Travel times in the INRIA Marmousi models

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Summary

The possibility and accuracy of the calculation of the first–arrival travel times and the ray–theory travel times is demonstrated on two INRIA bench–mark versions of the 2-D Marmousi model. The two models differ considerably in the degree of global smoothness.

Keywords

First–arrival travel times, ray–theory travel times, ray tracing, seismic models.

1 Introduction

In this paper, we are concerned with the theoretical travel times defined in terms of the eikonal equation. Within this class of travel times, we should especially distinguish between the first–arrival travel times and the ray–theory travel times. It is known that the first–arrival travel times differ considerably from the ray–theory travel times in general block models with structural interfaces.

The *first–arrival travel times* do not correspond to the ray–theory high–frequency asymptotic approximation, they describe the fastest possible propagation of information and are often connected with a negligible amount of energy. The first–arrival travel times are the property of the exact solution of elastodynamic equation. In this way, they are related rather to the full–wavefield finite differences than to the ray method. In dispersive media, the first–arrival travel times correspond to the maximum propagation velocities over the frequency range. The calculation of the first–arrival travel times is much easier than the calculation of the ray–theory travel times.

The *ray–theory travel times* ("multivalued" travel times) correspond to a selected elementary wave in the asymptotic decomposition of the wavefield. They describe the propagation of energy in the high–frequency asymptotic approximation. In dispersive media, the ray–theory travel times correspond to the velocities at the prevailing frequency.

In a special case of a nondispersive, smooth medium, the first–arrival travel times are the minimum travel times selected of the ray–theory travel times. Theoretically, not in practice. As a smooth medium becomes more complex, the number of multiple travel time branches exponentially increases with the number of smooth heterogeneities: each triplication of the wavefront caused by a small heterogeneity spreads further in a large angle, approaches many other small heterogeneities and is again triplicated at each of the heterogeneities and so on. In practice, we cannot calculate millions of travel–time branches and millions of multiple arrivals at a receiver. The multivalued ray–theory travel times may thus be calculated just in models which are *sufficiently globally smooth*, i.e. they are not only locally smooth but also contain just a reasonably small number of distinct heterogeneities. In more complex models, the statistical properties

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of the multivalued ray–theory travel times instead of the individual values have to be estimated.

In this paper, the results of the calculation of the first–arrival travel times and the ray–theory travel times in two smooth models, which differ considerably in the degree of global smoothness, are briefly summarized.

The two smooth continuous bench–mark 2-D models without structural interfaces, based on the Marmousi model, have been prepared for the workshop "Computation of multi–valued traveltimes" held in INRIA Rocquencourt, France, on September 16–18, 1996. The "hard" model is globally quite rough and is smooth on the scales of the order of several metres, whereas the "smoothed" model is a considerably smoothed version of the "hard" model, see Klimeš & Bulant (1996).

The point source is situated at distance $x_1 = 6.000\,\mathrm{km}$ from the left–hand side of each 2-D model of dimensions $9.192\,\mathrm{km} \times 2.904\,\mathrm{km}$, at the depth of $x_2 = 2.800\,\mathrm{km}$. The receivers are located along the top of the model ($x_2 = 0.000\,\mathrm{km}$), with spacing 24 metres ($x_1 = 0.000\,\mathrm{km}$, $0.024\,\mathrm{km}$, $0.048\,\mathrm{km}$, ..., $9.192\,\mathrm{km}$).

2 First–arrival travel times in the Marmousi "smoothed" and "hard" models

The first–arrival travel times are, as a rule, calculated on regular rectangular grids of points, using the discretized versions of seismic models. The calculation by means of a stable causal algorithm usually makes no problems, with exception of the accuracy. The accuracy is dependent on the algorithm used (and, naturally, on the efficient coding) and on the density of the grid.

The author used, in the first place, the very accurate second–order method by Klimeš (1996). Although the equations for the errors of the method are known, they are not presently coded. The accuracy has thus been roughly estimated using the combination of the two approaches: (a) analysis of the convergence of the calculated first–arrival travel times with respect to decreasing grid spacing; (b) comparing the calculated first–arrival travel times with the minima selected of the ray–theory travel times which are very accurately calculated by the shooting method. The (b) approach could, of course, be applied only in the "smoothed" model.

