Comparison of synthetic seismograms calculated by QI approximations of the coupling ray theory with the Chebyshev spectral method

Ivan Pšenčík [1)], **Ekkehart Tessmer** [2)]

1) Geophysical Institute, Acad. Sci. of Czech Republic, Boční II, 141 31 Praha 4, Czech Republic. E-mail ip@ig.cas.cz
2) Institut für Geophysik, Universität Hamburg, Bundesstrasse 55, 20146 Hamburg, Germany. E-mail ekkehart.tessmer@dkrz.de

Summary

The quasi-isotropic (QI) approximation of the coupling ray theory is numerically compared with the Chebyshev spectral method. Synthetic seismograms of qS waves propagating in a model of vertically inhomogeneous weakly transversely isotropic medium with horizontal axis of symmetry are compared. Synthetics are calculated for directions, in which the phase velocities of the two qS waves differ as well as for directions, in which they coincide (kiss singularity).

Keywords
Weak anisotropy, qS waves, QI approximation, coupling ray theory, Chebyshev spectral method

1 Introduction

The quasi-isotropic (QI) approximation of the coupling ray theory was designed for study of qS-wave propagation in inhomogeneous weakly anisotropic media, in which neither the ray theory for isotropic media nor the ray theory for anisotropic media work properly. As its name indicates, the QI approximation is a simplified version of a more accurate coupling ray theory. Detailed discussion of various approximations of the coupling ray theory and estimates of their accuracy by comparison with an exact solution has been performed by Bulant and Klimeš (2004), where also further references can be found. The approximations are tested there on the case of propagation of a plane qS wave along the axis of spirality in the 1D "oblique twisted crystal" model.

Here, the results of the QI approximation used by Pšenčík and Dellinger (2001) are compared with the results of the Chebyshev spectral method (CSM) by Tessmer (1995).

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The inhomogeneous 1D model of "stronger" anisotropy used by Pšenčík and Dellinger (2001) is used for the comparison. In the following synthetics, only the qS waves propagating in directions of distinct as well as coinciding qS-wave phase velocities are considered. Presented seismograms are non-scaled results of each of the used methods. The observed fit is only due to equal specification of input data for both methods.

2 Comparison

For the following tests, we use the VSP configuration and the model WA used by Pšenčík and Dellinger (2001). The source, a vertical point force directed downwards, is situated at the surface, 1 km from the mouth of the borehole. The effects of the free surface are neglected throughout. The source time function is a windowed symmetric Gabor wavelet. There are 29 receivers in the borehole, distributed with a uniform step size of 0.02 km, with receiver depths ranging from 0.01 km to 0.57 km. The 3-component receivers record the vertical (positive downwards), transverse, and radial (positive away from the source) components of the wave field. The recording system is right-handed. All calculated seismograms are shown with no differential scaling between components and traces, so true relative amplitudes can be seen.

The medium is vertically inhomogeneous and transversely isotropic with a horizontal axis of symmetry (HTI). In the coordinate system with the axis of symmetry parallel to the $x-$axis, the matrices of density-normalized elastic parameters A_{ij}, in units of (km/s)^2, for the VWA model are as follows: At a depth of 0. km,

$$\begin{pmatrix} 13.39 & 4.46 & 4.46 & 0.00 & 0.00 & 0.00 \\ & 15.71 & 5.04 & 0.00 & 0.00 & 0.00 \\ & & 15.71 & 0.00 & 0.00 & 0.00 \\ & & & 5.33 & 0.00 & 0.00 \\ & & & & 4.98 & 0.00 \\ & & & & & 4.98 \end{pmatrix}$$

and at a depth of 1. km,

$$\begin{pmatrix} 20.42 & 6.80 & 6.80 & 0.00 & 0.00 & 0.00 \\ & 23.96 & 7.69 & 0.00 & 0.00 & 0.00 \\ & & 23.96 & 0.00 & 0.00 & 0.00 \\ & & & 8.13 & 0.00 & 0.00 \\ & & & & 7.60 & 0.00 \\ & & & & & 7.60 \end{pmatrix}.$$

