Local inversion for anisotropy from qP-wave slowness and polarization vectors

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Abstract

Data from multi-azimuthal walkaway VSPs are very useful for detection and determination of local anisotropy at individual receivers in a borehole. Various inversion techniques have been proposed. They differ by assumptions about properties of the medium surrounding receivers, by types of waves involved and by computational methods used for the inversion. We present a technique based on the first-order perturbation theory for anisotropic media. Basic equations of our technique are applicable to the data of qP waves propagating in laterally inhomogeneous, weakly anisotropic media of arbitrary symmetry. The data are components of polarization and slowness vectors of qP waves generated at surface sources and recorded at a given receiver. For each source, we thus have an equation. The SVD algorithm is used to solve an overdetermined system of equations for weak anisotropy (WA) parameters. From them qP-wave phase velocity can be reconstructed. We test the technique on a set of synthetic multi-azimuthal walkaway VSP data and on a set of walkaway VSP data measured in the Java Sea region. In the former case, true anisotropy of the model is known, in the latter case, the results of inversion can be compared with results of other techniques. In both cases good agreement is found.

Introduction

Let us consider a multi-azimuthal multiple-source offset VSP experiment with three-component downhole recordings unaffected by a free surface. Attempts to determine anisotropy parameters from data measured in such an experiment are not new. It is quite common to derive slowness components from such data and use them for the determination of anisotropy (White et al., 1983; Gaiser, 1990). If only a component of the slowness vector is available, the use of polarization information becomes a necessity, see Zheng and Pšenčík (2001). In most attempts, VTI anisotropy (transverse isotropy with vertical axis of symmetry) has been assumed (Hsu et al., 1991; de Parscau, 1991; Williamson et al., 2000). Horne et al. (1998) and Horne and Leaney (2000) proposed a method based on the simultaneous use of components of slowness and polarization vectors, which can be used in media with arbitrary structural complexities in the overburden, and can be applied, in principle, to anisotropy of arbitrary symmetry. Their method minimizes the difference between observed and calculated components of slowness vectors corresponding to the observed polarization directions. Forward modelling uses a numerical search for the slowness direction corresponding to the observed polarization. Adaptive simulated annealing is used to minimize the objective function. Horne and Leaney (2000) apply the method to qP and qS direct and reflected waves.

The algorithm used in this paper represents an alternative to the above approach. It is based on the first-order perturbation theory for anisotropic media, see Jech and Pšenčík (1989). Zheng and Pšenčík (2001) have shown that the perturbation formulae relating slowness and polarization vectors of a qP wave to 15 weak anisotropy (WA) parameters describing this wave (Pšenčík and Gajewski, 1998) can be transformed into a single linear equation if only the component of the slowness vector along the borehole is known (this is the case in most applications). This equation corresponds to a single source-receiver pair. Multi-azimuthal walkaway experiments thus produce large systems of such equations, which can be solved for WA parameters at a receiver. The medium in which a receiver is situated can be laterally inhomogeneous (no requirement of local homogeneity is necessary) and anisotropy of arbitrary symmetry can be dealt with. The only assumption is that the anisotropy is weak. Thanks to the use of the perturbation theory the equations have a very simple and transparent form. Since weak anisotropy is common for many real materials (Thomsen, 1986), the equations have a broad applicability.

The algorithm proceeds as follows. For the estimation of components of slowness and polarization vectors various approaches can be used. One way is the parametric inversion approach proposed by Esmersoy (1990) for two downgoing wavefields and extended to N wavefields by Leaney (1990). Given slowness and polarization data we find the isotropic reference medium from which we seek the weakly anisotropic medium and construct rays in this medium connecting sources and the receiver. Information about these rays and about components of slowness and polarization vectors is used for the formation of an overdetermined system of linear equations at each receiver, from which the WA parameters can be determined.

