Estimating the correlation function of a self-affine random medium

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Summary

The medium covariance function is of principal importance in refraction travel–time tomographic inversion, especially when estimating the accuracy of the seismic model, its relation to the geological structure, or the covariance matrix describing the statistics of synthetic travel times. The medium correlation function for the travel–time tomography should be obtained from travel times.

Since a geological structure contains heterogeneities of all sizes, very similar on various scales, a self–affine random medium is a mathematical model very suitable for approximating the statistics of a geological structure. A particular class of self–affine random media, composed of a heterogeneous mean value and a stationary self–affine random function, is considered. The self–affine random function is assumed to be realized in terms of a white noise filtered by the power–law spectral filter of amplitude proportional to a reasonable power of the wavenumber. The corresponding power–law medium correlation function depends on two parameters: the Hurst exponent and the reference standard deviation.

The corresponding geometrical travel–time covariances are derived. The geometrical travel–time variances are proportional to the power of ray lengths. A method designed to estimate the parameters of the medium correlation function using field travel times is proposed, and applied to data from the Western Bohemia region.

The determination of the Hurst exponent from field travel times is very difficult and sensitive to numerical parameters selected for the inversion. The medium correlation functions with the values of the Hurst exponent like $N = -0.1$ or $N = -0.2$ are equally acceptable to statistically describe the travel times measured in the Western Bohemia region. On the other hand, for the fixed Hurst exponent, the determination of the reference standard deviation of slowness is easy and reliable.

Keywords

Travel times, self–affine random medium, correlation function, scaling geology, fractal geology, inversion.

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1 Introduction

In order to estimate the relation of a seismic model to the geological structure and, in consequence, to estimate the relation of the synthetic quantities, calculated in the seismic model, to reality, it is important to have an estimate of the medium covariance function. The medium covariance function is of principal importance in refraction travel–time tomographic inversion (Tarantola 1987; Maurer, Holliger & Boerner 1998; Klimeš 2002a), especially when estimating the accuracy of the seismic model, its relation to the geological structure, or the covariance matrix describing the statistics of synthetic travel times.

In a self–affine random medium, the material parameters may be scaled simultaneously with scaling the spatial dimensions in such a way that the statistical properties remain unchanged by the scaling. Since a geological structure contains heterogeneities of all sizes, very similar on various scales, a self–affine random medium is a mathematical model very suitable for approximating the statistics of a geological structure (Pilkington & Todoeschuck 1990). We thus assume the slowness distribution in the geological structure to be a particular representation of the self–affine random medium in this paper. For an overview and brief discussion of other types of random media used in geophysics, refer to Klimeš (2002b).

Assuming a stationary (statistically homogeneous) medium, the medium covariance function may be expressed in terms of the medium correlation function. In Section 2, a particular class of self–affine random media, composed of a heterogeneous mean value and a stationary random function, is considered, and the corresponding medium correlation function is derived. The medium correlation function depends on two parameters: the Hurst exponent and the corresponding reference standard deviation.

Section 3 is devoted to the dependence of the a priori geometrical covariance matrix of field travel times (Tarantola 1987) on the medium covariance function. The a priori geometrical covariance matrix of field travel times describes the deviations of travel times from the mean travel–time curve. The deviations are caused by the heterogeneities, especially the lateral ones.

The scales of applicability of the empirical medium correlation function are limited by the inner and outer scales of the correlation function (Mandelbrot 1977), which depend on the nature of experimental data used to estimate the correlation function. For example, the correlation function determined from well logs (e.g., Wu 1982; Wu, Xu & Li 1994; Holliger 1997; Goff & Holliger 1999), attenuation or scattering (e.g., Wu 1982; Wu & Aki 1985b, 1988; Wu 1989a, 1989b; Shapiro 1992; Shapiro & Kneib 1993; Kneib & Shapiro 1995; Shapiro, Schwarz & Gold 1996; Sato & Fehler 1998; Müller & Shapiro 2001) has usually too small outer scale to be applied in the travel–time tomography. The medium correlation function for the travel–time tomography should be obtained from travel times. Application of the correlation functions estimated from quite different data, e.g., from the surface geological maps (Holliger & Levander 1994; Levander et al. 1994) can hardly be statistically justified.

The medium correlation function derived in Section 2 depends on two parameters: the Hurst exponent and the corresponding reference standard deviation. Section 4 is devoted to the method for estimation of these two parameters, essential for travel–time tomography, from field travel times. The determination of the reference standard deviation for the fixed Hurst exponent is easy, and there is a plenty of geophysical papers doing it, e.g., Mintzer (1953), Aki (1973), Berteussen et al. (1974), Capon (1974), Gudmundsson, Davies & Clayton (1990), Wu & Xie (1991), or Roth (1997). The problem is to estimate the Hurst exponent on the regional scales. Flatté & Wu (1988) and Wu & Flatté (1990) demonstrated that decreasing the Hurst exponent from $N = \frac{1}{2}$ to $N = 0$ improves the fit of the arrival–time and amplitude fluctuations at the NORSAR. They did not continue with the negative Hurst exponents, but found the combination of two layers with the Hurst exponents of $N = -\frac{3}{2}$ and $N = \frac{1}{2}$ yielding better fit than $N = 0$. Note that their combination of the two correlation functions can reasonably be fit within the relevant inner and outer cutoff scales by a single correlation function with the negative Hurst exponent $-\frac{1}{2} < N < 0$. On the other hand, Wu & Aki (1985a) used the seismic coda waves to determine the Hurst exponents from $N = 0.23$ for shallow events to $N = 0.5$ for deep events.

The method proposed in Section 4 is applied to field data from the Western Bohemia region in Section 5 to demonstrate the possibilities of estimating the medium correlation function.

The reader should be aware that the Einstein summation does not apply to the equations anywhere in this paper.

2 Correlation function of a self–affine random medium

Random medium $u(\mathbf{x})$ is *stationary* if mean value $\langle u(\mathbf{x})\rangle$ is constant and the *medium covariance function*

$$C(\mathbf{x}_1,\mathbf{x}_2) = \langle u(\mathbf{x}_1)\, u(\mathbf{x}_2)\rangle - \langle u(\mathbf{x}_1)\rangle \, \langle u(\mathbf{x}_2)\rangle \tag{1}$$

depends only on distance $\mathbf{x}_1 - \mathbf{x}_2$,

$$C(\mathbf{x}_1,\mathbf{x}_2) = c(\mathbf{x}_1 - \mathbf{x}_2) \quad , \tag{2}$$

where $c(\mathbf{x})$ is the *medium correlation function* (Tarantola 1987).

