Effects of 1-D versus 3-D velocity models on moment tensor inversion in the Dobrá Voda locality at the Malé Karpaty region, Slovakia

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Summary

To retrieve the parameters of a seismic source from seismograms means to deconvolve the response of the medium from the seismic records. Thus, in general, source parameters are determined from both the seismograms and the Green function describing the properties of the medium in which the earthquake focus is buried. The quality of each of these two data sets is equally significant for a successful determination of the source characteristics. As a rule, both the sets are subject to contamination by effects, which decrease the resolution of the source parameters. Seismic records are usually contaminated by noise which appears as a spurious signal not related to the source. Error in the Green function is usually caused by the use of an improper model of the medium due to generally common poor knowledge of the seismic velocity of the area under study. Then, the phenomena of the structure which are not modelled in the Green function are assigned to the source where they distort the source mechanism.

To demonstrate these effects, we performed a synthetic case study simulating seismic observations in Dobrá Voda locality at Malé Karpaty region, Slovakia. A simplified 1-D and 3-D laterally inhomogeneous structural models were constructed, and the synthetic data were calculated in the 3-D model. We then used both the models during moment tensor inversion. The synthetic data were contaminated by a random noise up to 10% and 20% of maximum signal amplitude. We compared the influence of these two effects on retrieving moment tensors. It turned out that a poor structural model could be compensated by a high quality data, and, in similar way, a lack of data could be compensated by a detailed model of the medium. As an example, three local events of magnitude higher than 2 from Dobrá Voda locality were processed.

Keywords

Ray tracing, 3-D velocity model, earthquake mechanism, amplitude inversion.

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1. Introduction and motivation

The mountain region of Malé Karpaty in western Slovakia, especially the zone around Dobrá Voda, suffers from moderate seismicity with earthquakes of magnitudes up to M 3.5. We study the mechanisms of the events in the complete moment tensor description, which is a useful tool for the identification of the active fault systems and for the determination of the tectonic stress. The moment tensors are sensitive to a mislocation of the hypocenters, low signal-to-noise ratios, insufficient focal sphere coverage, and especially to a quality of structural model (Jechumtálová & Šílený, 2005; Šílený, 2009). It is very common that a detailed 3-D laterally inhomogeneous model of the medium in the hypocenter area is not available, and it is thus usually substituted by simplified 1-D model. In view of the fact that the velocity data available for the Dobrá Voda locality enable to construct such a 3-D structural model, we can study the effects of using 1-D or 3-D models on the resulting moment tensors.

In the first part of the paper, we shortly review the methodology of the used moment tensor inversion and of the applied velocity model building algorithm.

Chapter 3 of the paper is devoted to the description of the available velocity data and to the description of building 1-D and 3-D velocity models of the locality.

In chapter 4, we present a synthetic test showing possible distortions of moment tensors caused by using a 1-D velocity model for calculating the response of the medium instead of using the true 3-D model. In the synthetic test, we study also the influence of random noise added to the data for the moment tensor inversion.

Chapter 5 is devoted to moment tensor inversion of three real earthquakes in the Dobrá Voda locality and to the comparison of results obtained using 1-D or 3-D velocity models.

2. Methodology

2.1. Smoothing velocity models for ray tracing

As described in the previous section, one of the steps of the MT inversion method applied is the modelling of ray-theory Green functions from the earthquake hypocenters to the stations of the seismic network. In order to calculate the ray-theory Green function, the construction of a velocity model of the locality is the first step. If the discrete values of velocity are known, we need to fit them by a continuous velocity model. In order to successfully perform ray tracing, proper smoothing of the velocity model is a key issue. We use the method of the construction of a velocity model by fitting the given values of velocity while minimizing the Sobolev norm of the model composed of second velocity derivatives (Bulant, 2002). The velocity in the constructed model is interpolated by B-splines. The values of velocity in the prescribed spline points of the constructed model are calculated during smoothing, based on the given velocity data and on the applied amount of smoothing.

