Ray tracing and inhomogeneous dynamic ray tracing for anisotropy specified in curvilinear coordinates

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Abstract

Ray tracing has recently been expressed for anisotropy specified in a local Cartesian coordinate system, which may continuously vary in a model. It takes advantage of the fact that anisotropy has often a simpler nature locally (and is thus specified by a less number of elastic parameters) and that the orientation of its symmetry elements may vary. We extend this approach by replacing the local Cartesian coordinate system with a curvilinear one of global extent and by applying the approach to ray tracing and inhomogeneous dynamic ray tracing. Our formulation has several advantages compared to standard ray tracing and inhomogeneous dynamic ray tracing for anisotropy specified in global Cartesian coordinates. Among these are improved efficiency, lower consumption of computer memory, and conservation of anisotropic symmetry throughout the model.

Introduction

In recent years, there has been increased focus on modelling and imaging in the presence of highly complicated anisotropic structures of general orientation. The objective of this contribution is to formulate ray-tracing and dynamic ray-tracing procedures applicable to such complicated structures and thereby take advantage of the local anisotropic properties, so that modelling can be performed faster and with less memory usage compared to conventional approaches for ray tracing and dynamic-ray tracing.

Iversen and Pšenčík (2007) proposed a ray tracing procedure, in which anisotropy was specified in a local Cartesian coordinate system, which could vary continuously in a model. In this contribution, we replace the local Cartesian coordinate system with a curvilinear one of global extent and generalize the procedure to include inhomogeneous dynamic ray tracing. The main motive for introducing the curvilinear coordinate system is the same as in the above-mentioned paper: to reduce the number of volumetric model functions required for simulation of wave propagation in a realistic model with higher-symmetry anisotropy. The orientation of basic symmetry elements, e.g., the symmetry axis of a transversely isotropic (TI) medium or symmetry planes of an orthorhombic (OR) medium, are allowed to vary spatially. A major reduction of the number of involved volumetric model functions has a significant impact on efficiency of computations as well as on consumption of computer memory. Furthermore, if we ensure, e.g., that the symmetry axis of a TI medium is always parallel to a selected coordinate axis of the curvilinear coordinate system or that any symmetry plane of an OR medium is normal to a coordinate axis of the curvilinear coordinate system, it will be guarantied that the anisotropic symmetry is conserved during volumetric model function evaluations. The effect of using a curvilinear coordinate system in the context of continuously varying anisotropy is equivalent to using a local Cartesian coordinate system with continuously changing orientations of the axes; see Iversen and Pšenčík (2007).

The reformulated ray-tracing and dynamic ray-tracing equations presented here are valid for any kind of anisotropy. Their broadest application is, however, expected in media of higher anisotropic symmetry like TI or OR symmetry. There are several advantages of the reformulated ray tracing and dynamic ray tracing: faster computations, lower memory requirements, enhanced user-friendliness, and conservation of anisotropic symmetry properties when evaluating the material parameters in an arbitrary point of the model. The latter is not guaranteed for standard ray tracing and dynamic ray tracing based on evaluation of 21 elastic moduli given in Cartesian coordinates. A basic assumption in our approach is the existence of elastic moduli belonging to the curvilinear coordinate system as C2 continuous functions of the Cartesian coordinates. In addition, we assume to have access to a number of functions parameterizing the transformation between the curvilinear and Cartesian coordinates everywhere within the region of interest . As with the elastic moduli, such functions must be C2 continuous in the Cartesian coordinate system.

Coordinate systems

Consider a Cartesian coordinate system, (x_1, x_2, x_3) and an orthogonal curvilinear coordinate system, (ξ_1, ξ_2, ξ_3). Using Voigt notation, we denote elastic moduli belonging to Cartesian and curvilinear coordinates, respectively, as $A^{(x)}_{\alpha\beta}$ and $A^{(\xi)}_{\alpha\beta}$, where α and β take the values 1,2,...,6.

