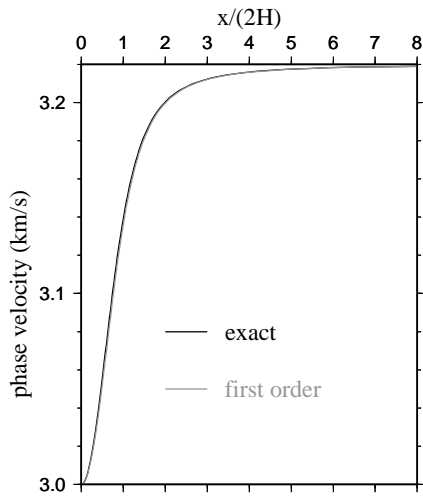


Figure 1: Variations of exact (black) and first-order (grey) P-wave phase-velocities in the Limestone model (anisotropy  $\sim 8\%$ ) as a function of the normalized offset  $\bar{x} = x/2H$ .

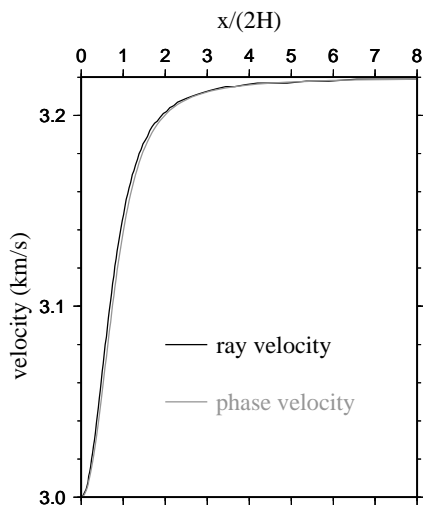


Figure 2: Variations of exact ray (black) and phase (grey) P-wave velocities in the Limestone model (anisotropy  $\sim 8\%$ ) as a function of the normalized offset  $\bar{x} = x/2H$ .

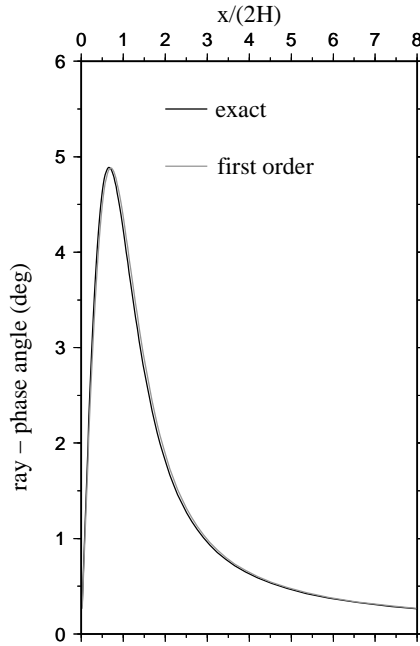


Figure 3: Angular difference of exact (black) and first-order (grey) P-wave ray- and phase-velocity directions  $\mathbf{N}$  and  $\mathbf{n}$  in the Limestone model (anisotropy  $\sim 8\%$ ) as a function of the normalized offset  $\bar{x} = x/2H$ .

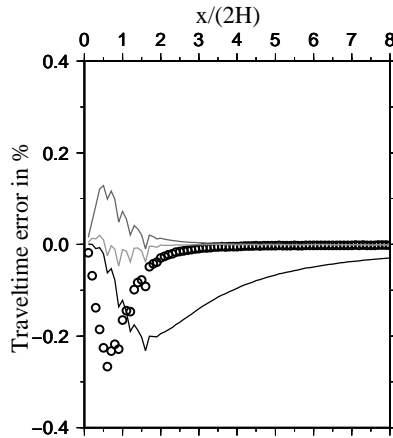


Figure 4: Relative traveltime errors in the Limestone model. Traveltime calculated from the first-order equation (16) ignoring different directions of ray- and phase-velocity vectors  $\mathbf{N}$  and  $\mathbf{n}$  - open circles; the first-order equation (22) taking into account different directions of  $\mathbf{N}$  and  $\mathbf{n}$  - grey; the second-order equation (25) - light grey; the long-spread moveout equation (26) - black.

