Approximate P-wave ray tracing and dynamic ray tracing in weakly orthorhombic media of varying symmetry orientation

Nabil Masmoudi [1] and Ivan Pšenčík [2]

[1]Physical Sciences and Engineering Division, King Abdullah University of Science and Technology, Saudi Arabia, (nabil.masmoudi@kaust.edu.sa)
[2]Institute of Geophysics, Acad. Sci. of CR, Boční II, 141 31 Praha 4, Czech Republic (ip@ig.cas.cz)

76th Annual International Meeting, EAGE, Expanded Abstracts, Amsterdam

Summary

We present an approximate, but efficient and sufficiently accurate P-wave ray tracing and dynamic ray tracing procedure for 3D inhomogeneous, weakly orthorhombic media with varying orientation of symmetry planes. In contrast to commonly used approaches, the orthorhombic symmetry is preserved at any point of the model. The model is described by six weak-anisotropy parameters and three Euler angles, which may vary arbitrarily, but smoothly, throughout the model. We use the procedure for the calculation of rays and corresponding two-point traveltimes in a VSP experiment in a part of the BP benchmark model generalized to orthorhombic symmetry.

Seismic Waves in Complex 3–D Structures, **24** (2014), 159–163 (ISSN 2336–3827, online at http://sw3d.cz)

Introduction

In recent years, seismic modeling and imaging in orthorhombic models with varying symmetry planes have drawn increased attention. Ray tracing in such media has many potential applications, among them prestack depth migration, traveltime tomography and even full waveform inversion. We present an approximate, but efficient and sufficiently accurate ray tracing procedure for orthorhombic media whose symmetry planes may vary throughout the medium, and test it on the BP model (Shah, 2007).

We start from a ray tracing procedure specified in a curvilinear orthogonal coordinate system valid for anisotropy of arbitrary symmetry (Iversen and Pšenčík, 2008). The coordinate system is constructed so that the coordinate lines are perpendicular to the symmetry planes of an orthorhombic medium. Advantages of this approach are the conservation of orthorhombic symmetry throughout the model and reduction of the number of parameters specifying the model. We combine this procedure with first-order ray kinematic and dynamic ray tracing equations for P waves propagating in smooth, inhomogenous, weakly anisotropic media (Pšenčík and Farra, 2005, 2007). The first-order ray tracing and dynamic ray tracing equations are derived from the exact ones by replacing the exact P-wave eigenvalue of the Christoffel matrix by its first-order approximation. In orthorhombic media, such equations are controlled by 6 weak anisotropy (WA) parameters, which represent a linearized generalization of the Thomsen (1986) parameters.

The accuracy of such a procedure was tested on simple models, for which exact results were available. Observed relative traveltime differences were less than 0.2% in models of the sizes comparable with the size of the model used in this study. In the following, we apply the proposed procedure to the computation of two-point P-wave rays and corresponding traveltimes in a generalization of the BP model (Shah, 2007). The generalized BP model is weakly orthorhombic, with one of the symmetry planes tangent to the structural elements of the model.

We use Einstein summation convention for repeated subscripts.

Curvilinear coordinate system and transformation matrix

Let us introduce an orthogonal curvilinear coordinate system, (ξ_1,ξ_2,ξ_3) with its origin $(\xi_1,\xi_2,\xi_3) = (0,0,0)$ fixed relative to the origin of the Cartesian coordinate system (x_1,x_2,x_3). We define the coordinate lines of the curvilinear system so that they are perpendicular to the symmetry planes of an orthorhombic medium at any point of the medium. The transformation matrix $\mathbf{H}$ from curvilinear to Cartesian coordinates in terms of Euler angles ϕ, θ and ν reads:

$$\mathbf{H} = \begin{pmatrix} cos\phi cos\theta cos\nu - sin\phi sin\nu & cos\phi cos\theta sin\nu + sin\phi cos\nu & cos\phi sin\theta \\ -sin\phi cos\theta cos\nu - cos\phi sin\nu & -sin\phi cos\theta sin\nu + cos\phi cos\nu & -sin\phi sin\theta \\ -sin\theta cos\nu & -sin\theta sin\nu & cos\theta \end{pmatrix} . \tag{1}$$

Angle ϕ controls the rotation around x_3 axis of the Cartesian coordinates. It transforms axes x_1 and x_2 to $x_1^{'}$ and $x_2^{'}$, and is positive if the rotation is anticlockwise. Angle θ controls the rotation around $x_2^{'}$ axis. It transforms axis $x_1^{'}$ to $x_1^{''}$ and axis x_3 to $x_3^{'}$, and is positive if the rotation is anticlockwise. Angle ν controls the rotation around the $x_3^{'}$ axis.