The transformation between the Cartesian and curvilinear coordinates is assumed to be one to one and described by a 3 x 3 matrix **H** with elements

$$H_{mk} = \frac{\partial x_m}{\partial \xi_k} . \qquad (1)$$

The matrix **H** is assumed to be accessible everywhere in the region of interest as a continuous function with continuous derivatives of first and second order. For

models of TI symmetry, the minimum number of functions needed to parameterize the transformation matrix **H** is two. Here, we use two Euler angles, λ and μ, for this purpose. The angles λ and μ yield the orientation of the symmetry axis. For anisotropy of lower symmetry than TI, we use an additional angle, ν, for parameterization of the matrix **H**. The angles λ and μ have an analogy with longitude and latitude specifying a point on the Earth's surface. In our context, the "longitude" λ and "latitude" μ are positive toward the "east" and "south", respectively, and the transformation matrix **H** thus reads

$$\mathbf{H}=\begin{pmatrix} \cos\lambda & \sin\lambda\sin\mu & \sin\lambda\cos\mu \\ 0 & \cos\mu & -\sin\mu \\ -\sin\lambda & \cos\lambda\sin\mu & \cos\lambda\cos\mu \end{pmatrix}. \qquad (2)$$

The ray-coordinate system, (γ_1, γ_2, γ_3), is a coordinate space in which one can represent ray-field attributes, e.g., position vectors, slowness vectors, paraxial ray-tracing matrices, amplitudes, etc., as single-valued functions of three ray coordinates. The coordinates γ_1 and γ_2 are typically position vector coordinates, slowness vector components, etc., at the starting point of the ray, whereas γ_3 is the variable along the ray. We use traveltime, i.e., $\gamma_3 = T$, as this variable. Ray coordinates are of particular importance within dynamic ray tracing.

Ray tracing and inhomogeneous dynamic ray tracing in Cartesian coordinates

Ray tracing in Cartesian coordinates is governed by the system of differential equations for the position vector **x** and the slowness vector $\mathbf{p}^{(x)}$,

$$\frac{d}{dT}\begin{pmatrix}\mathbf{x}\\ \mathbf{p}^{(x)}\end{pmatrix}=\begin{pmatrix}\mathbf{v}^{(x)}\\ \boldsymbol{\eta}^{(x)}\end{pmatrix}. \qquad (3)$$

In equation (3), travel time T is the variable along the ray, and $\mathbf{v}^{(x)}$ and $\boldsymbol{\eta}^{(x)}$ are the ray-velocity vector and the so-called eta vector, with components

$$v_i^{(x)}=\frac{1}{2}\frac{\partial G^{(x)}}{\partial p_i^{(x)}}, \qquad \eta_i^{(x)}=-\frac{1}{2}\frac{\partial G^{(x)}}{\partial x_i}, \qquad (4)$$

where $G^{(x)}$ is an eigenvalue of the *generalized* Christoffel matrix (it is called generalized when its argument is the slowness vector $\mathbf{p}^{(x)}$ rather than the unit vector parallel to $\mathbf{p}^{(x)}$). On the ray of a selected elementary wave, the corresponding eigenvalue is one; $G^{(x)}=1$.

So-called inhomogeneous dynamic ray-tracing in Cartesian coordinates (Červený, 2001, equation 4.3.40) is formulated by the following system of linear differential equations:

$$\frac{d}{dT}\begin{pmatrix}\mathbf{Q}\\ \mathbf{P}\end{pmatrix}=\begin{pmatrix}\mathbf{S}^T & \mathbf{T}\\ -\mathbf{R} & -\mathbf{S}\end{pmatrix}\begin{pmatrix}\mathbf{Q}\\ \mathbf{P}\end{pmatrix}+\begin{pmatrix}\mathbf{E}\\ -\mathbf{F}\end{pmatrix}. \qquad (5)$$