A rotation of 45° about the $z-$axis and linear interpolation of elastic parameters between the depths of 0. and 1. km has to be carried out to obtain the WA model of Pšenčík and Dellinger (2001). The axis of symmetry of the resulting model points out of the vertical plane containing the source and the borehole. The seismograms calculated for the WA model using the QI approach were compared with the seismograms calculated by the reflectivity method for anisotropic media by Pšenčík and Dellinger (2001). It was found that the reflectivity and QI seismograms were very similar; the synthetics nearly coincided for most receivers. There was only a slight phase shift on the vertical and transverse components, and it was not clear whether it was caused by the reflectivity method or the QI approximation. Comparison with the Chebyshev spectral method (as well as the results of the tests made by Bulant and Klimeš, 2004) shows that the shift was caused by inaccuracies of the QI approximation. In Fig.1, the synthetics calculated by the CSM are shown by black, the QI synthetics by red. The upper plot shows radial (X), the middle plot transverse (Y), and the bottom plot vertical (Z) component. As in the comparison with the reflectivity method, the QI approximation yields faster waves, mainly for shallow receivers. In remaining parts of the sections, the fit is perfect.

In the next experiment, the above matrices of elastic parameters are rotated by 30° about the $y-$axis. In this way, the vertical plane containing the source and the borehole coincides with the symmetry plane, with the symmetry axis dipping by 30°. In this case, the two qS waves are decoupled and propagate independently. One qS wave, the qSV wave, is polarized within the vertical plane, the other, the SH wave, is polarized perpendicularly to it. Since the source is the vertical point force, only the qSV wave is generated. Thus the section with transverse components is empty in this case. Due to the dip of the axis of symmetry, many of the calculated rays are somewhere along their paths parallel to the axis of symmetry, i.e., their direction corresponds to the kiss singularity. Standard ray method for anisotropic media would fail in such a case. In the QI approximation, the basic equations are decoupled but yield finite results. Fig.2 shows that, except certain phase shift, the results of the QI approximation approximate reasonably well the results of the CSM. It is interesting to note that for shallow receivers the QI approximation yields the qSV wave which is faster than the qSV wave yielded by the CSM. For deeper receivers the situation is opposite.

Fig.3 shows comparison for a model close to the model of Fig.2. The axis symmetry is again dipping under 30° but it points out of the vertical plane containing the source and the borehole. The angle between the axis of symmetry and the vertical plane makes 20°. Vertical and radial components have the same character as in Fig.2. Due to the axis of symmetry pointing out of the vertical plane, both qS waves are generated. We can see slight delay of the transverse component behind the vertical and radial ones indicating shear-wave splitting.

3 Conclusions

Despite all the approximations of the QI approach, it yields reasonably accurate results for experiments of the size of the one considered in this contribution. It was shown that the QI approximation yields reasonable results also in the vicinity of the kiss singularity where the standard ray method for anisotropic media breaks down. The most pronounced discrepancy in all comparisons is the phase shift caused by the QI approximation of the travel time, see Sec.3.4 of Bulant and Klimeš (2004).

Further tests of the accuracy of the QI approach (or its modifications) in vicinities of other singular directions in inhomogeneous weakly anisotropic media of arbitrary anisotropy are planned. Since computational efforts required by the CSM seem not to be as high as of the reflectivity (the QI approach is, however, still superior in this respect), the CSM seems to be a proper candidate for such comparisons. Its great advantage is that it is, like the QI approach, applicable to laterally inhomogeneous media of unrestricted anisotropy.