Ray tracing equations

The ray tracing system is governed by a system of differential equations for position vector $\mathbf{x}$ and slowness vector $\mathbf{p}$ in Cartesian coordinates:

$$\frac{d}{dt}\begin{pmatrix} \mathbf{x} \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathbf{H} & \mathbf{0} \\ -\mathbf{K} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \upsilon^{\xi} \\ \eta^{\xi} \end{pmatrix} . \tag{2}$$

In equation (2), **H** is the transformation matrix (1), **I** is a 3 $\times$ 3 identity matrix and **K** is a 3 $\times$ 3 matrix with elements:

$$K_{mk} = \frac{\partial H_{ik}}{\partial x_m} p_i. \tag{3}$$

The vectors υ^{ξ} and η^{ξ} have components:

$$\upsilon_i^{\xi} = \frac{1}{2}\frac{\partial G^{\xi}}{\partial p_i^{\xi}}, \qquad \eta_i^{\xi} = -\frac{1}{2}\frac{\partial G^{\xi}}{\partial x_i}. \tag{4}$$

Symbol G^{ξ} denotes the first-order P-wave eigenvalue of the Christoffel matrix, expressed in curvilinear coordinates ξ. For the orthorhombic case, G^{ξ} reads

$$\begin{aligned} G^{\xi} = \alpha^2 \Big(& p_k^{\xi} p_k^{\xi} + 2[\varepsilon_x (p_1^{\xi})^2 + \varepsilon_y (p_2^{\xi})^2) + \varepsilon_z (p_3^{\xi})^2] \\ & + 2(p_k^{\xi} p_k^{\xi})^{-1} [\eta_x (p_2^{\xi})^2 (p_3^{\xi})^2 + \eta_y (p_1^{\xi})^2 (p_3^{\xi})^2 + \eta_z (p_1^{\xi})^2 (p_2^{\xi})^2] \Big) , \end{aligned} \tag{5}$$

where

$$\eta_x = \delta_y - \varepsilon_y - \varepsilon_z, \quad \eta_y = \delta_x - \varepsilon_x - \varepsilon_z, \quad \eta_z = \delta_z - \varepsilon_x - \varepsilon_y. \tag{6}$$

For further details and definitions of weak-anisotropy (WA) parameters ε_x, ε_y, ε_z, δ_x, δ_y and δ_z see (Pšenčík and Farra, 2005, 2007). The quantity α in equation (5) is a constant reference velocity used in the definition of WA parameters. Equation (5) and the ray tracing equations are independent of α. Thus α can be chosen arbitrarily. We use α that makes the WA parameters as small as possible.

Second order traveltime correction

Ray tracing (2) provides first-order traveltime t. To increase the accuracy of the traveltime computation, we compute a second-order traveltime correction Δt along the ray Ω (Pšenčík and Farra, 2005):

$$\Delta t = -\frac{1}{2}\int_{\Omega} c^{-2} \frac{B_{13}^2 + B_{23}^2}{V_P^2 - V_S^2} dt. \tag{7}$$

Here c is the phase velocity, V_P and V_S are P- and S-wave velocities in the reference isotropic medium closely approximating the studied weakly anisotropic medium. Symbols B_{13} and B_{23} denote elements of the Christoffel matrix projected into a local coordinate system connected with the ray. For details, see Pšenčík and Farra (2005).

Dynamic ray tracing equations

For planned computation of ray amplitudes, for two-point ray tracing and for many other useful applications, we need dynamic ray tracing. The dynamic ray tracing system for the above specification reads (Iversen and Pšenčík, 2008):

$$\frac{d}{dt}\begin{pmatrix} \mathbf{Q} \\ \mathbf{P} \end{pmatrix} = \begin{pmatrix} \mathbf{H} & \mathbf{0} \\ -\mathbf{K} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{S}^{\top} & \mathbf{T} \\ -\mathbf{R} & -\mathbf{S} \end{pmatrix} \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{K}^{\top} & \mathbf{H}^{\top} \end{pmatrix} \begin{pmatrix} \mathbf{Q} \\ \mathbf{P} \end{pmatrix} + \begin{pmatrix} \mathbf{V}^{\top} & \mathbf{0} \\ -\mathbf{U}^{\top} & -\mathbf{V} \end{pmatrix} \begin{pmatrix} \mathbf{Q} \\ \mathbf{P} \end{pmatrix} . \tag{8}$$

Here **Q** and **P** are 3 $\times$ 1 matrices with elements:

$$Q_i = \frac{\partial x_i}{\partial \gamma}, \quad P_i = \frac{\partial p_i}{\partial \gamma}. \tag{9}$$