Formulae

For the determination of WA parameters we use Eq.(22) of Zheng and Pšenčík (2001):

$$(\alpha^2-\beta^2)^{-1}(B_{13}X^{(2)}-B_{23}X^{(1)})-\frac{1}{2}\alpha^{-2}B_{33}\eta =$$
$$(X^{(2)}e_i^{(1)}-X^{(1)}e_i^{(2)})g_i+\alpha\Delta\eta(X^{(1)}e_3^{(2)}-X^{(2)}e_3^{(1)}+\alpha\eta^2). \quad (1)$$

The symbols α and β are the P- and S-wave velocities of the reference medium, the symbols g_i the polarization vectors. The vectors $e_i^{(1)}$, $e_i^{(2)}$ are two mutually perpendicular unit vectors, perpendicular to the vector $e_i^{(3)}$, the phase normal of the P wave in the reference isotropic medium. The symbols B_{mn} denote elements of the *weak anisotropy* matrix,

$$B_{mn}=\Delta a_{ijkl}e_i^{(m)}e_j^{(3)}e_l^{(3)}e_k^{(n)}, \quad (2)$$

in which Δa_{ijkl} is the deviation of elastic parameters of the anisotropic medium from elastic parameters of the isotropic reference medium. The elements B_{13}, B_{23} and B_{33} can be expressed in terms of 15 WA parameters, see Zheng and Pšenčík (2001). The WA parameters are the sought unknowns in Eq.(1). The symbols $X^{(K)}$, $(K=1,2)$ are defined as

$$X^{(K)}=\xi e_2^{(K)}-\zeta e_1^{(K)}. \quad (3)$$

The symbols ξ, ζ and η denote sizes of radial, transverse and vertical components of the slowness vector in the reference medium, $\Delta\eta$ is the deviation of the vertical component of the observed slowness vector from the reference medium.

Synthetic test

For this test we use the VSP configuration shown schematically in Fig. 1. The model is confined in a model box whose horizontal dimensions are 10 × 10 km. The borehole is situated in the center of the box. The model is laterally inhomogeneous and of TI symmetry. The qP-wave anisotropy is about 8%. The elastic parameters are chosen so that the symmetry axis at the horizontal top surface (the depth of 0 km) is inclined by 10^o from horizontal and rotated by 25^o from the x-axis. The axis of symmetry at the bottom surface is horizontal and parallel with the x-axis. The bottom surface is an inclined plane. Between the top and the bottom surfaces the elastic parameters are determined by linear interpolation. The inclined bottom surface of the TI model introduces lateral inhomogeneity into the model.

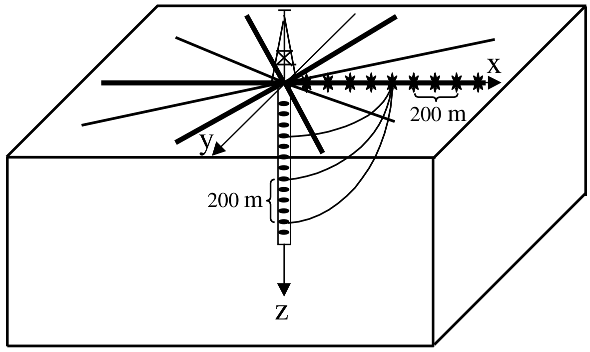

Figure 1: Multi-azimuthal multiple-source offset VSP experiment configuration: 5 radial surface profiles (the 3-profile experiment by thick lines) with 9 shots to each side of the borehole, with 0.1 km separation; 13 three-component receivers in the borehole with 0.05 km separation.

We consider 3 or 5 surface profiles with their center at the mouth of the borehole, see Fig. 1. The 3 profiles are distributed uniformly with the step of 60^o (0^o, 60^o and 120^o), measured from the positive x-axis. The 5 profiles are associated with angles 0^o, 30^o, 60^o, 120^o and 150^o. Each profile contains 9 sources at each side of the borehole, starting at 0.1 km from the mouth of the borehole and separated by 0.1 km. The vertical borehole contains 13 three-component receivers starting at a depth of 0.1 km and separated by 0.05 km. The free surface effects are neglected throughout the experiment. For the 3 profile experiment 54 equations (1) are constructed to solve for the 15 WA parameters. For the 5 profile experiment there are 90 equations.