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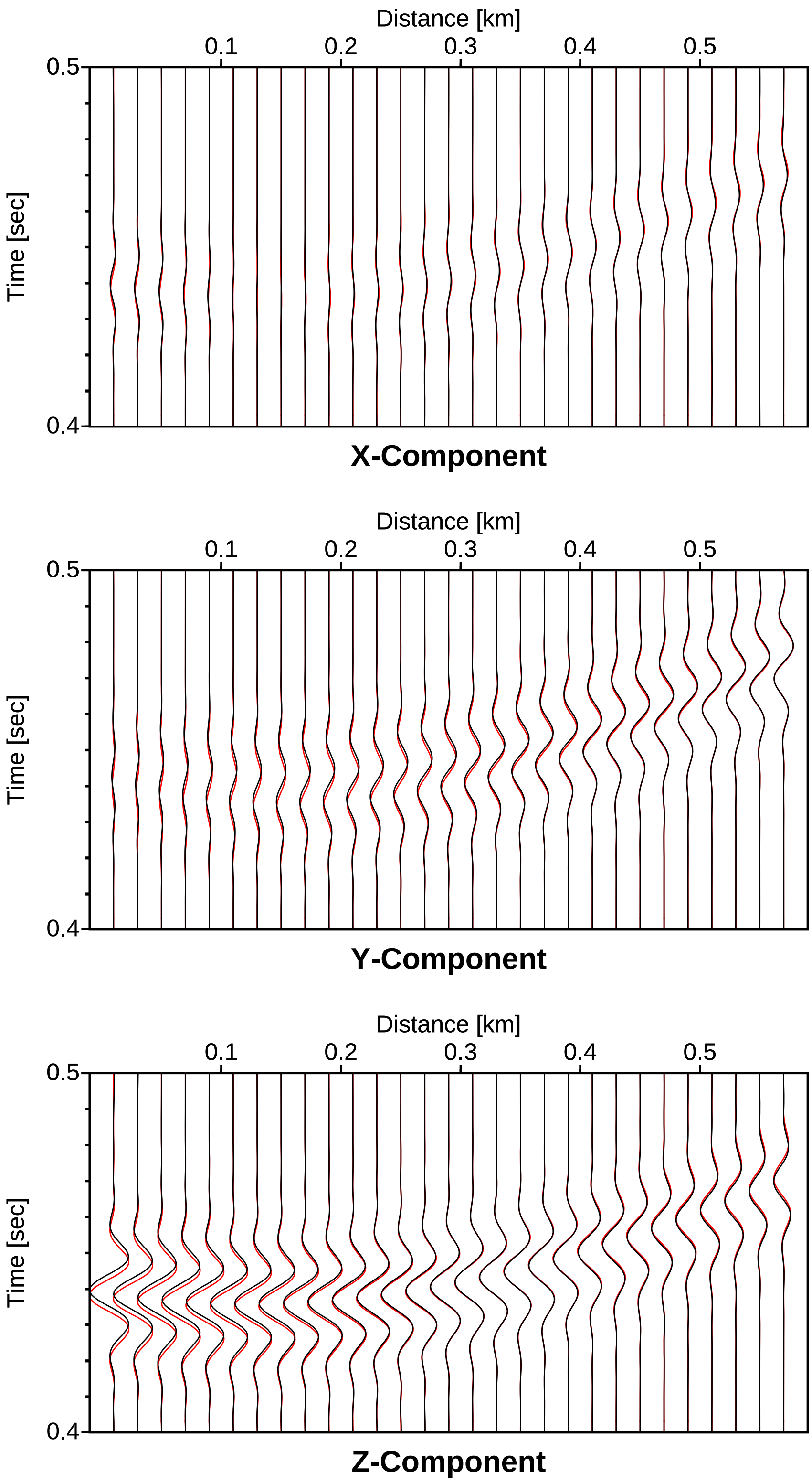
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Fig.1