2.2. Moment tensor inversion

The description of an earthquake mechanism by a complete moment tensor (MT) allows us to search for general dipole sources, i.e. not only for a double couple (DC), and at the same time the MT inversion remains linear. The MT could be retrieved from complete waveforms (e.g. Šílený et al., 1992, 1996), amplitudes of seismic waves (e.g. Šílený & Milev, 2008) or even from their ratios (e.g. Snoke, 2003; Jechumtálová & Šílený, 2005). If the structural model is not detailed enough, the waveforms may not be correctly modelled by the synthetic seismograms, and the result of the waveform inversion may be significantly biased. The impact of uncertainty about the medium may be partly reduced by using only amplitudes instead of complete waveforms, as demonstrated by Šílený & Milev (2008). Therefore we employed amplitude inversion. The response of the medium to elementary dipole excitation, i.e. the Green functions, with regard to the amplitude of direct P and S waves was constructed by means of the ray method using the software packages MODEL and CRT (Červený, Klimeš, & Pšenčík, 1988). To solve the linear system of equations in the MT inversion, we use the singular value decomposition method and apply the library routine from Numeric Recipes by Press et al. (1992).

The complete MT is commonly split into the volumetric component that can be either explosive or implosive and deviatoric component. The decomposition of the latter part is not unique, traditionally it is split into a double-couple (DC) and compensated linear-vector dipole that can be oriented either along a tension axis (T-axis) or a pressure axis (P-axis). In this paper we apply the evaluation of the percentage of individual components defined by Vavryčuk (2001).

3. 1-D and 3-D velocity models of the Dobrá Voda locality

3.1. Data for the P and S-wave velocity

For the Dobrá Voda locality, the P-wave and S-wave velocity data are available in the form of a very sparsely sampled 3-D model created by Geofyzika Brno (1985), consisting of $7 \times 8 \times 8$ discrete values of P and S-wave velocity. The data grid is rectangular but irregular, namely in the vertical direction where 6 grid points are available for depths from 0 km to 4.8 km, with two remaining grid points in depths 25 km and 50 km. The values at depths 0 to 4.8 km display lateral variation of the velocity, whereas the velocity at the remaining two depth levels is laterally invariant, see Figure 1. Vertically the data consist of two very different parts, the upper densely sampled part with a strong velocity gradient and strong lateral variation, and the lower sparsely sampled part with a weak gradient.

3.2. 1-D model of Dobrá Voda locality

Using the algorithm of the velocity model construction described above, and properly selecting the amount of smoothing, the optimum 1-D P-wave velocity model of the Dobrá Voda locality was prepared by Bulant (2010). In a very similar way, the 1-D S-wave velocity model was constructed, see Figure 1.