The resulting rough estimates of the accuracy of the first–arrival travel times calculated by two different methods at two different grid spacings are presented in Table 1.

Program	Grid spacing	"smoothed" model	"hard" model
`TTT-2D`	$\frac{24}{9}\,\mathrm{m}$	$10^{-5}\,\mathrm{s}$	$10^{-4}\,\mathrm{s}$
`TTT-2D`	$24\,\mathrm{m}$	$10^{-3}\,\mathrm{s}$	N/A
`NET`	$\frac{24}{9}\,\mathrm{m}$	$10^{-3}\,\mathrm{s}$	$10^{-2}\,\mathrm{s}$
`NET`	$24\,\mathrm{m}$	$4 \times 10^{-3}\,\mathrm{s}$	$4 \times 10^{-2}\,\mathrm{s}$

Table 1. The order of the absolute errors of the first–arrival travel times in the "smoothed" and "hard" INRIA bench–mark versions of the 2-D Marmousi model, calculated by the second–order grid travel–time tracing according to Klimeš (1996) (program `TTT-2D` version `0.17`), and by the first–order network shortest–path ray tracing according to Klimeš & Kvasnička (1994) (program `NET` version `2.00`).

The first–arrival travel times calculated in both models are displayed in Figures 1 and 4 in different scales.

3 Ray–theory travel times in the Marmousi "smoothed" model

The ray–theory travel times at given receivers in the "smoothed" model were calculated by the shooting method. Even the "smoothed" model is not sufficiently smooth for the shooting method: the geometrical spreading, especially at the leftmost part of the receiver profile, is so great that there is no take–off angle available within the single precision real numbers to shoot the ray to that region, see Figures 2 and 5. Thus, the shooting method cannot catch all the two–point rays in this model. It may miss about 20% two–point rays. On the other hand, if the two–point ray is found, the calculated ray–theory travel time is probably very accurate.

The wavefront tracing would be probably reliable in this model. The question remains whether the price for it would be the loss of accuracy.

4 Ray–theory travel times in the Marmousi "hard" model

The ray–theory travel time curve in the "hard" model is so complex that the two–point ray tracing makes no sense. For illustration, travel times corresponding to several of the initial–value rays are shown in Figures 3 and 5. The cloud of points in Figures 3 and 5 is far from starting to form the complete travel–time curve.

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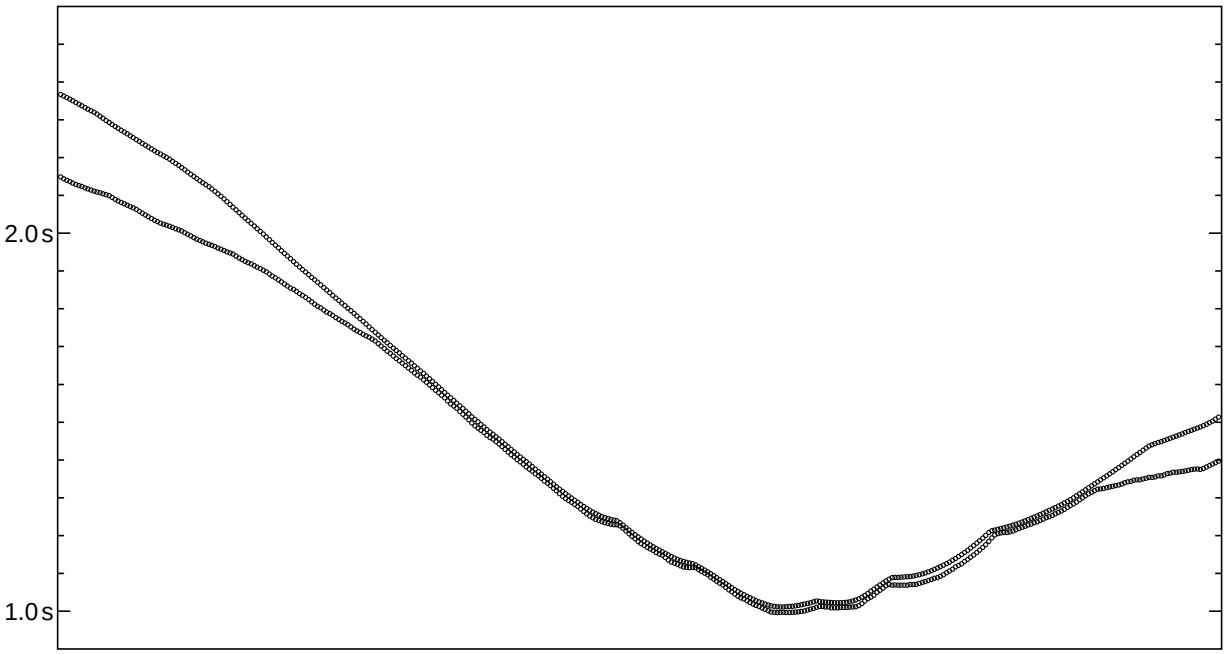