Realizations of a statistically isotropic stationary *self–affine random medium*, uniformly scalable over all lengths, may be obtained by multiplying the Fourier transform of the realizations of a stationary *white noise* by spectral filter

$$\widehat{F}(\mathbf{k}) = \kappa k^{-\frac{d}{2}-N} \tag{3}$$

with

$$k = (\mathbf{k}^{\mathrm{T}}\mathbf{k})^{\frac{1}{2}} \quad , \tag{4}$$

and inversely Fourier transforming the products back into the space domain. Here d is the Euclidean dimension of the space, $d = 3$ in 3-D, constant N is called the *Hurst exponent*, and κ is a constant proportional to the reference standard deviation of resulting self–affine random functions.

Assuming that the white noise has a unit standard deviation, the medium correlation function is then

$$c(\mathbf{x}) = (2\pi)^{-d} \int_{-\infty}^{+\infty} \mathrm{d}k_1 \cos(k_1 x_1) ... \int_{-\infty}^{+\infty} \mathrm{d}k_d \cos(k_d x_d)\; [\widehat{F}(\mathbf{k})]^2 \quad . \tag{5}$$

Since spectral filter (3) is rotationally symmetric, we may rotate, before integrating, the k_1 axis into the direction of vector $\mathbf{x}$ to arrive at

$$c(\mathbf{x}) = \kappa^2(2\pi)^{-d} \int_{-\infty}^{+\infty} \mathrm{d}k_1 \cos(k_1 x) \int_{-\infty}^{+\infty} \mathrm{d}k_2 ... \int_{-\infty}^{+\infty} \mathrm{d}k_d \quad k^{-d-2N} \quad , \tag{6}$$

where

$$x = (\mathbf{x}^{\mathrm{T}}\mathbf{x})^{\frac{1}{2}} \quad . \tag{7}$$

For $d > 1$, we introduce distance r from the k_1 axis,

$$r = [(k_2)^2 + ... + (k_d)^2]^{\frac{1}{2}} \quad , \tag{8}$$

and recall the equation

$$V_d(r) = \frac{\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2}+1)} \quad r^d \tag{9}$$

for the volume of the d–dimensional sphere of radius r. Differentiating the volume with respect to the radius, we get the surface of the sphere,

$$S_d(r) = \frac{d \ \pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2}+1)} \quad r^{d-1} = \frac{2 \ \pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})} \quad r^{d-1} \quad . \tag{10}$$

Integrating (6) over the surface of the $(d-1)$–dimensional sphere of radius r in subspace $k_1 = constant$, and taking into account that the integrands are constant along the surface, we arrive at

$$c(\mathbf{x}) = \kappa^2 \ (2\pi)^{-d} \frac{2 \ \pi^{\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})} \int_{-\infty}^{+\infty} \mathrm{d}k_1 \cos(k_1 x) \int_{0}^{+\infty} \mathrm{d}r \ r^{d-2} [(r)^2 + (k_1)^2]^{-\frac{d}{2}-N} \quad . \tag{11}$$

For $0 < d-1 < d+2N$, the integral with respect to r may be calculated,

$$c(\mathbf{x}) = \kappa^2 \ (2\pi)^{-d} \ \pi^{\frac{d-1}{2}} \ \frac{\Gamma(\frac{1}{2}+N)}{\Gamma(\frac{d}{2}+N)} \int_{-\infty}^{+\infty} \mathrm{d}k_1 \cos(k_1 x) \ |k_1|^{-1-2N} \quad . \tag{12}$$

For $d = 1$, equation (6) reads

$$c(\mathbf{x}) = \kappa^2(2\pi)^{-1} \int_{-\infty}^{+\infty} \mathrm{d}k_1 \cos(k_1 x) \ |k_1|^{-d-2N} \quad . \tag{13}$$

For $0 \leq d-1 < d+2N$, equations (12) and (13) take the common form of

$$c(\mathbf{x}) = \kappa^2 2^{-d} \pi^{-\frac{d+1}{2}} \ \frac{\Gamma(\frac{1}{2}+N)}{\Gamma(\frac{d}{2}+N)} \int_{-\infty}^{+\infty} \mathrm{d}k_1 \cos(k_1 x) \ |k_1|^{-1-2N} \quad . \tag{14}$$

For $-\frac{1}{2} < N < 0$, the integral with respect to k_1 comes out as

$$c(\mathbf{x}) = \kappa^2 2^{-d} \pi^{-\frac{d+1}{2}} \frac{\Gamma(N+\frac{1}{2})}{\Gamma(N+\frac{d}{2})} \, 2\,\Gamma(-2N) \sin[\pi(N+\tfrac{1}{2})] \quad x^{2N} \quad . \tag{15}$$

We define *reference standard deviation* σ, related to arbitrarily selected *reference distance* L, by equation

$$\sigma^2 = \kappa^2 L^{2N} 2^{-d+1} \pi^{-\frac{d+1}{2}} \frac{\Gamma(N+\frac{1}{2})}{\Gamma(N+\frac{d}{2})} \; \Gamma(-2N) \cos(-\pi N) \quad . \tag{16}$$

Reference distance L is introduced here to define σ consistently with respect to physical units. The medium covariance function (2) may then be expressed in the form of

$$\boxed{C(\mathbf{x}_1,\mathbf{x}_2) = \sigma^2 \; \left(\frac{|\mathbf{x}_1 - \mathbf{x}_2|}{L}\right)^{2N} \quad , \qquad (17)}$$

where the Hurst exponent satisfies

$$-\tfrac{1}{2} < N < 0 \quad . \tag{18}$$

In 3-D space, $d=3$, which is of particular interest in travel–time tomography, equation (16) reads

$$\sigma^2 = \kappa^2 L^{2N} 2^{-1} \pi^{-2} \frac{\Gamma(-2N)}{1+2N} \cos(-\pi N) \quad . \tag{19}$$

Note that the limiting case of Hurst exponent $N = -\frac{1}{2}$ and the cases of $-\frac{d}{2} < N < -\frac{1}{2}$ are somewhat unstable (see the above derivation) and their statistical properties resemble a *white noise* produced by $N = -\frac{d}{2}$ in (3), with the medium covariance function

$$C(\mathbf{x}_1,\mathbf{x}_2) = \kappa^2 \delta(\mathbf{x}_1 - \mathbf{x}_2) \quad . \tag{20}$$

Negative Hurst exponents $-\frac{1}{2} < N < 0$ are suitable to characterize the material parameters of the geological structures.