The quantities **Q** and **P** in equation (5) are 3 x 1 matrices describing absolute or relative changes of the position and slowness vectors of the ray tracing system (3). The quantities **R**, **S**, and **T** on the right-hand side of equation (5) are 3 x 3 coefficient matrices defined as

$$R_{ij}=\frac{1}{2}\frac{\partial^2 G^{(x)}}{\partial x_i \partial x_j}, \quad S_{ij}=\frac{1}{2}\frac{\partial^2 G^{(x)}}{\partial x_i \partial p_j^{(x)}}, \quad T_{ij}=\frac{1}{2}\frac{\partial^2 G^{(x)}}{\partial p_i^{(x)} \partial p_j^{(x)}}. \qquad (6)$$

The quantities **E** and **F** are 3 x 1 coefficient matrices. The definition of **E** and **F** is context-dependent, i.e., their definition depends on the definition of the dependent-variable matrices **Q** and **P**. In the following, we discuss various specifications for the matrices **Q**, **P**, **E**, and **F**.

One possibility is to define **Q** and **P** in terms of the partial derivatives

$$Q_i=\frac{\partial x_i}{\partial \chi}, \qquad P_i=\frac{\partial p_i^{(x)}}{\partial \chi}, \qquad (7)$$

where χ is an arbitrary parameter. We consider two specific choices for the parameter χ. The first choice is to let $\chi = \gamma_i$, where γ_i is one selected ray coordinate. In this case the model is assumed to be fixed and the coefficient matrices **E** and **F** are set equal zero. The dynamic ray-tracing equations are homogeneous in this case. The second choice is to let χ be one of the parameters, m, constituting the model, i.e., $\chi = m$. In this case all three ray coordinates are fixed, and the quantities **E** and **F** are defined as

$$E_i=\frac{1}{2}\frac{\partial^2 G^{(x)}}{\partial p_i^{(x)} \partial \chi}, \qquad F_i=\frac{1}{2}\frac{\partial^2 G^{(x)}}{\partial x_i \partial \chi}. \qquad (8)$$

In equation (8), the differentiation is subjected to the requirement that the quantities x_i, $p_i^{(x)}$, and χ (which is equal to m) are independent variables.

As an alternative to defining **Q** and **P** by the partial derivatives in equation (7) one may specify

$$Q_i=\Delta x_i, \qquad P_i=\Delta p_i^{(x)}, \qquad (9)$$

where the quantities Δx_i and $\Delta p_i^{(x)}$ can be defined in various ways. Again, we limit ourselves to two options. The first option is to let $\Delta\mathbf{x}$ and $\Delta\mathbf{p}^{(x)}$ denote the differences in the position and slowness vectors between the current ray (central ray) and a neighbouring ray (paraxial ray), for a given travel time, T, on the current ray. The medium parameters are assumed to be fixed, and the matrices **E** and **F** are zero. As the second option, we require that $\Delta\mathbf{x}$ and $\Delta\mathbf{p}^{(x)}$ are the changes in the position and slowness vectors resulting from the perturbation of the anisotropic velocity model. The perturbations $\Delta\mathbf{x}$ and $\Delta\mathbf{p}^{(x)}$ are performed under the condition that the three ray coordinates are fixed, and the matrices **E** and **F** have the definitions

$$E_i=\frac{1}{2}\frac{\partial \Delta G^{(x)}}{\partial p_i^{(x)}}, \qquad F_i=\frac{1}{2}\frac{\partial \Delta G^{(x)}}{\partial x_i}. \qquad (10)$$

The perturbation $\Delta G^{(x)}$ in equation (10) is performed under the condition that the vectors **x** and $\mathbf{p}^{(x)}$ are fixed.

Transformation relations for coefficients of the differential equations

The numerical integration of the ray-tracing equations (3) and inhomogeneous dynamic ray-tracing equations (5) is performed in Cartesian coordinates. The topic of this section is to express the coefficients on the right-hand sides of the equations in Cartesian coordinates in terms of corresponding coefficients belonging to a mixed phase-space domain. This domain is formed by the Cartesian position vector $\mathbf{x}$ and the slowness vector $\mathbf{p}^{(\xi)}$ in curvilinear coordinates. In the mixed domain, the eigenvalue of the generalized Christoffel matrix is denoted $G^M(\mathbf{x}, \mathbf{p}^{(\xi)})$. As with the eigenvalue $G^{(x)}$, the eigenvalue G^M equals one along the ray.

