Calculation of multivalued ray–theory travel times in 3D -
- interpolation within individual ray cells

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Summary

Recently there is a number of ray modelling methods based on decomposition of the model volume into ray cells and in further interpolation within the cells. This paper addresses the problem of interpolation within single ray cell. We propose bilinear and bicubic interpolation schemes, which should be applicable in any modelling method based on interpolation within ray cells. The bicubic interpolation is designed for travel times and the bilinear interpolation for other quantities.

1 Introduction

3-D ray modelling, especially the computation of ray-theory travel times at nodes of dense grids of points becomes to be quite important in recent years. The possibility to compute the travel times is of principal importance in imaging of depth structures, seismic tomography or hypocenter determination techniques. Several methods based on decomposition of the model volume into ray cells, and on further interpolation within individual cells, were introduced. Let us mention, e.g., the *wavefront construction method* by Vinje *et al.* (1996a, 1996b), the *recursive method* by Moser and Pajchel (1997), and the *controlled initial–value ray tracing method* by Bulant (1997).

Whereas the above mentioned methods differ in the manner of the decomposition of the model volume into ray cells, they all contain some procedure of interpolation of computed quantities to the receivers (usually nodes of a receiver grid) located within a single ray cell. We propose bilinear and bicubic interpolation schemes. The bilinear interpolation is easy and robust. It incorporates both the decision, whether a receiver lies in the ray cell, and the interpolation of quantities computed along the rays to the receiver. The scheme uses only the values of quantities at the vertices of the ray cell. Thus the scheme should be applicable in any modelling method based on interpolation within ray cells. If the partial derivatives are known in addition to the functional values at the vertices of the ray cell, the bicubic interpolation scheme may be used to increase the accuracy of the interpolation. The bicubic interpolation is designed for travel times and the bilinear interpolation for other quantities.

2 Determination of a ray cell

By determination of a ray cell we understand the decision, whether an arbitrary point lies within the ray cell, or not. Assume a ray cell defined by six points $\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3, \mathbf{C}_1, \mathbf{C}_2, \mathbf{C}_3$ on three rays, where points $\mathbf{B}_i = ({B_i}^1, {B_i}^2, {B_i}^3)^T$ are at the bottom, and points $\mathbf{C}_i$ at the top of the ray cell, see figure 1. We introduce a local coordinate system parametrized by parameters w_1, w_2, w_B. The coordinates of arbitrary point $\mathbf{X}$ are then given as

$$\mathbf{X} = \mathbf{A}_3 + w_1(\mathbf{A}_1 - \mathbf{A}_3) + w_2(\mathbf{A}_2 - \mathbf{A}_3) \tag{1}$$

$$\text{where} \qquad \mathbf{A}_i = \mathbf{C}_i + w_B(\mathbf{B}_i - \mathbf{C}_i),\ i = 1, 2, 3. \tag{2}$$

Equations (1) and (2) may also be expressed as

$$\mathbf{X} = w_B w_i \mathbf{B}_i + w_C w_i \mathbf{C}_i \tag{3}$$

with $w_C = 1 - w_B, \quad w_3 = 1 - w_1 - w_2$. Equation (1) is the parametric description of planar triangle $\mathbf{A}_1\mathbf{A}_2\mathbf{A}_3$. Equations (2) are parametric descriptions of straight-line segments $\mathbf{B}_1\mathbf{C}_1$, $\mathbf{B}_2\mathbf{C}_2$ and $\mathbf{B}_3\mathbf{C}_3$, approximating the ray segments. Note that only in the homogeneous parts of the model the straight segments lie on the exact rays.

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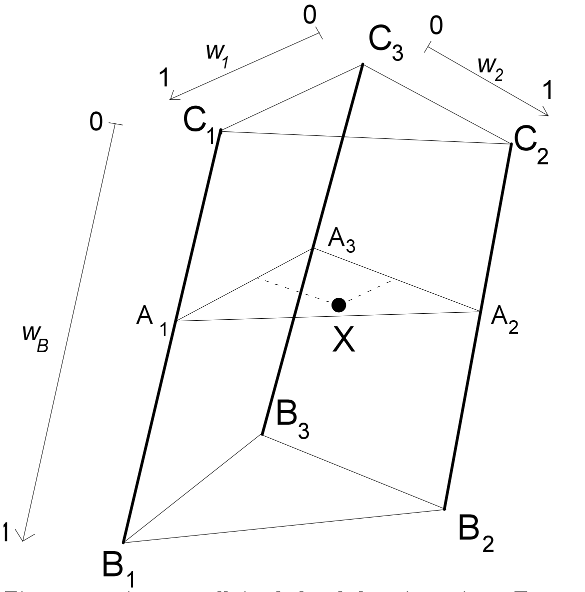