For Hurst exponents $N \geq 0$, the medium cannot be self–affine at all scales, but only at scales sufficiently smaller than a finite *correlation length*, which may be selected arbitrarily large. The limiting case of $N=0$ is called a *flicker noise* (Schottky 1926). *Fractional Brown noises* obtained for positive Hurst exponent $0 < N < 1$ (Addison 1997) may be divided into *antipersistent fractional Brown noises* with $0 < N < \frac{1}{2}$ (Mandelbrot 1977), *Brown noise* with $N = \frac{1}{2}$ (e.g., Mandelbrot 1977; Turcotte 1989; Addison 1997), and *persistent fractional Brown noises* with $\frac{1}{2} < N < 1$ (Mandelbrot 1977), called also *black noises* (Addison 1997). In spite of that, in 2-D space, $d=2$, Crossley & Jensen (1989) refer to the case of $N = -\frac{1}{2}$ as flicker noise and to the case of $N=0$ as brown noise. If the correlation length limits to infinity, all self–affine medium covariance functions with Hurst exponents $N \geq 0$ limit to covariance function

$$C(\mathbf{x}_1,\mathbf{x}_2) = \sigma^2 \tag{21}$$

of a *random homogeneous medium*, obtained as the limiting case of (17) for $N=0$.

Hereinafter, we shall assume a statistically homogeneous medium covariance function, see (2), as in a stationary random medium, but allow for a *spatially variable mean value*. We shall assume that the medium covariance function is of isotropic power-law form (17).

3 A priori geometrical covariance matrix of travel times

Assuming the ray–theory linearization approach, field travel times may be expressed in the form of

$$T_I = \tau_I + \delta T_I \tag{22}$$

where

$$\tau_I = \int_0^{s_I} \mathrm{d}s \; u\,(\mathbf{x}(s)) \tag{23}$$

is the integral of the slowness $u(\mathbf{x})$ along the corresponding ray of length s_I, and δT_I is the error in determining field travel time T_I.

The *a priori geometrical covariance* (Tarantola 1987)

$$\Theta_{KL} = \left\langle (\tau_K - \tau_K^0)(\tau_L - \tau_L^0) \right\rangle \quad , \tag{24}$$

of the K^{th} and L^{th} travel times is then given by

$$\Theta_{KL} = \int_0^{s_K} \mathrm{d}s'_K \int_0^{s_L} \mathrm{d}s'_L \; C(\mathbf{x}(s'_K), \mathbf{x}(s'_L)) \tag{25}$$

(Chernov 1960), where the integration is performed along the corresponding rays of lengths s_K and s_L. Here τ_K^0 are the reference travel times corresponding to mean value $u^0(\mathbf{x}) = \langle u(\mathbf{x}) \rangle$ of the random slowness. Notice that, for medium covariance function (17), $\sigma^{-2}\Theta_{KL}$ is independent of σ and is determined by a single medium parameter, N.

The derivative of geometrical covariance Θ_{KL} with respect to N is then

$$\frac{\partial \Theta_{KL}}{\partial N} = \int_0^{s_K} \mathrm{d}s'_K \int_0^{s_L} \mathrm{d}s'_L \; C(\mathbf{x}(s'_K), \mathbf{x}(s'_L)) \;\; 2\ln\left(\frac{|\mathbf{x}(s'_K) - \mathbf{x}(s'_L)|}{L} \right) \tag{26}$$

and $\sigma^{-2}\frac{\partial \Theta_{KL}}{\partial N}$ is again independent of σ.

3.1 A priori geometrical variances of travel times

If we approximate the distance between ray points $\mathbf{x}(s_1)$ and $\mathbf{x}(s_2)$ of the same ray by

$$|\mathbf{x}(s_1) - \mathbf{x}(s_2)| \approx |s_1 - s_2| \tag{27}$$

(Bergmann 1946), equation (17) may be inserted into (25) to arrive at

$$\Theta_{KK} \approx \sigma^2 L^{-2N} \int_0^{s_K} \mathrm{d}s_1 \int_0^{s_K} \mathrm{d}s_2 |s_1 - s_2|^{2N} \quad . \tag{28}$$

For $-\frac{1}{2} < N$, the integrals may be calculated to read

$$\begin{aligned}\Theta_{KK} &\approx \sigma^2 L^{-2N} \int_0^{s_K} \mathrm{d}s_1 \left[\int_0^{s_1} \mathrm{d}s_2 \ (s_2)^{2N} + \int_0^{s_K - s_1} \mathrm{d}s_2 \ (s_2)^{2N} \right] \\ &= \sigma^2 L^{-2N} \int_0^{s_K} \mathrm{d}s_1 \left[\frac{(s_1)^{2N+1}}{2N+1} + \frac{(s_K - s_1)^{2N+1}}{2N+1} \right] \quad ,\end{aligned} \tag{29}$$

and finally

$$\Theta_{KK} \approx \frac{2 \ \sigma^2 \ L^2}{(2N+1)(2N+2)} \left(\frac{s_K}{L} \right)^{2N+2} \quad . \tag{30}$$

The derivative of variance Θ_{KK} with respect to N is then

$$\frac{\partial \Theta_{KK}}{\partial N} \approx \frac{2 \ \sigma^2 \ L^2}{(2N+1)(2N+2)} \left(\frac{s_K}{L} \right)^{2N+2} \ 2 \ \ln \left(\frac{s_K}{L} \right) \quad . \tag{31}$$

Unfortunately, off–diagonal elements $\Theta_{K \neq L}$ of the geometrical travel–time covariance matrix have to be calculated numerically.

4 Determination of the medium correlation function from the field travel times

4.1 Differences of the relative field travel times and their statistical moments

Let us study the mutual differences of the field travel times. Since the travel times are strongly dependent on hypocentral distances, it is possible to compare only the travel times T_K and T_L along the rays of similar lengths s_K and s_L. This restriction may, to some extent, be reduced if we relate the travel times to some reference travel–time curve

$$\tau^0 = \tau^0(s) \quad . \tag{32}$$

We may than compare the relative travel times T_K/τ^0_K and T_L/τ^0_L, where

$$\tau^0_I = \tau^0(s_I) \quad . \tag{33}$$

The differences of the relative travel times then depend on the local value of the reference travel–time curve in terms of a multiplication factor which has practically no influence on the statistics. The differences of the relative travel times are distorted especially by the error in the derivative of the reference travel–time curve multiplied by $|s_K - s_L|$. We assume that the local error in the derivative of the reference travel–time curve compared to the exact mean travel–time curve is locally negligible in intervals defined by

$$q\,T_L < T_K < T_L \tag{34}$$

with given parameter q, $0 \le q < 1$.