Figure 1. The first–arrival travel times in the "smoothed" (upper curve) and "hard" (lower curve) INRIA bench–mark versions of the 2-D Marmousi model.

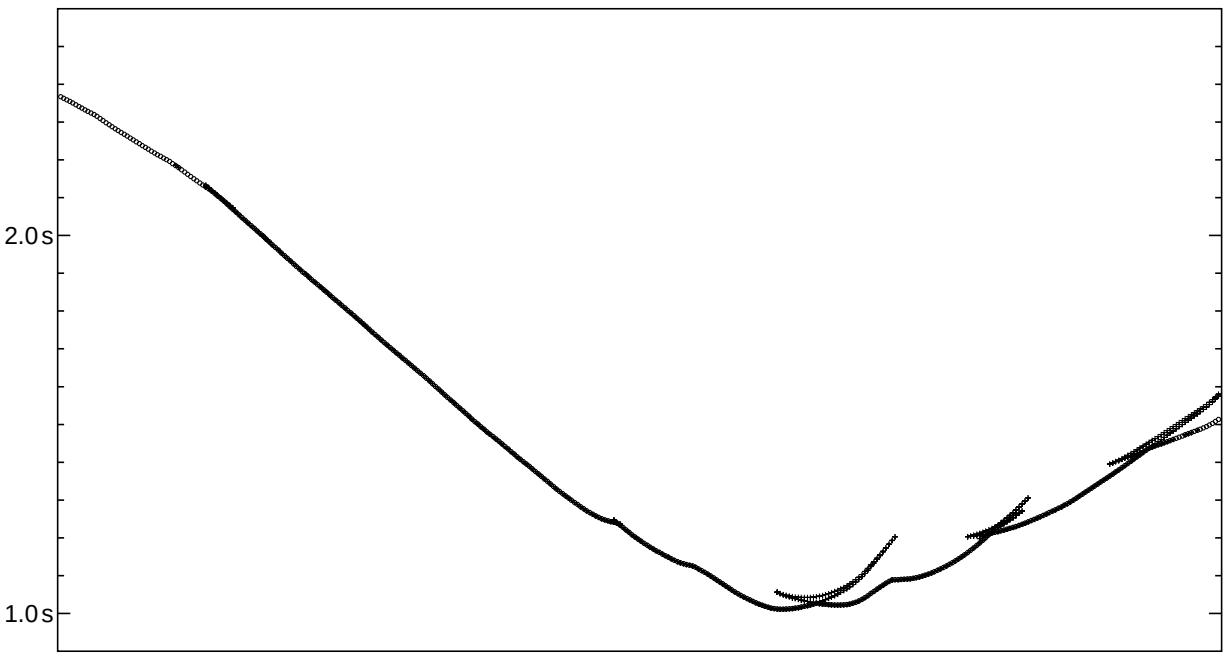

Figure 2. The first–arrival travel times (o) and the two–point ray–theory travel times (+) in the "smoothed" INRIA bench–mark version of the 2-D Marmousi model. The first–arrival travel times at receivers where the shooting method failed appear as empty circles.