The transformation formula for the coefficients of the ray-tracing equations (3) reads

$$\begin{pmatrix} \mathbf{v}^{(x)} \\ \boldsymbol{\eta}^{(x)} \end{pmatrix} = \mathbf{C} \begin{pmatrix} \mathbf{v}^{M} \\ \boldsymbol{\eta}^{M} \end{pmatrix}. \tag{11}$$

Here we have introduced the ray-velocity and eta vectors, $\mathbf{v}^M$ and $\boldsymbol{\eta}^M$, in the mixed phase-space domain. These vectors have the components

$$v_i^M = \frac{1}{2}\frac{\partial G^M}{\partial p_i^{(\xi)}}, \qquad \eta_i^M = -\frac{1}{2}\frac{\partial G^M}{\partial x_i}. \tag{12}$$

The 6 x 6 matrix

$$\mathbf{C} = \begin{pmatrix} \mathbf{H} & \mathbf{0} \\ -\mathbf{K} & \mathbf{I} \end{pmatrix} \tag{13}$$

in equation (11) contains the following 3 x 3 matrices: the transformation matrix $\mathbf{H}$ defined in equation (1), the zero matrix $\mathbf{0}$, the identity matrix $\mathbf{I}$, and a matrix $\mathbf{K}$ with elements

$$K_{mk} = \frac{\partial H_{ik}}{\partial x_m} p_i^{(x)}. \tag{14}$$

The transformation formula for the coefficient matrices $\mathbf{R}$, $\mathbf{S}$, and $\mathbf{T}$ of the inhomogeneous dynamic ray-tracing system (5) reads

$$\begin{pmatrix} \mathbf{S}^T & \mathbf{T} \\ -\mathbf{R} & -\mathbf{S} \end{pmatrix} = \mathbf{C} \begin{pmatrix} \mathbf{S}^{M\,T} & \mathbf{T}^{M} \\ -\mathbf{R}^{M} & -\mathbf{S}^{M} \end{pmatrix} \mathbf{D} + \begin{pmatrix} \mathbf{V}^T & \mathbf{0} \\ -\mathbf{U} & -\mathbf{V} \end{pmatrix}, \tag{15}$$

where the 3 x 3 matrices $\mathbf{R}^M$, $\mathbf{S}^M$, $\mathbf{T}^M$, $\mathbf{U}$ and $\mathbf{V}$ are defined by

$$R_{nm}^M = \frac{1}{2}\frac{\partial^2 G^M}{\partial x_n \partial x_m}, S_{nm}^M = \frac{1}{2}\frac{\partial^2 G^M}{\partial x_n \partial p_m^{(\xi)}}, T_{nm}^M = \frac{1}{2}\frac{\partial^2 G^M}{\partial p_n^{(\xi)} \partial p_m^{(\xi)}},$$

$$U_{nm} = v_k^M \frac{\partial^2 H_{ik}}{\partial x_n \partial x_m} p_i^{(x)}, \qquad V_{nm} = v_k^M \frac{\partial H_{mk}}{\partial x_n}. \tag{16}$$

The matrix $\mathbf{C}$ is given in equation (13) and the 6 x 6 matrix $\mathbf{D}$ reads

$$\mathbf{D} = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{K}^T & \mathbf{H}^T \end{pmatrix}. \tag{17}$$

Finally, the transformation formula for the coefficients $\mathbf{E}$ and $\mathbf{F}$ of the inhomogeneous dynamic ray-tracing system (5) reads

$$\begin{pmatrix} \mathbf{E} \\ -\mathbf{F} \end{pmatrix} = \mathbf{C} \begin{pmatrix} \mathbf{E}^M \\ -\mathbf{F}^M \end{pmatrix}. \tag{18}$$

Numerical examples

Models and configuration

We illustrate the application of the above-described procedures on continuous single-layered models of slightly heterogeneous media of varying TI anisotropy. The heterogeneity is formed by use of two isosurfaces. The upper surface is a horizontal plane ($x_3 = 0$); the lower surface is only slightly curved with maximum dip approximately along the line $x_2 = x_1$. The elastic moduli are required to be constant along each isosurface, and in between they vary linearly in the vertical direction. Since the depth (x_3) of the lower isosurface varies laterally, any elastic moduli function created from two different constants at the two isosurfaces will vary laterally, as well.