We now define the variances of the relative travel–time differences

$$D_{KL,KL} = \left\langle \left[\frac{T_K}{\tau^0_K} - \frac{T_L}{\tau^0_L} \right]^2 \right\rangle \quad , \tag{35}$$

and the fourth–order variances

$$D_{KL,KL,KL,KL} = \left\langle \left[\left(\frac{T_K}{\tau^0_K} - \frac{T_L}{\tau^0_L} \right)^2 - D_{KL,KL} \right]^2 \right\rangle = \left\langle \left(\frac{T_K}{\tau^0_K} - \frac{T_L}{\tau^0_L} \right)^4 \right\rangle - \left[D_{KL,KL} \right]^2 , \tag{36}$$

describing the standard deviations of the squared differences of the relative field travel times from variances (35).

We assume picking errors δT_I statistically independent of travel times τ_I. The variances

$$D_{KL,KL} = \left\langle \left[\frac{\tau_K + \delta T_K}{\tau^0_K} - \frac{\tau_L + \delta T_L}{\tau^0_L} \right]^2 \right\rangle \quad , \tag{37}$$

of the relative travel–time differences may then be expressed in terms of the first two statistical moments of the relative travel times τ_I/τ^0_I,

$$\theta_K = \left\langle \frac{\tau_K}{\tau^0_K} \right\rangle \quad , \qquad \theta_{KL} = \left\langle \frac{\tau_K}{\tau^0_K} \, \frac{\tau_L}{\tau^0_L} \right\rangle \quad , \tag{38}$$

and the first two statistical moments of the relative picking errors $\delta T_I/\tau_I^0$,

$$t_K = \left\langle \frac{\delta T_K}{\tau_K^0} \right\rangle \quad , \qquad t_{KL} = \left\langle \frac{\delta T_K}{\tau_K^0} \; \frac{\delta T_L}{\tau_L^0} \right\rangle \quad , \tag{39}$$

as

$$D_{KL,KL} = \theta_{KK} - 2\theta_{KL} + \theta_{LL} + t_{KK} - 2t_{KL} + t_{LL} + 2\left[\theta_K t_K - \theta_K t_L - \theta_L t_K + \theta_L t_L\right] \;. \tag{40}$$

We assume zero mean value of picking errors δT_K,

$$\langle \delta T_K \rangle = 0 \quad . \tag{41}$$

Then

$$t_K = 0 \tag{42}$$

and variances (40) become independent of the mean values θ_K of the reduced travel times,

$$D_{KL,KL} = \theta_{KK} - 2\theta_{KL} + \theta_{LL} + t_{KK} - 2t_{KL} + t_{LL} \quad . \tag{43}$$

We assume the *data covariance matrix*

$$\langle \delta T_K \; \delta T_L \rangle = T_{KL} \tag{44}$$

to be known, at least approximately. Inserting (38) with (24) for θ_{MN} and (39) with (44) for t_{MN} into (43), we see that variances (43) depend on parameters σ and N of the medium covariance function (17) through

$$D_{KL,KL} = D^0_{KL,KL} + \sigma^2 D^1_{KL,KL}(N) \tag{45}$$

with

$$D^0_{KL,KL} = \frac{T_{KK}}{\tau_K^0 \tau_K^0} - 2\frac{T_{KL}}{\tau_K^0 \tau_L^0} + \frac{T_{LL}}{\tau_L^0 \tau_L^0} \tag{46}$$

and

$$D^1_{KL,KL}(N) = \frac{\sigma^{-2}\Theta_{KK}}{\tau_K^0 \tau_K^0} - 2\frac{\sigma^{-2}\Theta_{KL}}{\tau_K^0 \tau_L^0} + \frac{\sigma^{-2}\Theta_{LL}}{\tau_L^0 \tau_L^0} \quad . \tag{47}$$

Values (46) are constants with respect to N and σ. Functions (47) of N are independent of σ.

To be able to approximate the fourth–order moments in (36) using the second–order moments, we assume Gaussian probability distributions for both the the self–affine random medium and the picking errors. If all probability distributions in the problem are Gaussian, the marginal probability distribution describing the relative travel–time differences is Gaussian too. If the probability distribution is Gaussian,

$$\left\langle \left(\frac{T_K}{\tau_K^0} - \frac{T_L}{\tau_L^0} \right)^4 \right\rangle = 3 \left\langle \left(\frac{T_K}{\tau_K^0} - \frac{T_L}{\tau_L^0} \right)^2 \right\rangle^2 \tag{48}$$

(Beran 1968; Goff & Jordan 1988), and equation (36) reads

$$D_{KL,KL,KL,KL} = 2\left[D_{KL,KL}\right]^2 \quad . \tag{49}$$

4.2 Objective function

We select the objective function in the form of

$$y = \left[\sum_{K,L} 1\right]^{-1} \sum_{K,L} \left[\left(\frac{T_K}{\tau_K^0} - \frac{T_L}{\tau_L^0}\right) - D_{KL,KL}\right]^2 \left[D_{KL,KL,KL,KL}\right]^{-1} \quad , \tag{50}$$

and minimize it with respect to parameters σ and N of the medium covariance function (17). Let us emphasize that the minimum has to be sought for constant fourth–order variances $D_{KL,KL,KL,KL}$.