"Observed" synthetic seismograms were generated using the program package ANRAY (Gajewski and Pšenčík, 1990). From the seismograms, arrival times were picked and particle motion diagrams were constructed. Picked travel times from five neighbouring receivers were used to determine the best fitting hyperbola, from which the partial derivative of the travel time with respect to the $z-$coordinate was found. Polarization vectors, g_i, are computed from least square fits to the observed particle motions of the first arrivals.

The observed travel times were used to determine an approximate 1-D distribution of the P-wave velocity α. The S-wave velocity β was taken as $\beta=\alpha/\sqrt{3}$. Then the rays from the source to individual receivers were traced in the reference medium. Normalized slowness vectors at receivers yielded the phase normal $e_i^{(3)}$, with respect to which, the vectors $e_i^{(1)}$ and $e_i^{(2)}$ were defined.

For solving the resulting system of equations (1), the SVD method was used. In Fig. 2, equal area plots of isolines of the qP-wave phase velocity obtained from inversion (thin lines) and from the true model (thick lines) are compared. Solid circles denote a longitudinal direction which coincides with the symmetry axis in the true model. The open circles indicate longitudinal direction obtained from inversion. The left column in Fig. 2 shows results for 3 profiles, the right column for 5 profiles. In each column, results for the receivers 4, 7 and 10 situated at depths z=0.25, 0.4 and 0.55 km are shown.

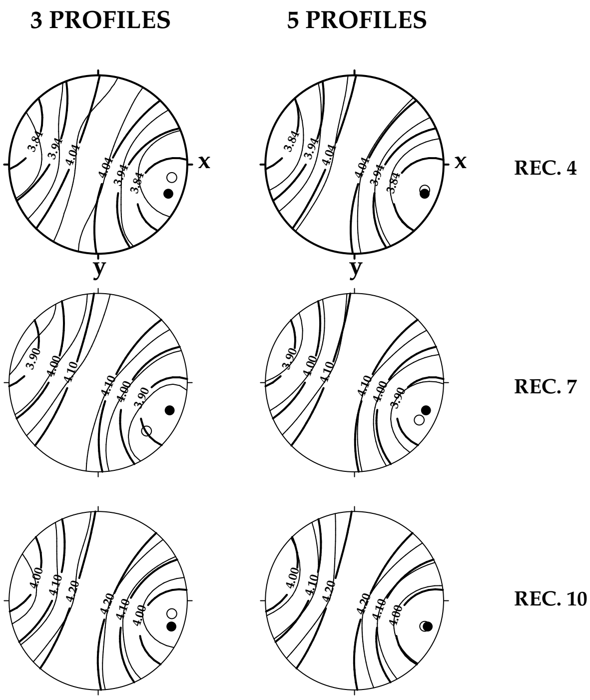

Figure 2: Equal area plots of phase velocity obtained from inversion (thin lines) and for the true model (thick lines). Solid and open circles denote the true and inverted longitudinal directions.

We can see that the thin curves in Fig. 2 have a more complicated form than the thick curves, which correspond to the TI model. This means that the inverted model no longer possesses TI symmetry. As expected, the fit between true and inverted phase velocities is better for 5 profiles. The best overall fit is observed for receiver 4. Isolines of the true and inverted model fit each other well not only in the centre of the circle, i.e., for vertical direction, but also at the border of the circle, i.e., for horizontal directions. This picture changes when moving to deeper receivers. For the receiver 10, a good fit can be observed at the centre of the circle. Towards the border of the circle the misfit increases. This indicates insufficient illumination of receivers with increasing depth. Longitudinal directions are obviously better determined for 5 profiles.