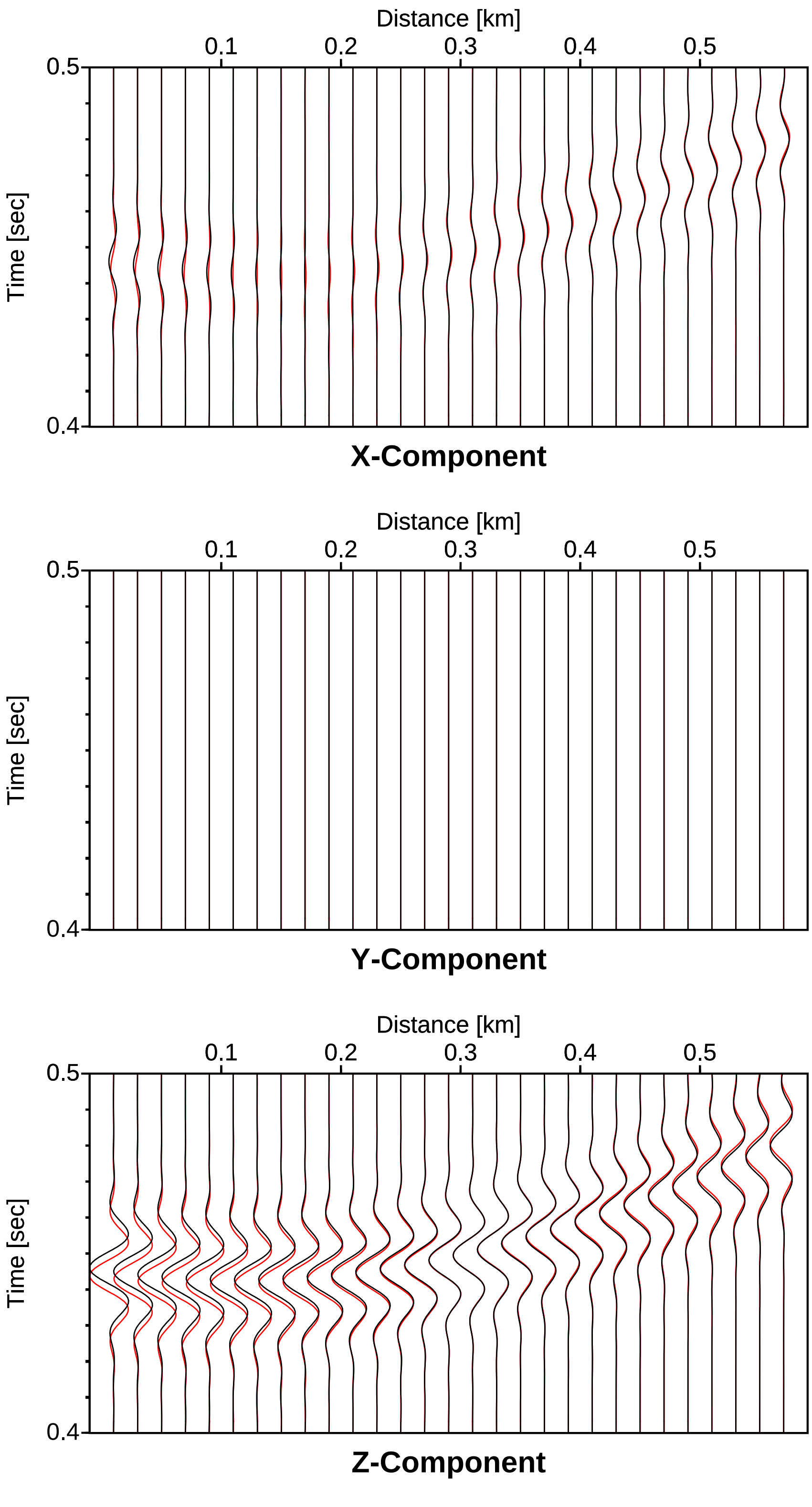
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Fig.2