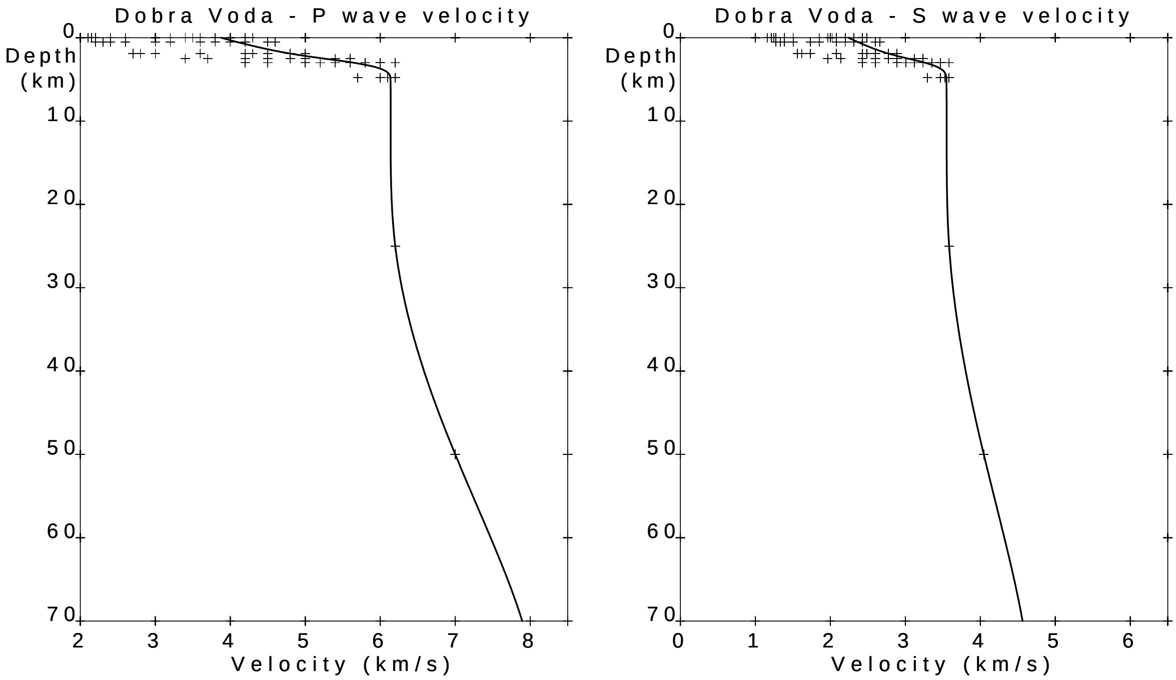

Figure 1: 1-D P-wave (left plot) and S-wave (right plot) velocity models of Dobrá Voda locality. The vertical axis is the depth, horizontal axis is the P-wave or S-wave velocity. The crosses show the values of the velocity in the data points. The solid line shows the velocity in the constructed smooth 1-D velocity models.

3.3. 3-D model of Dobrá Voda locality

Construction of the 3-D smooth velocity model using the above described method of fitting the given values of velocity while minimizing the Sobolev norm of the model composed of second velocity derivatives should be, in principle, very similar to the construction of the 1-D model. However, in the case of Dobrá Voda locality, due to the fact that the data consist of two very different parts, the construction of the 3-D model based on the velocity data only was not possible. This is mainly due to the abrupt change in the vertical velocity gradient in the data at the depth of 4.8 km combined with the lack of the data in the depths under 4.8 km. This lack of the data causes appearance of unacceptable low and high velocity channels in the depth around 12 km, as already indicated by Bulant (2010, Chapter 5).

The velocity data were thus completed with the values of velocities obtained from the 1-D velocity model in the depths of 5 to 45 km, and a 3-D model was created. Completing of the data may be justified by the fact, that the original data below 4.8 km are available only at two depths of 25 and 50 km, and are laterally invariant. Forcing the 3-D model to be similar to 1-D model under 4.8 km thus seems to be reasonable.

Several sections through the resulting 3-D P-wave velocity model are shown in Figure 2. The basic features of the locality, like higher velocities at the Malé Karpaty mountains and lower velocities at the sedimentary basin around river Váh, are nicely described by the model. Two-point rays calculated from one of the hypocentres of the earthquakes to the stations of the seismic network are displayed in Figure 2, and they illustrate that the 3-D model is suitable for calculation of ray-theory Green functions for the purposes of the moment tensor inversion. The 3-D S-wave velocity model was constructed analogously to the construction of the 3- D P-wave velocity model, and it displays in general similar behaviour to the 3-D P-wave velocity model.