Figure 1: A ray cell is defined by six points $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{C}_3$, situated on three rays (bold lines). Point $\mathbf{X}$ is examined, whether it is located within the ray cell. Its position may be expressed using the local coordinate system parametrized by parameters w_1, w_2, w_B. If all of the quantities w_1,w_2,w_B,$1-w_1-w_2$ are in range $<0,1>$, point $\mathbf{X}$ lies within the ray cell.

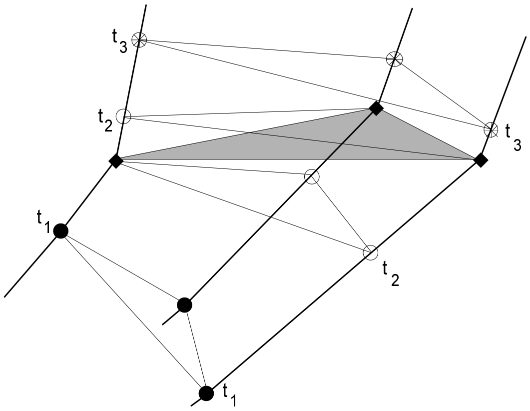

Figure 2. A ray tube transmits through a structural interface. Points on the rays forming the ray tube at times t_1, t_2, t_3 are denoted by circles. Points of transmission of the rays through the structural interface are marked by diamonds.

There are four ray cells shown on the plot. We can see one regular ray cell formed by six points, then two degenerate cells formed by five and four points, and again a regular ray cell. The bottom of the first ray cell and the top of the fourth cell are formed by triangles which approximate wavefronts at travel-time levels t_1 and t_3. The shaded triangle, which is the top of the second cell and the bottom of the third cell, approximates the structural interface.

If point $\mathbf{X}$ lies within the ray cell, all values w_1, w_2, w_3, w_B, w_C are within range $<0,1>$. Thus, for each receiver, which might be located within the selected ray cell, we compute its local coordinates w_1, w_2, w_B and quantities w_3, w_C. If all of them display values from range $<0,1>$, point $\mathbf{X}$ is within the ray cell, and the values of all required quantities will be determined by interpolation within the ray cell.

If the ray cell touches a caustic, two or three (in 3-D) different sets of values w_1, w_2, w_B may be found within range $<0,1>$ for the same point $\mathbf{X}$. Different coordinates w_1, w_2, w_B of the same point $\mathbf{X}$ correspond to different ray coordinates $\gamma_1, \gamma_2, \gamma_3$, and the interpolation should be performed for each set of coordinates w_1, w_2, w_B.

2.1 Computation of w_B

First we compute the value of local coordinate w_B. Point $\mathbf{X}$ must lie in the plane determined by points $\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3$. Thus the volume of the parallelepiped defined by vectors $\mathbf{A}_1-\mathbf{X}$, $\mathbf{A}_2-\mathbf{X}$, $\mathbf{A}_3-\mathbf{X}$ should be equal to zero. According to equation (2)

$$\mathbf{A}_i - \mathbf{X} = \mathbf{C}_i - \mathbf{X} + w_B(\mathbf{B}_i - \mathbf{C}_i), \;\; i = 1,2,3. \tag{4}$$

We thus arrive at the cubic equation for parameter w_B in the form of

$$\det(\widehat{\mathbf{Z}} + w_B\widehat{\mathbf{Y}}) = a{w_B}^3 + b{w_B}^2 + cw_B + d = 0, \tag{5}$$

where $\widehat{\mathbf{Z}}$ is the 3×3 matrix composed of vectors $\mathbf{C}_i - \mathbf{X}$, $i = 1,2,3$, and $\widehat{\mathbf{Y}}$ is the 3×3 matrix composed of vectors $\mathbf{B}_i - \mathbf{C}_i$, $i = 1,2,3$.