Inserting (45), and (49) with constant $\sigma = \sigma_0$ and $N = N_0$, objective function (50) reads

$$y(\sigma, N) = \tfrac{1}{2}\left[\sum_{K,L} 1\right]^{-1}$$

$$\times \sum_{K,L} \left[\left(\frac{T_K}{\tau_K^0} - \frac{T_L}{\tau_L^0}\right)^2 - D^0_{KL,KL} - \sigma^2 D^1_{KL,KL}(N)\right]^2 \left[D^0_{KL,KL} + (\sigma_0)^2 D^1_{KL,KL}(N_0)\right]^{-2} \quad . \tag{51}$$

Here parameters σ_0 and N_0 are fixed during the minimization, but should be selected close to the final solution,

$$\sigma_0 \approx \sigma \quad , \qquad N_0 \approx N \quad . \tag{52}$$

Objective function (51) has its minimum with respect to σ for

$$\sigma^2(N) = \frac{F_1(N)}{F_2(N)} \tag{53}$$

with

$$F_0(N) = \sum_{K,L} \left[\left(\frac{T_K}{\tau_K^0} - \frac{T_L}{\tau_L^0}\right)^2 - D^0_{KL,KL}\right]^2 \left[D^0_{KL,KL} + (\sigma_0)^2 D^1_{KL,KL}(N_0)\right]^{-2} \quad , \tag{54}$$

$$F_1(N) = \sum_{K,L} \left[\left(\frac{T_K}{\tau_K^0} - \frac{T_L}{\tau_L^0}\right)^2 - D^0_{KL,KL}\right] D^1_{KL,KL}(N) \left[D^0_{KL,KL} + (\sigma_0)^2 D^1_{KL,KL}(N_0)\right]^{-2} \,, \tag{55}$$

and

$$F_2(N) = \sum_{K,L} \left[D^1_{KL,KL}(N)\right]^2 \left[D^0_{KL,KL} + (\sigma_0)^2 D^1_{KL,KL}(N_0)\right]^{-2} \quad . \tag{56}$$

The minimum value of the objective function with respect to σ is

$$y(N) = \tfrac{1}{2}\left[\sum_{K,L} 1\right]^{-1} \left[F_0(N) - \frac{[F_1(N)]^2}{F_2(N)}\right] \quad . \tag{57}$$

Since inaccurate field travel times may severely distort the estimated statistics of the geological structure, it is reasonable to restrict the summation only to travel times satisfying inequality

$$T_{KK} \le (\sigma_{\mathrm{err}})^2 \; \sigma^{-2} \Theta_{KK} \quad , \tag{58}$$

where σ_{err} is a given constant. The right–hand side of (58) has to be calculated at fixed $N = N_0$ in order not to alter the data set during the minimization of the objective function.

4.3 Minimization of the objective function

First we select reasonable values of constants q and $\sigma_{\rm err}$. Then we select the value of N_0. The corresponding value of σ_0 may be found iteratively: for an initial estimate of σ_0 we calculate new $\sigma_0 = \sigma(N_0)$ using (53) to get a better estimate, rapidly approaching the value of σ_0 consistent with the chosen value of N_0. For fixed N_0 and σ_0 we calculate the values of the objective function (57) at different values of N in order to find the minimum. If the values of N depart from N_0, we should select new N_0 and determine new σ_0.

The minimum of the objective function with respect to N is not very pronounced and is very sensitive to bad mistakes in the data. It is also influenced by artificial numerical parameters such as q or $\sigma_{\rm err}$. This behaviour is due to the sensitivity of N to the fourth statistical moment of the field travel times. That is why N cannot be determined very accurately. An accuracy of the order of ± 0.05 in N may be thought to be an excellent result, difficult to achieve in practice. However, the author hopes that some small uncertainty in N will not influence the travel–time inversion considerably, see Pilkington & Todoeschuck (1990).

On the other hand, for given N, reference standard deviation σ depends on the second statistical moment of the field travel times and may be determined very accurately.

For the Gaussian probability distribution, the resulting minimum objective function should be close to 1.

5 Example: Western Bohemia

An attempt is made to estimate the medium correlation function for the region of Western Bohemia and the surrounding part of Germany, using the travel times from the refraction measurements performed during the years 1989 to 1991 (Bucha et al. 1992), see Figure 1.

5.1 Reference travel–time curve

The mean dependence of the travel times on the hypocentral distance has been roughly approximated by a rational function of the form

$$\tau^0(s) = \frac{a\,s + b\,s^2}{c + s} \quad . \tag{59}$$

At large distances s, the reference travel time (59) approaches the asymptotic line given by slowness b and the travel–time delay of $a - bc$. Constants a, b, and c have been fitted using least squares,

$$a = 0.50\,\mathrm{s} \quad , \qquad b = 0.17\,\mathrm{s\,km}^{-1} \quad , \qquad c = 1.25\,\mathrm{km} \quad . \tag{60}$$

The deviations $T_K - \tau(s_K)$ of field travel times T_K with respect to reference travel–time curve (59) are shown in Figure 2. The area of the dots in Figure 2 is proportional to the weights w'_K used in the least squares. Weights

$$w_K = \frac{1}{1 + \frac{T_{KK}}{(\delta_{\rm err} + \rho_{\rm err}\; T_K)^2}} \tag{61}$$