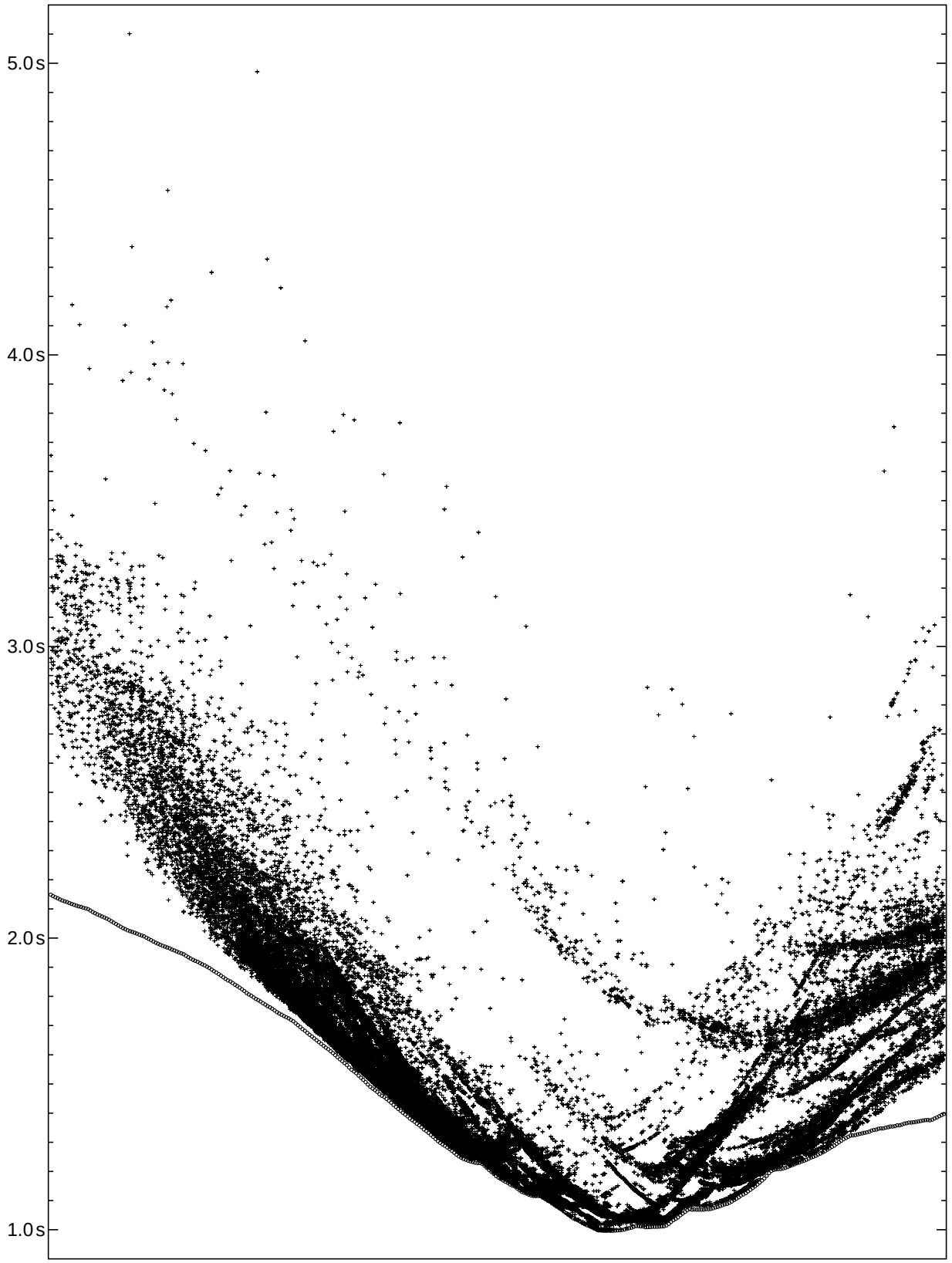

Figure 3. The first–arrival travel times (o) and 136 818 initial–value ray–theory travel times (+) in the "hard" INRIA bench–mark version of the 2-D Marmousi model. 136 818 of 244 302 initial–value rays, shot with the angular increment of 0.000 010 radians, hit the receiver profile.