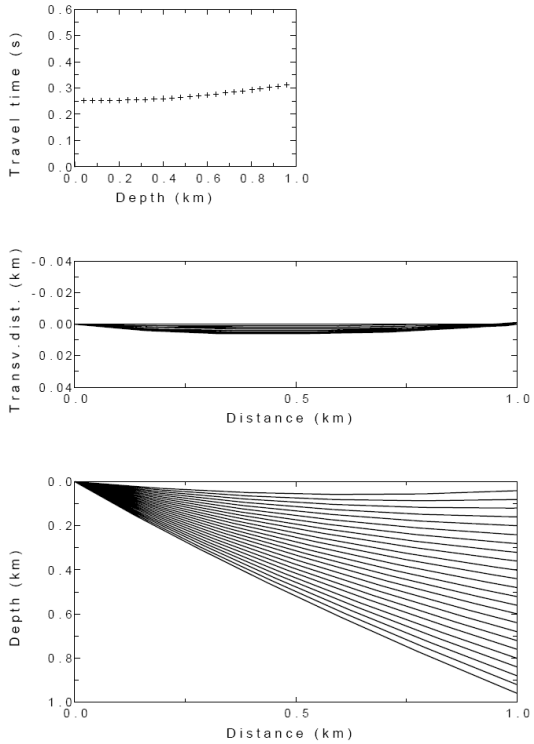

Figure 1. The numerical tests were performed for a VSP configuration. Top: Computed travel times for model HTI_{ROT}. Middle: Projection of rays in model HTI_{ROT} into a horizontal plane. Middle: Projection of rays in model HTI_{ROT} into a vertical plane.

Consider two models referred to as HTI_{FIX} and HTI_{ROT} whose anisotropy is of the HTI type, i.e., a TI medium with a horizontal symmetry axis. For both models, the elastic moduli belonging to the curvilinear coordinate systems at the upper and lower isosurfaces are identical. At the upper surface we have

$A^{(\xi)}_{11} = A^{(\xi)}_{22} = 15.71$,

$A^{(\xi)}_{33} = 13.90$,

$A^{(\xi)}_{12} = 5.05$,

$A^{(\xi)}_{13} = A^{(\xi)}_{23} = 4.46$,

$A^{(\xi)}_{44} = A^{(\xi)}_{55} = 4.98$, and

$A^{(\xi)}_{66} = 5.33\ km^2 s^{-2}$.

The values at the lower surface are

$A^{(\xi)}_{11} = A^{(\xi)}_{22} = 35.35$,

$A^{(\xi)}_{33} = 30.13$,

$A^{(\xi)}_{12} = 11.36$,

$A^{(\xi)}_{13} = A^{(\xi)}_{23} = 10.04$,

$A^{(\xi)}_{44} = A^{(\xi)}_{55} = 11.21$, and

$A^{(\xi)}_{66} = 11.99\ km^2 s^{-2}$.

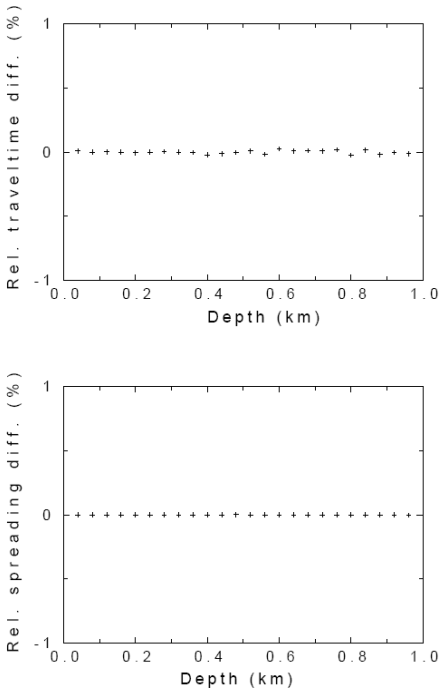

Figure 2. Relative differences in travel time and geometrical spreading (in %) for the procedures proposed above and for the standard procedures, using the model HTI_{FIX}.