The quantities Q_i and P_i describe variations along the wave front of the coordinates x_i and of the components p_i of the slowness vector due to the variation of the ray coordinte γ. We use two ray coordinates,

γ_1 and γ_2, which represent two shooting angles at the source. The matrices **R**, **S**, **T**, **U** and **V** in equation (8) are 3×3 matrices with elements:

$$R_{ij} = \frac{1}{2}\frac{\partial^2 G^\xi}{\partial x_i \partial x_j} \;, \qquad S_{ij} = \frac{1}{2}\frac{\partial^2 G^\xi}{\partial x_i \partial p_j^\xi} \;, \qquad T_{ij} = \frac{1}{2}\frac{\partial^2 G^\xi}{\partial p_i^\xi \partial p_j^\xi} \;,$$
$$U_{ij} = \frac{1}{2}\frac{\partial G^\xi}{\partial p_k^\xi}\frac{\partial^2 p_k^\xi}{\partial x_i \partial x_j} \;, \qquad V_{ij} = \frac{1}{2}\frac{\partial G^\xi}{\partial p_k^\xi}\frac{\partial^2 p_k^\xi}{\partial x_i \partial p_j^\xi} \;. \tag{10}$$

Symbol G^ξ denotes again the first-order P-wave eigenvalue of the Christoffel matrix, see (5).

Two point ray tracing

Since we deal with a single wave in a smooth medium, we use a simple two-point ray tracing procedure, in which, using the results of dynamic ray tracing, we convert deviations of a ray from the prescribed receiver position into the corrections of shooting angles at the source.

Numerical example

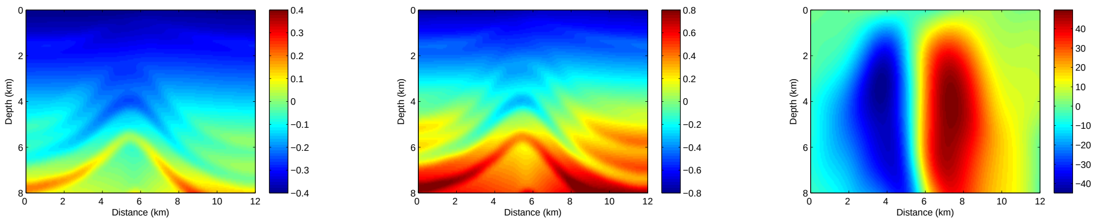
We apply the above-described procedure for ray tracing and traveltime computations in a 3D model of an orthorhombic medium. The model is a generalization of a part of the 2D BP transversely isotropic model with varying axis of symmetry (Shah, 2007). We extended this model to a 3D orthorhombic one by assuming that the structural features of the 2D model do not vary in the direction perpendicular to the plane of the 2D model, and by the appropriate choices of additional WA parameters. The parameters of the BP model are V_{P_0}, ε, δ (Thomsen, 1986) and a tilt angle θ specified in a regular grid with 0.05 km spacing in the vertical and the horizontal direction. In each grid point, we used the values of V_{P_0}, ε and δ and converted them into WA parameters ε_x, ε_y, ε_z, δ_x, δ_y and δ_z. The values and derivatives of WA parameters at an arbitrary point of the model were determined using the tricubic spline interpolation with smoothing. The velocity α was chosen, $\alpha = 3.5$ km/s. As to angles ϕ, θ and ν, we let only angle θ to vary as in the BP model and we kept angles ϕ and ν constant. We show results for three values of ϕ, specifically 15^o, 30^o and 45^o and we keep $\nu = 0^o$.

The distribution of parameters ε_z and δ_z in the plane (x_1, x_3) is shown in Figures 1a and 1b. The variation of remaining WA parameters has a similar character. Variation of the angle ϕ in the same plane is shown in Figure 1c. We can see in Figures 1a and 1b that model parameters decrease at the top of the salt dome which leads to a shadow zone effect. We can also notice in Figure 1c that θ varries from -45^o to $+45^o$.

We use a vertical seismic profiling (VSP) configuration, in which the source and the borehole are situated in a vertical plane (x_1, x_3). The borehole is parallel with the x_3-axis and the source is located on the surface ($x_3 = 0$km) at a 9 km distance from the intersection of the borehole with the surface. The spacing of receivers in the borehole is 0.2km. Figure 2a shows rays projected into the vertical plane containing the source and the borehole, while the projection of rays into the horizontal plane is shown in Figure 2b. Both Figures 2a and 2b correspond to the case $\phi = 15^o$. We can see that rays deviate from the vertical plane (x_1, x_3) due to the deviations of planes of symmetry from vertical and horizontal planes. Figure 1c illustrates effect of varying angle ϕ on the traveltimes. In this figure, one can see that traveltime decreases as the angle ϕ increases.