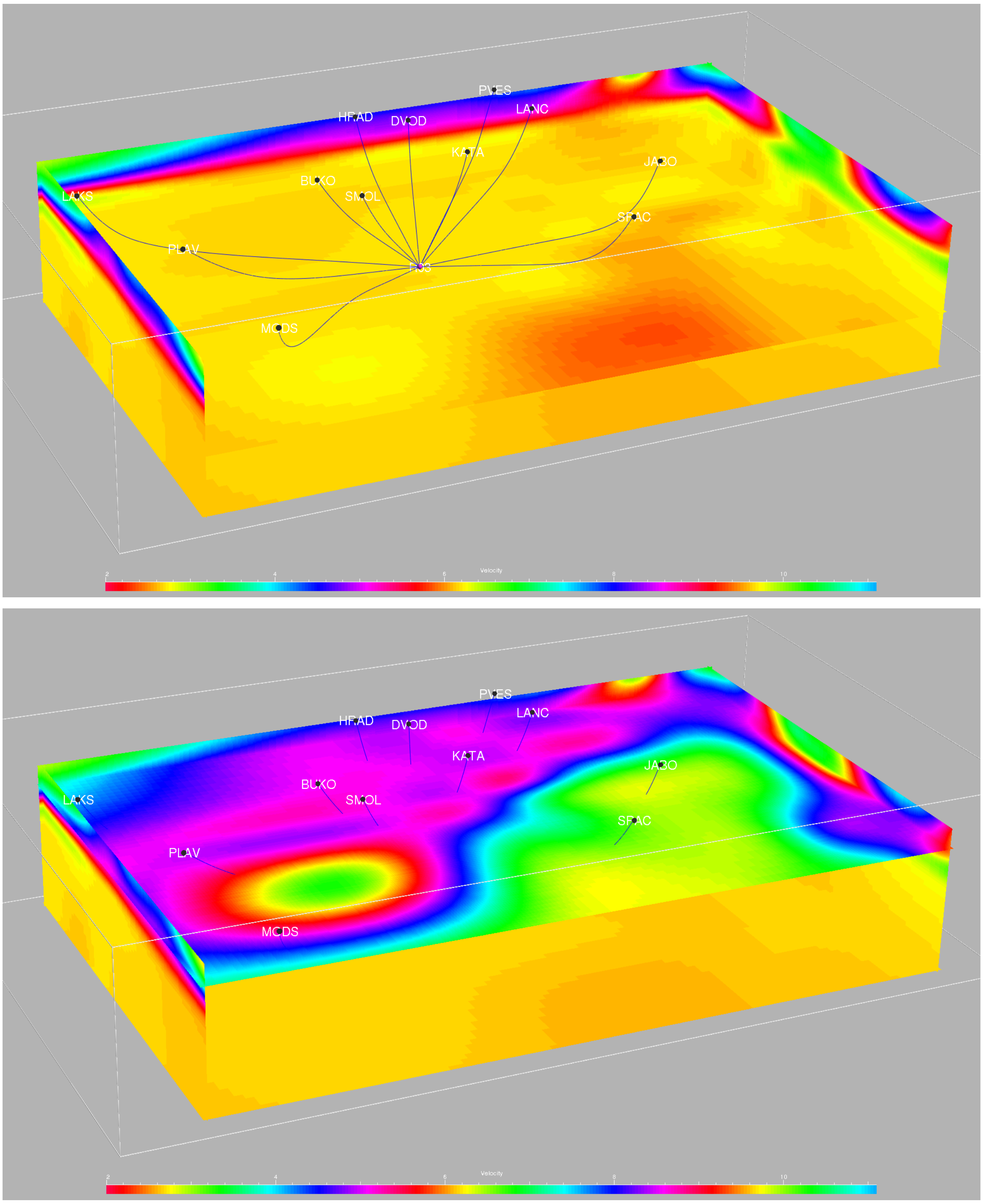

Figure 2: 3-D P-wave velocity model of the Dobrá Voda locality. Velocity sections at the west (left), north (back), east(right), and bottom side of the model are shown in both plots. The upper plot shows also horizontal velocity section at the depth of 23 km, the lower plot shows the section at 5 km depth. Positions of seismic stations are marked by black dots with station names shown in white. Two-point rays calculated from the hypocenter of one of the earthquakes to the seismic stations are also shown.