The models HTI_{FIX} and HTI_{ROT} differ by the way the curvilinear coordinate system is varied with respect to the Cartesian coordinate system. In the model HTI_{FIX}, the symmetry axis is parallel to the x_1 axis on both isosurfaces; hence, the direction of the symmetry axis is fixed throughout the model. In the model HTI_{ROT}, the symmetry axis is parallel to the x_1 axis along the lower isosurface, as in the model HTI_{FIX}, but on the upper isosurface the symmetry axis makes an angle of -45^0 with the x_1 axis. We therefore have a medium where the angle between the x_1 axis and the symmetry axis varies linearly from -45^0 to 0^0 along any vertical line between the upper and lower isosurface.

In the model HTI_{FIX}, the two angles parameterizing the matrix **H**, see equation (2), have the values $\lambda = 90^0$ and $\mu = 0^0$ on both isosurfaces. For the model HTI_{ROT}, the angles are $\lambda = 90^0$ and $\mu = -45^0$ on the upper isosurface and $\lambda = 90^0$ and $\mu = 0^0$ on the lower isosurface. In both models, the angles λ and μ and the elastic moduli $A^{(\xi)}_{\alpha\beta}$ are interpolated linearly along vertical lines between the two isosurfaces. In this way, we can obtain $H(\mathbf{x})$, $A^{(\xi)}_{\alpha\beta}(\mathbf{x})$ and their first- and second-order derivatives for every point **x**.

Consider a VSP survey configuration with a source point located in the origin of the Cartesian coordinate system and a vertical receiver line, for which $x_1 = 1$ and $x_2 = 0$ km. The receivers are distributed between depths $x_3 = 0.04$ km and $x_3 = 0.96$ km, with equidistant spacing of 0.04 km. Given this configuration, we perform two ray-tracing and dynamic ray-tracing computations of the direct P wave for each model. The first computation is based on the use of the formulae proposed above and makes use of the elastic moduli $A^{(\xi)}_{\alpha\beta}$, the angles λ and μ, and the first- and second-order derivatives of all these quantities with respect to the coordinates x_1, x_2, x_3. The second computation, based on the use of the program package ANRAY (Gajewski and Pšenčík, 1987), is performed in Cartesian coordinates. Before this computation, a model preparation step is needed, consisting of transforming the elastic moduli in curvilinear coordinates, $A^{(\xi)}_{\alpha\beta}$, into a full set of 21 elastic moduli in Cartesian coordinates, $A^{(x)}_{\alpha\beta}$, using the angles λ and μ. Ray diagrams and traveltimes for the simulation in model HTI_{ROT} are shown in Figure 1.

Simulation using model HTI_{FIX}

In the model HTI_{FIX}, the linear interpolation of elastic moduli $A^{(\xi)}_{\alpha\beta}$ is perfectly consistent with the linear interpolation of rotated elastic moduli $A^{(x)}_{\alpha\beta}$. As a consequence, the travel times and geometrical spreading values resulting from both computations should, in theory, be the same. This is confirmed in Figure 2. The largest relative travel time differences between the two computations are about 0.03%. The differences for the geometrical spreading are even smaller.

Simulation using model HTI_{ROT}

When the symmetry axes at the upper and lower isosurfaces are not parallel, linear interpolation of the transformed elastic moduli $A^{(x)}_{\alpha\beta}$ yields values which are generally different from values obtained by linear interpolation of the elastic moduli $A^{(\xi)}_{\alpha\beta}$. Interpolated values of $A^{(x)}_{\alpha\beta}$ become inconsistent with the TI symmetry.