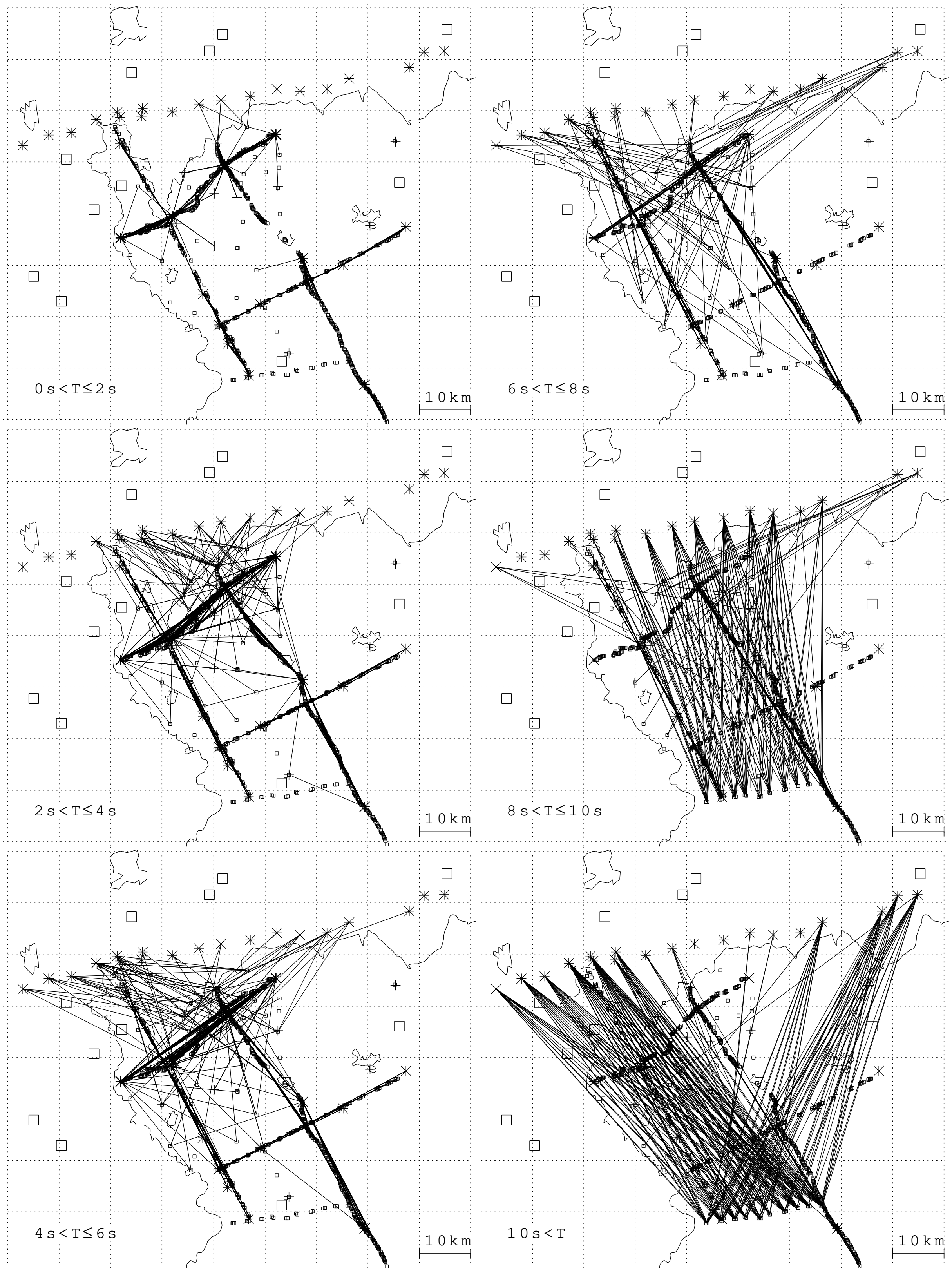

Figure 1. Two-point travel times T measured in the region of Western Bohemia and the surrounding part of Germany during the years 1989 to 1991, graphically represented by segments connecting sources (asterisks) and receivers (small squares). The travel times are sorted according to their length. The state border and greater towns have actual shapes, small towns are represented by the greater squares.

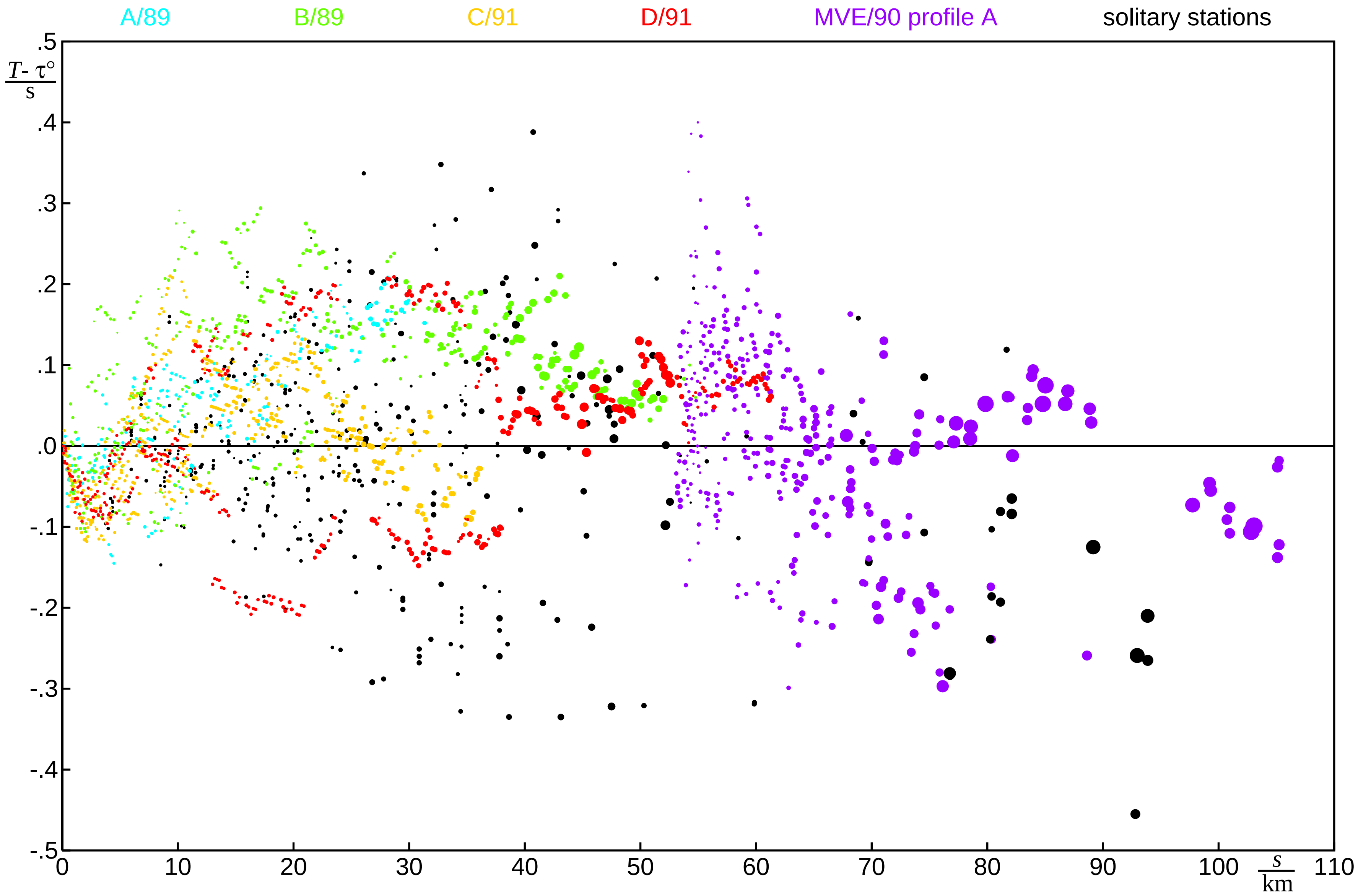

Figure 2. Deviations of field travel times from the reference travel time curve.

with

$$\delta_{\mathrm{err}} = 0.010\,\mathrm{s} \quad , \qquad \rho_{\mathrm{err}} = 0.005 \tag{62}$$

have been normalized separately in each interval of length

$$\Delta s = 1\mathrm{km} \tag{63}$$

using formula

$$w'_K = w_K \left[1 + \sum_L w_L \right]^{-1} \tag{64}$$

to get a relatively even coverage of all hypocentral distances s. The squared travel–time deviations have then been weighted with the least–squares weights

$$w''_K = w'_K \, s^{-p} \qquad \text{where} \qquad p = 0.5 \quad . \tag{65}$$

The colours in Figure 2 distinguish the travel times according to the receiver profiles. Black dots correspond to solitary receivers.