The velocity at the bottom of the model shown in brown color is approximately 6.2 km/s, the velocity under the station JABO in the lower plot shown in yellow is approximately 2.8 km/s; the velocity scales in the bottoms of the plots range from 2 to 12 km/s. The model nicely fits the basic features of the locality, like the low velocity in the sedimentary river basin in the area of stations SPAC and JABO, or the higher velocities in the area of Malé Karpaty mountains in the SW-NE direction from station MODS to station PVES. However, also some probably artificial artifacts like an extremely high velocity spot between stations MODS, PLAV and SMOL appear too.

4. Synthetic tests of moment tensor inversion

To assess the resolution power of the monitoring network, it is reasonable to perform synthetic experiments. We designed a series of synthetic tests simulating the real configuration of the seismic network Malé Karpaty, Slovakia, with the aim to test the importance of structural model used. Synthetic data were computed for 3-D model and subsequently inverted using the Green functions calculated in both 3-D and 1-D models. A random noise up to 10% and 20% of maximum amplitude was added to the input data in order to study the importance of the quality of the data. We simulated a tectonic event, i.e. pure double-couple, with dip 43°, strike 80° and rake 10°, see Figure 3. We checked the resolution of DC orientation and especially of the DC/non-DC contents, which is known to be sensitive to inexact modeling of the velocity in the crust. Three options of the data set representing varying amount of data were considered: in ascending order (i) both P and S waves on 3 components, (ii) complete P wave on 3 components, and (iii) vertical component of P wave.

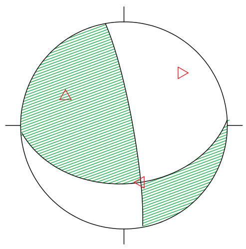

Figure 3: Source model of tectonic event for synthetic experiment. The mechanism in traditional fault plane solution, i.e. equal area projection of the lower hemisphere; black lines - nodal lines of the DC part; triangle up - T axis, triangle right - P axis, triangle left - N axis; green zone - compressions.

The results of the synthetic test, where we inverted 3-D input data using Green functions computed for 3-D structural model, i.e. the correct one, are displayed in the traditional fault plane solutions at Figure 4. Not surprisingly, we obtained almost exact seismic source for noise free input data. At this case we observe only the effect of network configuration here, which is very good and, thus, the distortion of the reconstructed MTs is negligible. The orientation of double-couple part of the mechanism was retrieved well also for noisy input data. As for the DC/non-DC contents of resulting MTs, rather large distortion however appears. The decompositions of complete moment tensor are summarised in Table 1. In the case when random noise up to 10% of maximum amplitude was added to the input data, the decomposition of MT was distorted only for inversion of vertical component of P wave. In case of 20% noise, the retrieved MT was distorted in both the options exploiting P amplitudes, and a good resolution is achieved from P & S wave amplitudes only.

The results of the synthetic test, where we inverted 3-D input data using 1-D structural model, i.e. the simplified one, are exhibited in Figure 5 in the traditional fault plane solution display. Satisfactory reconstruction of the orientation of double-couple part of the mechanism was achieved in all cases (even for 20% noise and inversion of scarce data, i.e. vertical component of P wave only). The decomposition of resulting MTs is expressed in percentages in Table 2. The DC vs. non-DC content was distorted unless both P & S wave amplitudes were inverted. The effect of incorrect velocity model overrides the effect of noise largely: we observe bigger distortion for noise free data than for data contaminated by noise. The similar feature appeared at a synthetic case study