Thus, the two simulations yield slightly different values of the elastic moduli at a given location, which in the end leads to different ray travel times and geometrical spreading values, see Figure 3. The largest relative difference in travel time is now about 0.37% and relative differences in the spreading slightly exceed 2 %. The results of ray tracing and dynamic ray tracing in the new formulation should be considered superior, because linear interpolation of the elastic moduli $A^{(\xi)}_{\alpha\beta}$ conserves the TI character in any point of the model. For the standard ray-tracing and dynamic ray- tracing approaches formulated in Cartesian coordinates, strict TI symmetry is only obtained along the isosurfaces on which the elastic moduli are specified. In between these surfaces the medium described by $A^{(x)}_{\alpha\beta}$ deviates slightly from TI.

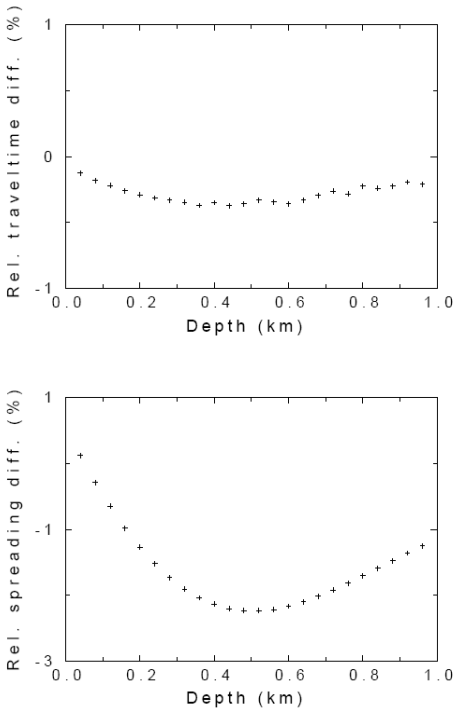

Figure 3. Relative differences in travel time and geometrical spreading (in %) for the procedures proposed above and for the standard procedures, using the model HTI_{ROT}.

Conclusions

We have introduced differential equations for ray tracing and inhomogeneous dynamic ray tracing, which allow efficient computations in media of higher-symmetry anisotropy with spatially varying symmetry elements (axes or planes). Our formulation, based on the knowledge of the minimum required number of elastic moduli $A^{(\xi)}_{\alpha\beta}$ in a curvilinear coordinate system, has a number of advantages compared to conventional ray tracing for anisotropic media, which is formulated with respect to the complete set of 21 elastic moduli $A^{(x)}_{\alpha\beta}$ in the Cartesian coordinate system. Although the proposed approach is generally applicable to media of arbitrary anisotropy, we foresee that its main use will be for anisotropic media of higher symmetry, e.g., TI or OR symmetry.

The main reasons for introducing elastic moduli $A^{(\xi)}_{\alpha\beta}$ in a curvilinear coordinate system are enhanced efficiency of computations, smaller requirements of computer memory, possibilities for improved user-friendliness by utilizing simple relations between the elastic moduli $A^{(\xi)}_{\alpha\beta}$ and dimensionless anisotropy parameters (Thomsen, 1986; Tsvankin,1997), and conservation of anisotropic symmetry during evaluation of the functions $A^{(\xi)}_{\alpha\beta}(\mathbf{x})$. We emphasize that the proposed procedures are quite simple to implement if one already has available ray-tracing and dynamic ray-tracing procedures for a TI medium with a vertical axis or an OR medium where the symmetry planes are aligned with the main planes of the Cartesian coordinate system.

In this contribution, we concentrated on ray-tracing and dynamic ray-tracing procedures for smoothly varying media and presented examples pertaining to P wave propagation. Generalizations for multiply reflected/transmitted and possibly converted elementary waves in layered media are expected to be quite straightforward. The procedure for the inhomogeneous dynamic ray tracing proposed above offers broad applications of various perturbation approaches. Combination of the procedures proposed here with the first-order ray tracing (Pšenčík and Farra, 2005) and its generalization offers further improvement of efficiency of the ray tracing and dynamic ray tracing in heterogeneous, weakly anisotropic media.

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