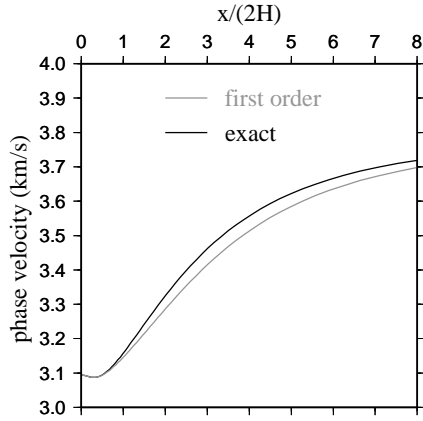


Figure 5: Variations of exact (black) and first-order (grey) P-wave phase-velocities in the Greenhorn shale model (anisotropy  $\sim 26\%$ ) as a function of the normalized offset  $\bar{x} = x/2H$ .

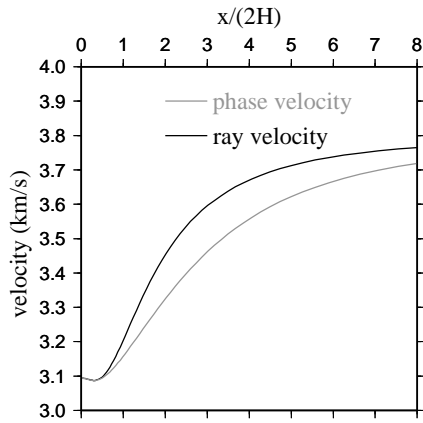


Figure 6: Variations of exact ray (black) and phase (grey) P-wave velocities in the Greenhorn shale model (anisotropy  $\sim 26\%$ ) as a function of the normalized offset  $\bar{x} = x/2H$ .

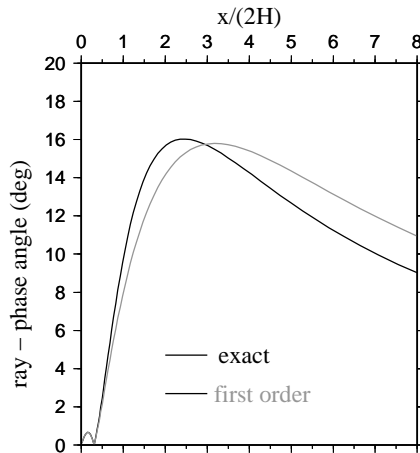


Figure 7: Angular difference of exact (black) and first-order (grey) P-wave ray- and phase-velocity directions  $\mathbf{N}$  and  $\mathbf{n}$  in the Greenhorn shale model (anisotropy  $\sim 26\%$ ) as a function of the normalized offset  $\bar{x} = x/2H$ .

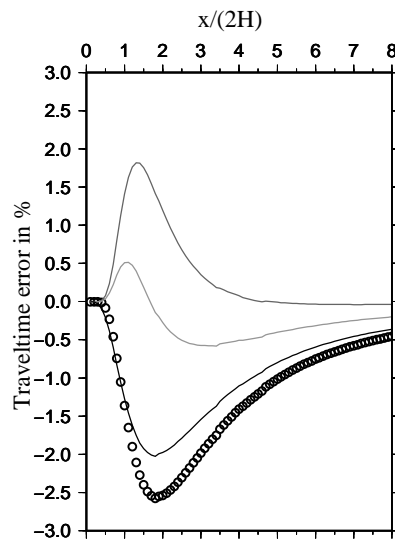


Figure 8: Relative traveltime errors for the Greenhorn shale model. Traveltime calculated from the first-order equation (16) ignoring different directions of ray- and phase-velocity vectors  $\mathbf{N}$  and  $\mathbf{n}$  - open circles; the first-order equation (22) taking into account different directions of  $\mathbf{N}$  and  $\mathbf{n}$  - grey; the second-order equation (25) - light grey; the long-spread moveout equation (26) - black.