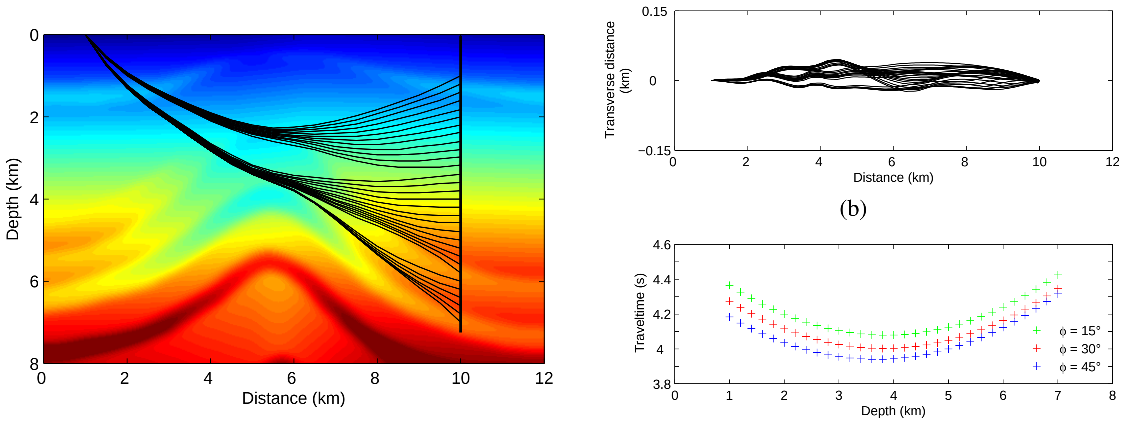
Conclusions

We presented a simple approach for ray tracing and dynamic ray tracing in inhomogeneous, weakly orthorhombic media with varying planes of symmetry. The approach guarantees the conservation of considered anisotropy symmetry (orthorhombic in our case) throughout the model and reduces the number of parameters necessary for the specification of the model. In the case of P-wave propagation in an orthorhombic medium, only six WA parameters and three Euler angles are necessary to be specified. The relative traveltime differences in models, which allowed comparison with exact computations, did not exceed 0.2%.



(a) ε_z (b) δ_z (c) θ

Figure 1: Distribution of WA parameters ε_z and δ_z and of the angle θ in the plane (x_1, x_3).



(a) (b) (c)

Figure 2: Results of ray tracing in the orthorhombic BP model with varying planes of symmetry. (a) Projection of rays into the vertical plane containing the source and the borehole for the case $\phi = 15^o$ with the distribution of V_{P_0} in the background. (b) Projection of rays into the horizontal plane for the case $\phi = 15^o$. (c) Traveltime curves for three values of angle ϕ.

Presented procedure can also be used, without any problem, for ray tracing and dynamic ray tracing in smooth TTI media. It is sufficient to take into account that $\varepsilon_x = \varepsilon_y = \delta_z/2$ and $\delta_x = \delta_y$. The number of independent WA parameters thus reduces to three: ε_x, ε_z and δ_x. Only two Euler angles, ϕ and θ are necessary to specify the orientation of the axis of symmetry. In smooth media, such a procedure is equivalent to the procedure proposed by Dehghan et al. (2007).

Acknowledgements

We thank Seismic Wave analysis Group (SWAG) and Professor Tariq Alkhalifah of KAUST for supporting this research. We also thank the Consortium Project "Seismic waves in complex 3-D structures" (SW3D) and the Research Project 210/11/0117 of the Grant Agency of the Czech Republic for support. We are also grateful to BP for making the benchmark synthetic model available.

References

Dehghan, K., Farra, V. and Nicolétis, L. [2007] Approximate ray tracing for qP-waves in inhomogeneous layered media with weak structural anisotropy. *Geophysics*, **72**(5), SM47–SM60.

Iversen, E. and Pšenčík, I. [2008] Ray tracing and inhomogeneous dynamic ray tracing for anisotropy specified in curvilinear coordinates. *Geophys. J. Int.*, **174**(1), 316–330.

Pšenčík, I. and Farra, V. [2005] First-order ray tracing for qP-waves in inhomogeneous, weakly anisotropic media. *Geophysics*, **70**(6), D65–D75.

Pšenčík, I. and Farra, V. [2007] First-order P-wave ray synthetic seismograms in inhomogeneous weakly anisotropic media. *Geophys. J. Int.*, **170**(3), 1243–1252.

Shah, H. [2007] The 2007 BP anisotropic velocity-analysis benchmark. 70th Anual EAGE meeting, workshop.

Thomsen, L. [1986] Weak elastic anisotropy. *Geophysics*, **51**(10), 1954–1966.