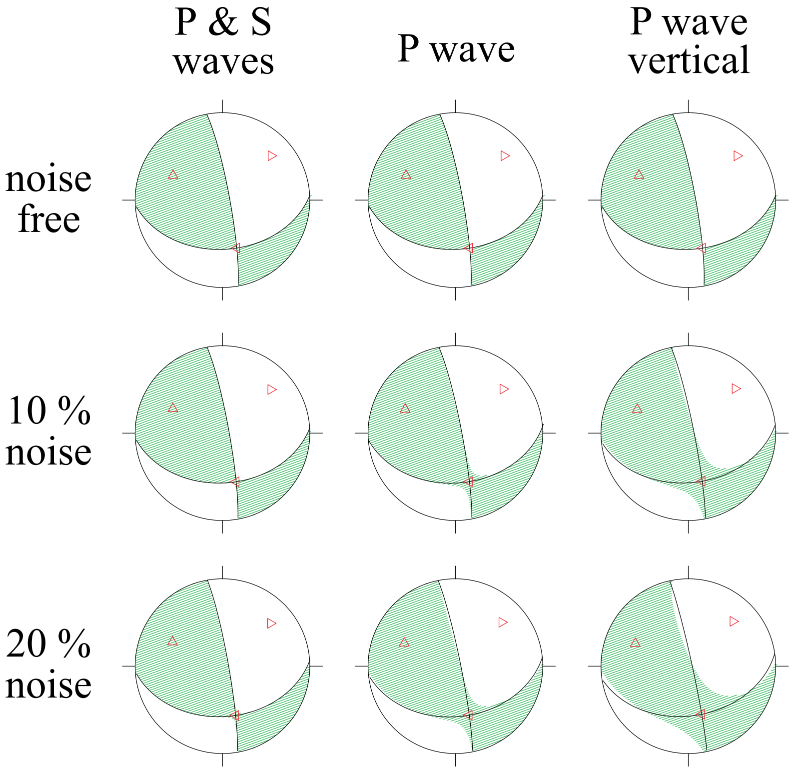

Figure 4: Synthetic experiment of moment tensor inversion using correct velocity model for calculation of responses of the medium. Upper row – noise free input data, middle row – random noise up to 10% of maximum amplitude added to the input data, bottom row – random noise up to 20% of maximum amplitude added to the input data. Columns from left to right: inversion of P together with S wave amplitude, inversion of P wave amplitude, inversion of vertical P wave amplitude only.

	P&S waves	P wave	P wave vertical
noise free	DC 100.0% V 0.0% CLVD 0.0%	DC 99.7% V 0.0% CLVD(T) 0.3%	DC 99.4% V 0.0% CLVD(T) 0.6%
10% noise	DC 97.2% V(imp) 0.8% CLVD(P) 2.0%	DC 91.6% V(imp) 1.3% CLVD(P) 7.1%	DC 77.5% V(imp) 1.4% CLVD(P) 21.1%
20% noise	DC 94.3% V(imp) 1.6% CLVD(P) 4.1%	DC 82.8% V(imp) 2.7% CLVD(P) 14.5%	DC 55.9% V(imp) 2.8% CLVD(P) 41.3%

Table 1. The decomposition of the moment tensors of the synthetic test with correct velocity model into the percentage of the double-couple (DC), implosive volumetric component (V(imp)) and the compensated linear-vector dipole oriented along the tension (CLVD(T)) or pressure axis (CLVD(P)).

simulating seismic observations at Soultz-sous-Forets, Alsace, hot dry rock site (Šílený, 2009), where inexact velocity structure caused bigger distortion than inexact hypocenter location.

These synthetic experiments with correct and simplified velocity models demonstrated that the more exact structural model we have the less data or lower quality data we can invert and still obtain reasonable results. Thus, we can determine source