5.2 Calculation of covariances between travel times

For the estimation of the parameters of the medium correlation function, we consider straight rays, as in a homogeneous medium. For the used refraction travel times whose rays do not penetrate very deep into the earth compared with the epicentral distance, it should be a reasonable approximation. Especially if the rays of considerably different lengths are not compared, see condition (34).

Geometrical covariance matrix (25) of travel times has been calculated numerically, dividing the rectangular integration area of dimensions $s_K \times s_L$ into small rectangular cells and replacing the integrand by a bilinear function in each cell. Since the integrand may reach infinity at some points, the integrand has been limited from above at each grid point in such a way as to get exact values of the integral in all square cells touched by the diagonal for the special case of variances Θ_{KK}.

Unfortunately, the first version of the code used for these tests is not sufficiently debugged, is slow and not sufficiently accurate. This may influence the reliability of the presented numerical results. However, the numerical tests will be improved further.

5.3 Medium correlation function

The reference distance of $L = 1$km is used.

First we attempted to find a good value of numerical parameter σ_{err} selecting the set of field travel times measured with sufficient accuracy, see (58). Table 1 shows the dependence of objective function $y(N)$ and number $\sum_K 1$ of the field travel times used on the selection of σ_{err}, for $q = 0.90$, $N_0 = -0.10$, and $N = -0.11$.

σ_{err}	$\sum_K 1$	$y(N)$
0.0008	1091	0.750
0.0009	1266	0.849
0.0010	1406	0.999
0.0011	1489	1.037
0.0012	1549	1.054
0.0013	1602	1.061
0.0014	1657	1.078
0.0015	1698	1.086
0.0016 $\leftarrow$	1728	1.084
0.0017	1759	1.095
0.0018	1792	1.138
0.0019	1854	1.222
0.0020	1881	1.274
0.0025	1980	1.719
0.0030	2054	2.007

Table 1. The dependence of number $\sum_K 1$ of the field travel times used and of the value of objective function $y(N)$ on the selection of σ_{err}, for $q = 0.90$, $N_0 = -0.10$, and $N = -0.11$.

Here the value of $\sigma_0 = 0.011330\,\mathrm{s\,km^{-1}}$ has been determined for $\sigma_{\mathrm{err}} = 0.001558\,\mathrm{s\,km^{-1}}$ and has been kept fixed in calculating $y(N)$ for different σ_{err}. The value of the objective function is relatively stable for $0.0010\,\mathrm{s\,km^{-1}} \leq \sigma_{\mathrm{err}} \leq 0.0017\,\mathrm{s\,km^{-1}}$ and increases considerably for larger σ_{err}. Such an increase probably indicates the influence of bad travel–time data. The author has chosen $\sigma_{\mathrm{err}} = 0.001558\,\mathrm{s\,km^{-1}}$ for the subsequent calculations.

The next task is to choose a reasonable value of the other numerical parameter q, which selects the pairs of field travel times according to (34). Unfortunately, the position $N = N_{\mathrm{min}}$ of the minimum of objective function $y(N)$ is influenced considerably by the choice of numerical parameter q. Table 2 shows the dependence of the minimum of objective function $y(N)$ on the selection of q, for $\sigma_{\mathrm{err}} = 0.001558\,\mathrm{s\,km^{-1}}$ and σ_0 corresponding to our choice of N_0.

q	N_0	$N_{\min}$	$\sigma(N_{\min})$	$y(N_{\min})$
0.50	-0.24	-0.24	0.0092	0.805
0.60	-0.22	-0.23	0.0092	0.850
0.70	-0.18	-0.21	0.0093	0.905
0.75 $\leftarrow$	-0.19	-0.20	0.0094	0.921
0.80	-0.14	-0.17	0.0097	1.006
0.85	-0.12	-0.15	0.0100	1.053
0.90 $\leftarrow$	-0.10	-0.12	0.0106	1.099
0.95	-0.10	-0.09	0.0117	1.121
0.98	-0.10	-0.08	0.0122	1.196

Table 2. The dependence of the minimum of objective function $y(N)$ on the selection of q, for $\sigma_{\rm err} = 0.001558\,{\rm s\,km}^{-1}$ and σ_0 corresponding to our choice of N_0. $N_{\min}$ and $\sigma(N_{\min})$ are the respective parameters of the medium correlation function. The arrows denote the range of acceptable values in author's estimation.

There are at least 3 different drawbacks of small values of q:
(a) For decreasing q, inaccurate reference travel–time curve $\tau^0(s)$ may begin to influence the results considerably.
(b) The number of differences $T_K/\tau^0_K - T_L/\tau^0_L$ of the relative travel times is much greater than the number of field travel times T_K, whereas we treat the differences as independent data in objective function (50). This processing need not be correct from the statistical point of view and may get worse for smaller values of q.
(c) Here we substituted curved rays with straight ones. It is probably a reasonable approximation for rays of similar lengths, but for rays of different lengths the straight approximations of rays may be much closer together than the correct rays, separated in depth, are. For small q, some geometrical covariances (25) may thus be calculated greater than correct, which may result in compensation by smaller (more negative) values of N.

On the other hand, the drawback of q approaching 1 consists in exclusion of field travel times not surrounded by other travel times, and consequently in considerable limitation of the amount of available information. This may be the case of the results obtained for values of $q = 0.95$ and $q = 0.98$.

The estimated statistical properties of the medium are applicable at distances between the *inner and outer cutoff scales* (Mandelbrot 1977), determined here by the epicentral distances. The reference travel–time curve (59) with (60) has been estimated using travel times at epicentral distances from 0.1 km to 105 km which serve as the inner and outer cutoff scales of the reference travel–time curve. The same travel times are used to estimate the medium covariance function. However, a small number of statistically independent travel times at the shortest and longest epicentral distances may require the *inner cutoff scale* of the medium correlation function to be increased towards 0.3 km and the *outer cutoff scale* to be decreased towards 60 km.