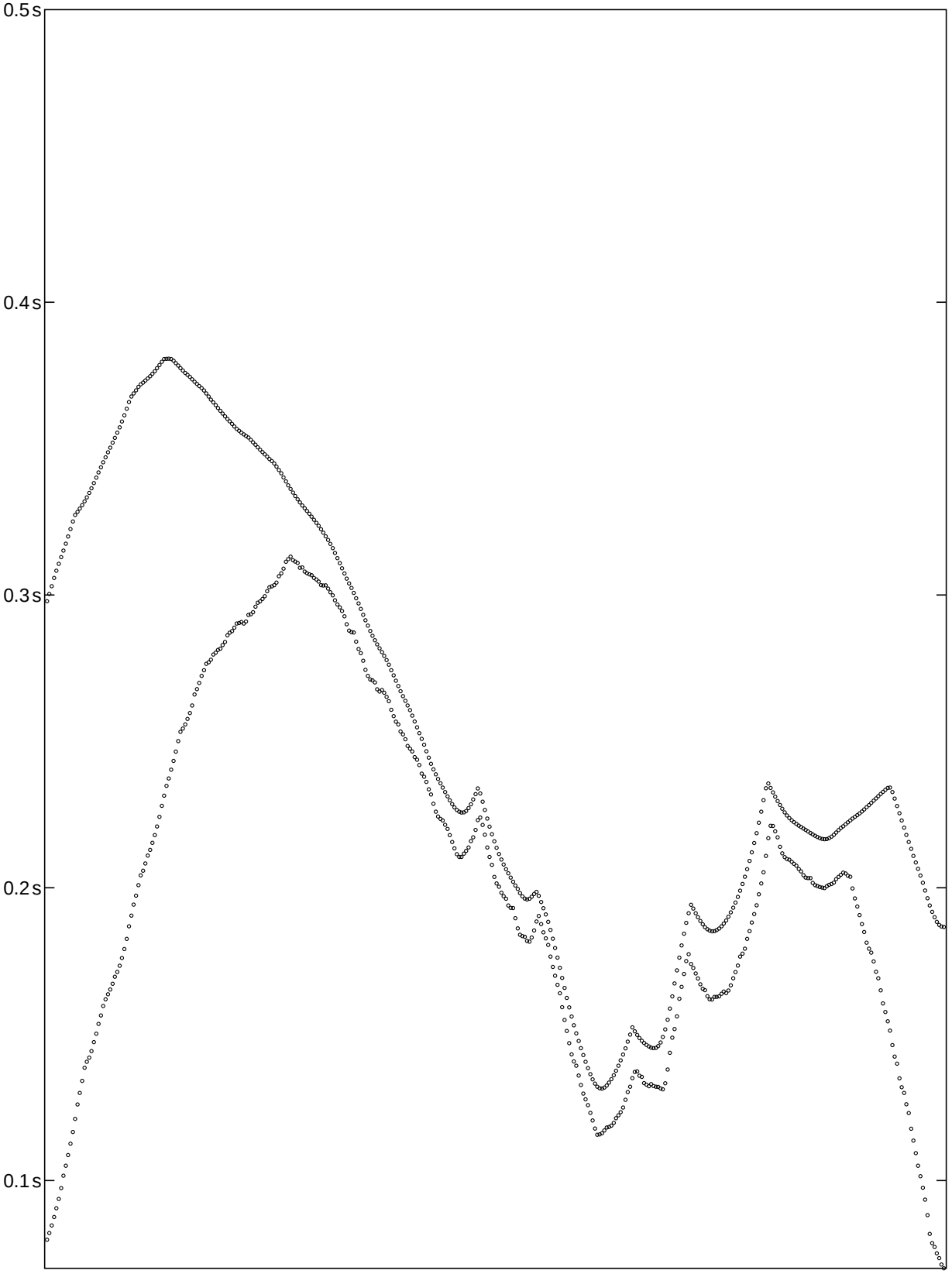

Figure 4. The first–arrival travel times in the "smoothed" (upper curve) and "hard" (lower curve) INRIA bench–mark versions of the 2-D Marmousi model. The travel times of Figure 1 are reduced here to velocity $3.2\,\mathrm{km\,s^{-1}}$, [reduced travel time]=[travel time]−[hypocentral distance]/[$3.2\,\mathrm{km\,s^{-1}}$], and are displayed in 10 times enlarged scale.