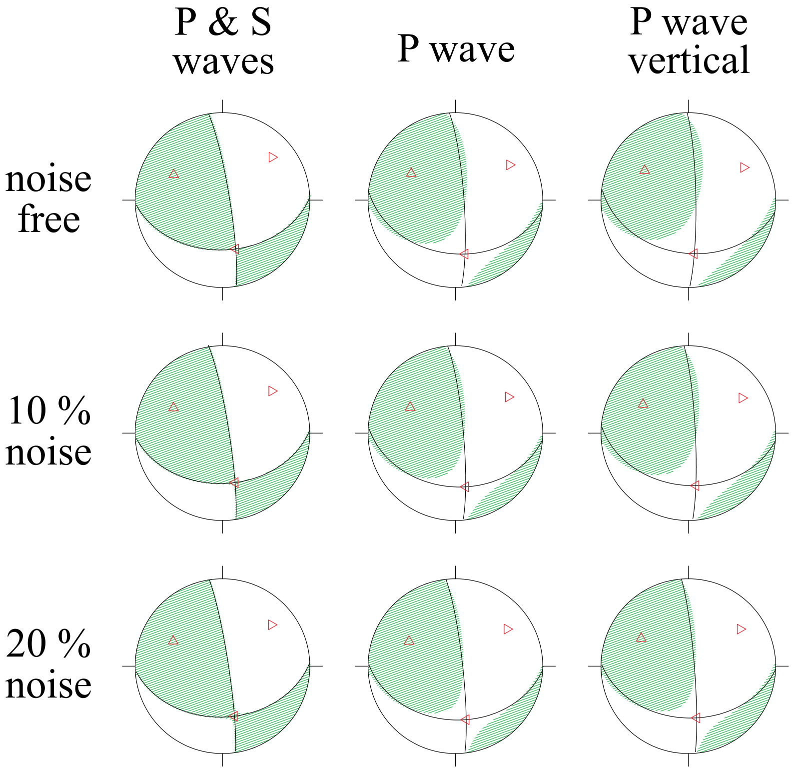

Figure 5: Synthetic experiment of moment tensor inversion using incorrect velocity model: 1-D model for calculation of responses of the medium, while the synthetic data correspond to 3-D model. For details see caption of Figure 4.

	P & S waves	P wave	P wave vertical
noise free	DC 91.4% V(exp) 2.9% CLVD(T) 5.7%	DC 51.7% V(exp) 4.4% CLVD(T) 49.9%	DC 34.6% V(exp) 5.6% CLVD(T) 59.8%
10% noise	DC 93.9% V(exp) 2.2% CLVD(T) 3.9%	DC 61.3% V(exp) 3.4% CLVD(T) 35.3%	DC 49.5% V(exp) 4.8% CLVD(T) 45.7%
20% noise	DC 96.5% V(exp) 1.5% CLVD(T) 1.9%	DC 72.5% V(exp) 2.4% CLVD(T) 25.1%	DC 68.4% V(exp) 3.7% CLVD(T) 27.9%

Table 2. The decomposition of the moment tensors of the synthetic test exploring incorrect velocity modelling into the percentage of the double-couple (DC), explosive volumetric component (V(exp)) and the compensated linear-vector dipole oriented along the T-axis (CLVD(T)).

parameters reliably even for smaller events provided that we are able to construct Greens functions in the 3-D laterally inhomogeneous model.

5. Dobrá Voda sample events

At the beginning of year 2011, the Malé Karpaty network was significantly innovated. Since that time several microearthquakes have occurred in the mountain region of Malé Karpaty. The three strongest events (see Table 3) that have local magnitude bigger than two were suitable for moment tensor inversion.

Date	Origin time	Latitude	Longitude	Depth	M_L
20.7.2011	18:30:58,0	48.620	17.870	16.0	2.1
21.10.2011	15:58:39,3	48.530	17.170	8.0	2.5
5.3.2012	22:56:57,1	48.550	17.120	14.0	3.1

Table 3. The catalogue of events with local magnitude $M_L > 2$. Data provided by ProgSeis.

The seismograms allow reliable estimation of P and sometimes also S wave amplitudes, which contributes to the confidence of the reconstructed MTs as we demonstrated in the synthetic modelling. When available, advantageously we inverted ground-displacement peak amplitudes of both P and S waves to retrieve the moment tensors of selected microearthquakes (Figure 6). For each event we inverted MT using 1-D and also 3-D structural model for calculation of responses of the medium. The decompositions of MTs are presented in Table 4. The resultant MTs indicate the dominance of the DC components, the non-DC part remaining low. Moreover, it is smaller for the inversion by using the 3-D velocity model than for the 1-D, the effect is well pronounced especially concerning the compensated linear-vector dipole (CLVD) component. Reminding the sensitivity of the non-DC to noise contamination, mislocation, irregular focal sphere coverage and the accuracy of the structural modelling demonstrated in the synthetic experiments, it suggests that they are probably insignificant here.