A priori geometrical standard deviations of travel times from the mean travel–time curve are

$$\sqrt{\Theta_{KK}} = \sigma\ L\ \sqrt{\tfrac{2}{(1+2N)(2+2N)}}\ \left(\frac{|\mathbf{x}_1-\mathbf{x}_2|}{L}\right)^{1+N} \quad , \qquad (66)$$

see (30). The dependence of geometrical standard deviations (66) on the selection of q is displayed in Table 3 for several epicentral distances $s = |\mathbf{x}_1 - \mathbf{x}_2|$.

q	$N_{\min}$	0.1 km	0.3 km	1 km	3 km	10 km	60 km	100 km
0.50	-0.24	0.0025	0.0058	0.0146	0.034	0.084	0.33	0.48
0.60	-0.23	0.0024	0.0057	0.0143	0.033	0.084	0.33	0.50
0.70	-0.21	0.0022	0.0053	0.0137	0.033	0.084	0.35	0.52
0.75 $\leftarrow$	-0.20	0.0021	0.0052	0.0136	0.033	0.086	0.36	0.54
0.80	-0.17	0.0019	0.0048	0.0131	0.033	0.089	0.39	0.60
0.85	-0.15	0.0018	0.0047	0.0130	0.033	0.092	0.42	0.65
0.90 $\leftarrow$	-0.12	0.0017	0.0045	0.0130	0.034	0.099	0.48	0.75
0.95	-0.09	0.0017	0.0045	0.0135	0.037	0.110	0.56	0.89
0.98	-0.08	0.0017	0.0046	0.0139	0.038	0.116	0.60	0.96

Table 3. The dependence of the geometrical standard deviations $\sqrt{\Theta_{KK}}$ of travel times on the selection of q. The geometrical standard deviations $\sqrt{\Theta_{KK}}$ in seconds are displayed for several epicentral distances. $N_{\min}$ is the respective coefficient of the medium correlation function. The arrows denote the range of acceptable values in author's estimation.

Taking into account the above considerations, the author considers the medium correlation functions obtained for the values of q from $q = 0.90$ (Klimeš 1996) to $q = 0.75$ as acceptable.

For $q = 0.90$, we have got Hurst exponent $N = -0.12$ and $\sigma = 0.0106\,\mathrm{s\,km^{-1}}$. Medium covariance function (17) then reads

$$C(\mathbf{x}_1, \mathbf{x}_2) \approx (0.0106\,\mathrm{s\,km^{-1}})^2 \left(\frac{|\mathbf{x}_1 - \mathbf{x}_2|}{\mathrm{km}}\right)^{-0.24} , \tag{67}$$

and geometrical standard deviations (66) are

$$\sqrt{\Theta_{KK}} \approx 0.0130\,\mathrm{s} \left(\frac{s_K}{\mathrm{km}}\right)^{0.88} . \tag{68}$$

For $q = 0.75$, we have got Hurst exponent $N = -0.20$ and $\sigma = 0.0094\,\mathrm{s\,km^{-1}}$. Medium covariance function (17) then reads

$$C(\mathbf{x}_1, \mathbf{x}_2) \approx (0.0094\,\mathrm{s\,km^{-1}})^2 \left(\frac{|\mathbf{x}_1 - \mathbf{x}_2|}{\mathrm{km}}\right)^{-0.40} , \tag{69}$$

and geometrical standard deviations (66) are

$$\sqrt{\Theta_{KK}} \approx 0.0136\,\mathrm{s} \left(\frac{s_K}{\mathrm{km}}\right)^{0.80} . \tag{70}$$

For the dependence of geometrical standard deviations (66) on $N = N_{\min}$ refer to Table 3. However, the tomographic inversion of the travel times should be performed with several different values of N, and the dependence of the resulting models on the uncertainty in N should be studied. Note that at least the inversion for 1-D electrical resistivities yields very similar results for the exact Hurst exponent and that decreased by 0.25 (Pilkington & Todoeschuck 1990).

Because the medium covariance function has been determined using the straight–ray approximation, it is applicable to horizontal directions, but not vertical. The vertical behaviour of the medium correlation function has been supplemented under the assumption of a statistically isotropic medium, which is obviously not the case of geological structures.

Note that standard deviations (66) also describe the accuracy of the synthetic travel times in the hypothetical best 1-D model of the 3-D geological structure under Western Bohemia, and may, e.g., be used to estimate the accuracy of the kinematic hypocentre determination in such a 1-D model.

Wu, Xu & Li (1994) studied the medium correlation function using the well–log data from the German continental deep–drilling project (KTB), situated very close to the Western Bohemia region (in the left–hand bottom corner of the region displayed in Figure 1). The relative standard velocity deviation on the sampling interval of 0.1524 m in the well–log data is smaller than the relative standard travel–time deviation on the epicentral distance of 1 km in the Western Bohemia region, see Table 3. This may occur if the KTB locality is considerably less heterogeneous than the Western Bohemia region. Wu, Xu & Li (1994) determined the Hurst exponent for the vertical direction close to $N = 0$, the Hurst exponent for the horizontal direction close to $N = \frac{1}{2}$ and the aspect ratio of the horizontal to vertical reference distances around 1.8. Goff & Holliger (1999) studied the same well–log data and determined the Hurst exponent for the P–wave velocity in the vertical direction also close to $N = 0$

6 Conclusions

Since a geological structure contains heterogeneities of all sizes, very similar on various scales, we have considered a particular class of self–affine random media, composed of a heterogeneous mean value and a stationary self–affine random function. The power–law medium correlation function then depends on two parameters, the Hurst exponent and the corresponding reference standard deviation.

A method designed to estimate the parameters of the medium correlation function of slowness distribution using field travel times has been proposed in Section 4, and applied to data from the Western Bohemia region in Section 5.

The determination of the Hurst exponent from field travel times is very difficult and sensitive to numerical parameters selected for the inversion. The medium correlation functions with the values of the Hurst exponent like $N = -0.1$ or $N = -0.2$ are equally acceptable to statistically describe the travel times measured in the Western Bohemia region. On the other hand, for the fixed Hurst exponent, the determination of the reference standard deviation of slowness is easy and reliable. The resulting minimum values of the objective function, chosen for the determination of the Hurst exponent and the corresponding reference standard deviation, indicate that the probability distribution of slowness in the Western Bohemia region may be close to Gaussian.

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