Field data

We apply the above inversion scheme to the walkaway VSP data of Horne and Leaney (2000). The main difference with the previous synthetic test is that we deal now with a single profile. We concentrate on direct and reflected qP waves only. The waves are generated by 228 sources distributed along a single profile on the Earth's surface. The sources extend from -2.5 to 2.5 km with, approximately, 25m spacing. We have at our disposal only vertical components of slowness and polarization vectors. No assumptions are made about the remaining components of the slowness vector. The polarization vector is assumed to be confined to the vertical plane containing the profile and the borehole. Because we have available data from a single profile only, we can recover only 5 of 15 WA parameters. For 228 sources, we have a system of 228 equations if only downgoing qP wave is considered and 456 equations if both downgoing and upgoing qP waves are considered. We solve these equations for the above 5 WA parameters.

Fig. 3 shows a plot of scaled vertical component of the slowness vector versus the vertical component of the polarization vector g_z. The quantity g_z is maximum when the polarization vector is vertical. To the sides, g_z decreases. The left-hand side corresponds to the positive, the right-hand side to the negative branch of the profile. The vertical coordinate is specified by the quantity αp_z, which specifies approximately the vertical component n_z of the phase normal. The data and the thin curve in the upper frame correspond to downgoing waves only. The data and the thin curve in the lower frame correspond to upgoing waves only. Remaining curves are the same in both frames and show results of inversion considering all qP waves (thick curve) and results of Horne and Leaney (2000) for slowness (slow) and slowness plus polarization (slow+pol) inversion. The dotted curve corresponds to the reference isotropic medium. Although only qP-waves were used, results are similar to results of Horne and Leaney (2000), which were obtained after sampling 10000 models.

Comparison of vertical sections of phase velocity reconstructed from results of Horne and Leaney (2000) and from the solution of system of equations

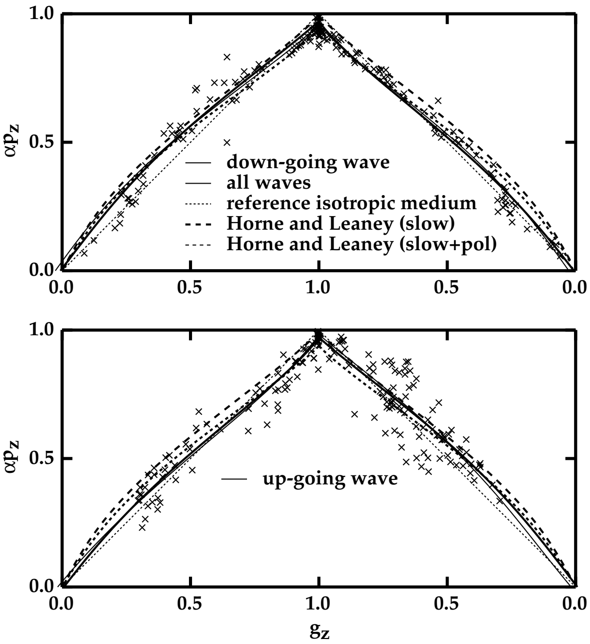

Figure 3: Scaled vertical components of slowness vector versus vertical component of the polarization vector. Crosses denote observed data.

(1) seems to indicate stronger anisotropy (about 14% and 19%) for the former inversion and only about 8% for the latter one, which also seems to indicate an inclined axis of symmetry. We are now analyzing if the differences are significant and if yes, whether they are caused by neglecting qS waves.

Conclusions

Techniques based on the first-order perturbation theory can be used for inversion of components of slowness and polarization vectors from multi-azimuthal or single profile walkaway VSPs for WA parameters. Consequently, the WA parameters can be used for the approximate construction of a phase velocity surface. Tests on synthetic and field data seem to indicate that the technique, although approximate, may yield good quality results.

In these tests, we assumed that the studied medium is generally anisotropic. If there are indications of higher symmetry, it is easy to specify the technique for such a higher symmetry anisotropy, see Zheng and Pšenčík (2001). Generalization for qS waves is under preparation.

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