Cubic term a equals zero if one of the vectors $\mathbf{B}_i-\mathbf{C}_i$ forming the ray tube is linearly dependent on the other two vectors (including the case in which the vector equals zero). Quadratic term b equals zero if two of the vectors $\mathbf{B}_i - \mathbf{C}_i$ are linearly dependent on the third. Thus for degenerate ray cells, when one or two of points $\mathbf{B}_i$ coincide with the corresponding points $\mathbf{C}_i$, cubic equation (5) reduces to a quadratic or linear equation, but the scheme remains valid. This usually happens when a ray tube interacts with a structural interface, see figure 2.

2.2 Computation of w_1, w_2

Once we have computed w_B from interval $<0,1>$, we proceed to the computation of w_1, w_2, w_3. We calculate the coordinates of points $\mathbf{A}_i$ according to equation (2). Then, considering equation (1), we immediately get a system of linear equations for w_1, w_2 and w_3,

$$\widehat{\mathbf{A}}\mathbf{W} = \mathbf{X} \tag{6}$$

where $\widehat{\mathbf{A}}$ is a 3×3 matrix composed of vectors $\mathbf{A}_i$, $i = 1,2,3$, and $\mathbf{W} = (w_1, w_2, w_3)^T$.

3 Interpolation within a ray cell

3.1 Bilinear interpolation

The simplest interpolation to fit the functional values at six points $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{C}_3$ is the bilinear interpolation with respect to w_B and (w_1, w_2). Linear interpolation

$$\tau^{\mathbf{A}_k} = w_B \tau^{\mathbf{B}_k} + w_C \tau^{\mathbf{C}_k} \tag{7}$$

with respect to w_B is used to obtain the values $\tau^{\mathbf{A}_k}$ of an interpolated quantity τ at points $\mathbf{A}_k$ from known values $\tau^{\mathbf{B}_k}$ and $\tau^{\mathbf{C}_k}$ at the vertices of the ray cell. Linear interpolation

$$\tau^{\mathbf{X}} = w_1 \tau^{\mathbf{A}_1} + w_2 \tau^{\mathbf{A}_2} + w_3 \tau^{\mathbf{A}_3} \tag{8}$$

with respect to (w_1, w_2) is then used to obtain the values at point $\mathbf{X}$. This interpolation fits the functional values at the six vertices of the ray cell. Since the values at each cell face are dependent on the values at the vertices of the face only, the interpolated values are continuous across the cell faces. This interpolation seems convenient in interpolating all the quantities resulting from ray tracing, except for the travel time which should be interpolated more accurately. See figure 3.

3.2 Bicubic interpolation

Assume that partial derivatives $\frac{\partial \tau}{\partial x^i}$ are known in addition to the functional values of τ at six vertices $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{C}_3$ of the ray cell. We may then wish to use the bicubic interpolation with respect to w_B and (w_1, w_2) to fit both the functional values and the first partial derivatives at the six vertices.

For the formulae for the bicubic interpolation refer to Bulant and Klimeš (1998). Here we will only mention some properties of the bicubic interpolation.

The interpolation fits the values and the first spatial partial derivatives at the six vertices of the ray cell. The functional values at each cell face depend on the functional values and derivatives at the vertices of the face only. The interpolated values are thus continuous across the cell faces. The gradient of the interpolated function is continuous at the triangular cell faces and along the cell edges, but may be discontinuous at the tetragonal cell faces separating ray tubes. The interpolation fits linear and quadratic functions exactly.

Bilinear interpolation. Bicubic interpolation.

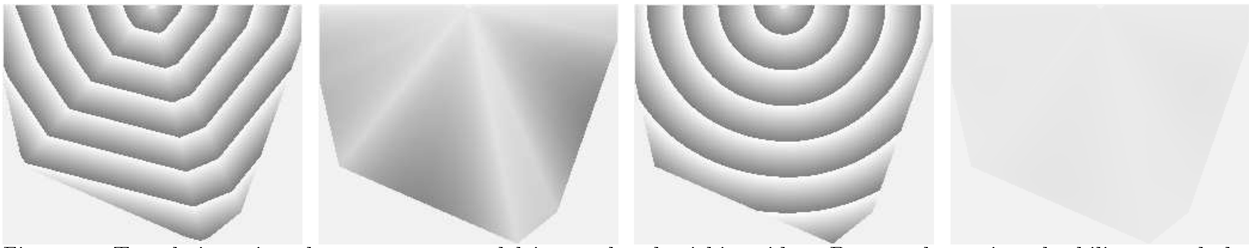

Figure 3: Travel times in a homogeneous model interpolated within wide 3-D ray tubes using the bilinear and the bicubic interpolations and their differences from the exact solution. One grey-scale period corresponds to 0.2 s, The maximum travel time is about 2.7 s. The maximum difference is about 0.308 s for the bilinear interpolation, and about 0.017 s for the bicubic interpolation.