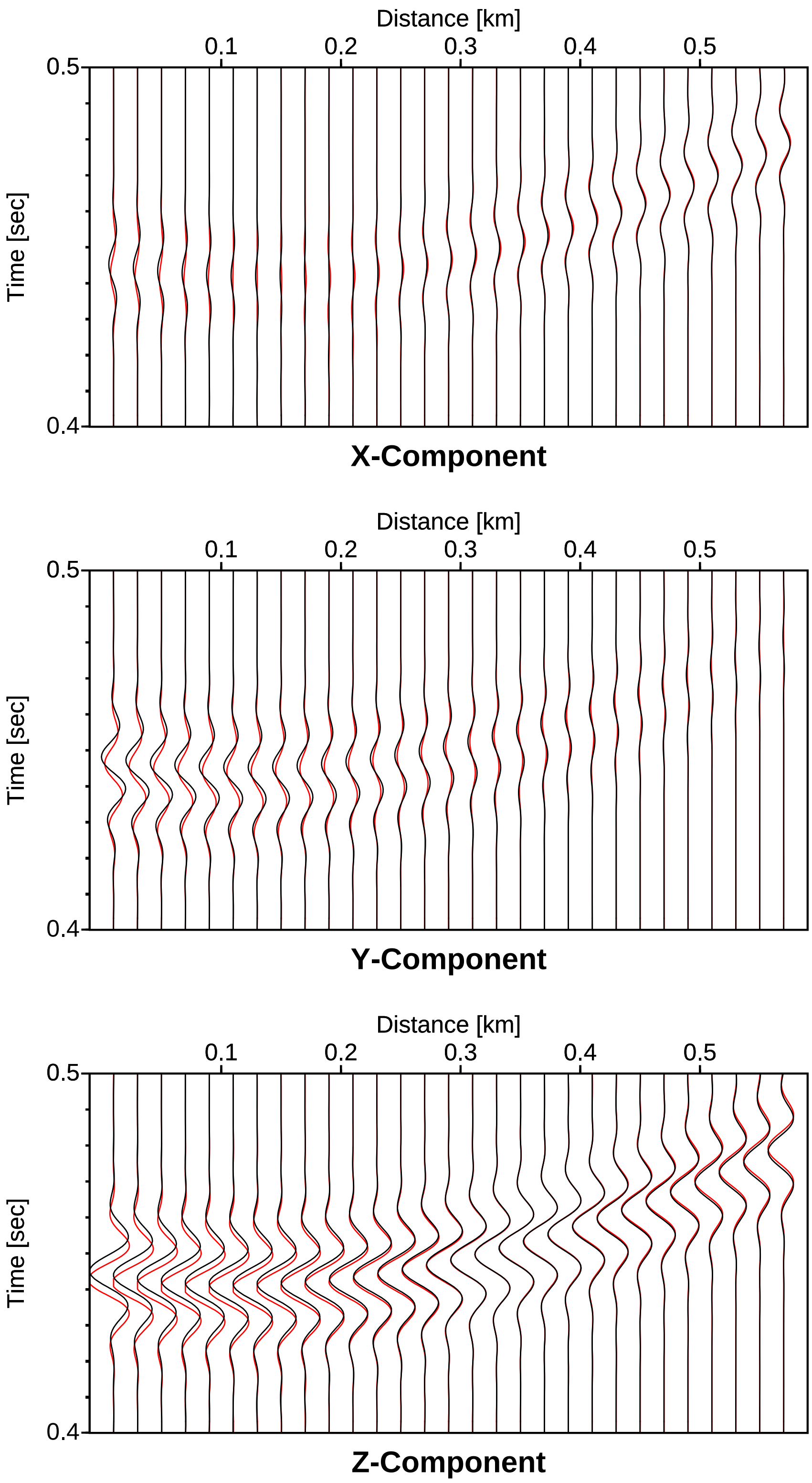
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Fig.3