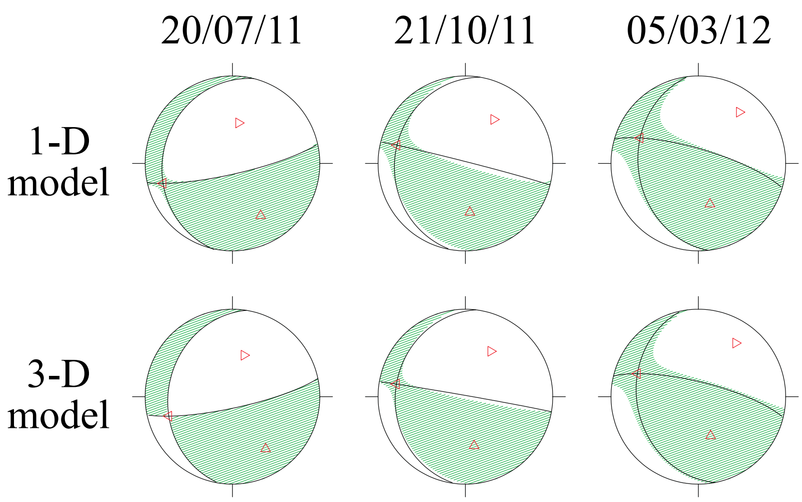

Figure 6: Moment tensor inversion of sample events from Dobrá Voda. Upper row – 1-D model used for calculation of responses of the medium, bottom row – 3-D model used for calculation of responses of the medium. Each column represents different seismic event from the mountain region of Malé Karpaty.

	20/07/11	21/10/11	05/03/12
1-D model	DC 95.9% V(imp) 0.5% CLVD(P) 3.6%	DC 64.2% V(imp) 8.2% CLVD(P) 27.6%	DC 75.9% V(exp) 6.2% CLVD(P) 17.9%
3-D model	DC 99.5% V(imp) 0.1% CLVD(P) 0.4%	DC 75.2% V(imp) 6.9% CLVD(P) 17.9%	DC 86.9% V(exp) 7.1% CLVD(P) 6.0%

Table 4. Decomposition of the moment tensors of sample events from Dobrá Voda into the percentage of the double-couple (DC), volumetric component (V) and the compensated linear-vector dipole (CLVD).

Conclusions

The quality of the data and the accuracy of the velocity model of the area of interest are the crucial for the moment tensor inversion. The velocity data available for the Dobrá Voda locality enabled both the 1-D and 3-D smooth structural velocity models to be constructed. The models are suitable for ray tracing and enable to calculate the response of the medium which is than used in the MT inversion. The 3-D model realistically describes the basic structural features of the locality. We performed a synthetic case study simulating seismic observations at Dobrá Voda site to demonstrate the influence of using 1-D approximation of the structure or exact 3-D structural model on the results of MT inversion. We have generated synthetic data in the 3-D model, added 0%, 10% and 20% of a random noise to the data, and inverted the data using both the 3-D and 1-D models. We showed that using inexact velocity model generates spurious non-DC components in the mechanism, which should be taken into account during the interpretation. Using of the exact 3-D model enables to use less data or lower quality data for MT inversion and still obtain reasonable results. Nevertheless, the double-couple component appears to be sufficiently accurate to identify the fault plane orientation properly even from noisy data and with a simple structural model. When a complete reading of P and S wave amplitudes in a high quality is available, a coarse structural model (even 1-D) may be sufficient.

Moment tensors of three $M_L > 2.0$ events observed during last year were retrieved by unconstrained moment tensor inversion of the peak amplitudes of the direct P and S waves, which were extracted from the seismograms of Malé Karpaty network. Using the 3-D laterally inhomogeneous model in the MT inversion provides smaller non-DC components of the mechanisms compared to 1-D model. The retrieved mechanisms of the events indicate that all three microearthquakes are nearly pure shear-slips.

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