4 Numerical example

A computation in synthetic 3-D model 98 is presented here as an example of interpolation of ray-theory travel times on a dense rectangular grid. Model 98 is composed of three layers separated by two interfaces. The upper interface is curved, the lower one is planar. The inclusion in the middle layer is approximately spherical. The first layer represents water with constant P-wave velocity and zero S-wave velocity. In other model blocks, the P-wave velocity has a vertical gradient and smaller lateral gradients, the S-wave velocity is a linear function of the P-wave velocity. See figure 4.

The ray-theory travel times are computed for a vertical 2-D grid of 251 times 201 points. The point source is located at the same plane as the receivers at a depth of 0.001 km. The elementary wave refracted at each interface is considered. The computed multi-valued travel-time map may be converted into the single-valued maps sorted according to the arrival time (i.e. first arrival at the gridpoints, second arrival at the gridpoints, ...), see figure 5.

Conclusions

We propose a scheme for interpolation within single ray cell. Using the scheme, all the quantities computed along rays may be interpolated. The scheme requires the values of the interpolated

P-wave. S-wave.

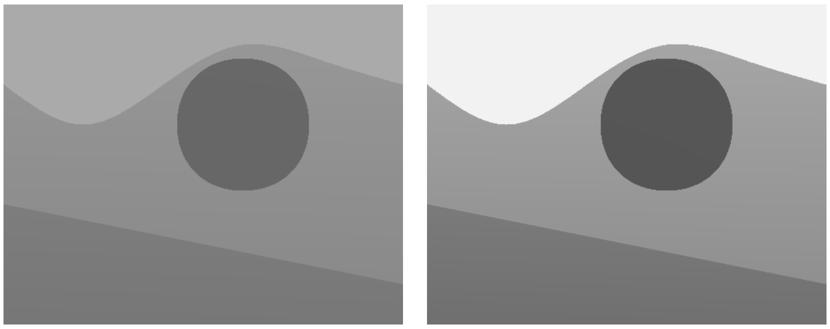

Figure 4: P-wave and S-wave velocity cross-sections through the middle of model 98. The P-wave velocity is lowest at the top water layer and highest within the spherical body. The S-wave velocity equals zero in the first layer and it is a linear function of the P-wave velocity in the other layers.

First arrival. Second arrival. Third arrival.

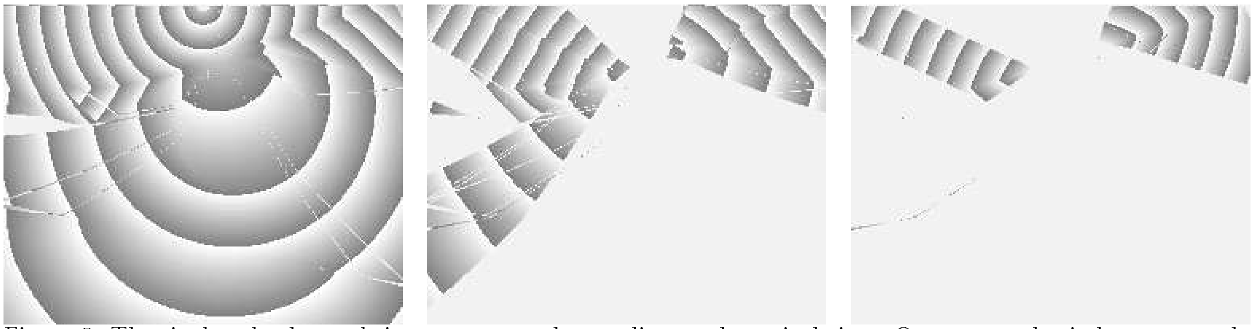

Figure 5: The single-valued travel-time maps sorted according to the arrival time. One gray scale circle corresponds to 0.2 s.

quantities only in vertices of the ray cell. It incorporates both the decision, whether a receiver lies in the ray cell, and the interpolation to the receiver. The scheme is applicable to all ray cells formed by six, five or four points. Further investigation should concentrate on numerical tests of the scheme and on comparisons with other schemes.

The numerical example presented shows the applicability of the method to a 3-D model with interfaces.

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