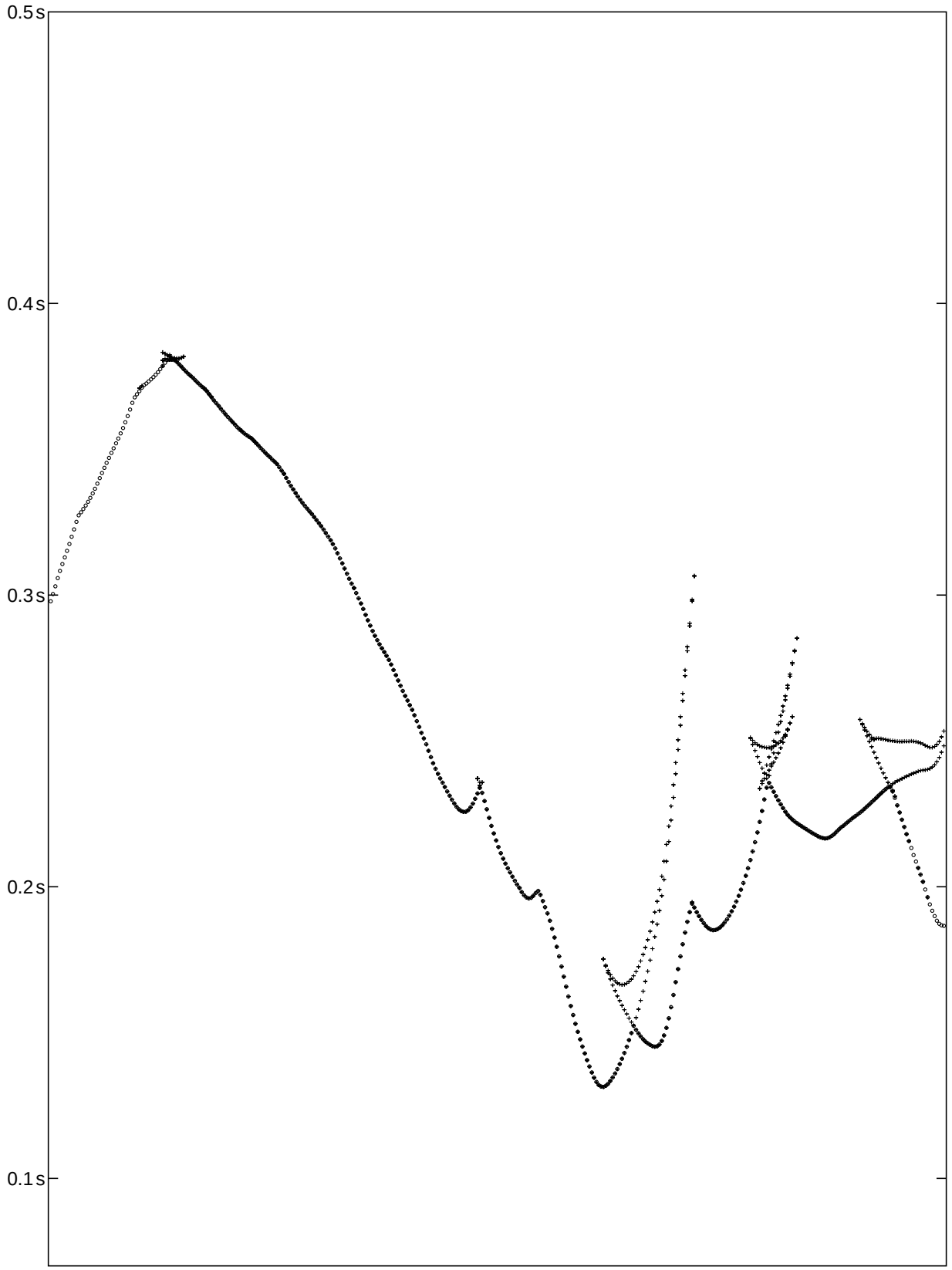

Figure 5. The first–arrival travel times (o) and the two–point ray–theory travel times (+) in the "smoothed" INRIA bench–mark version of the 2-D Marmousi model. The first–arrival travel times at receivers where the shooting method failed appear as empty circles. The travel times of Figure 2 are reduced here to velocity $3.2\,\mathrm{km\,s^{-1}}$ and are displayed in 10 times enlarged scale.

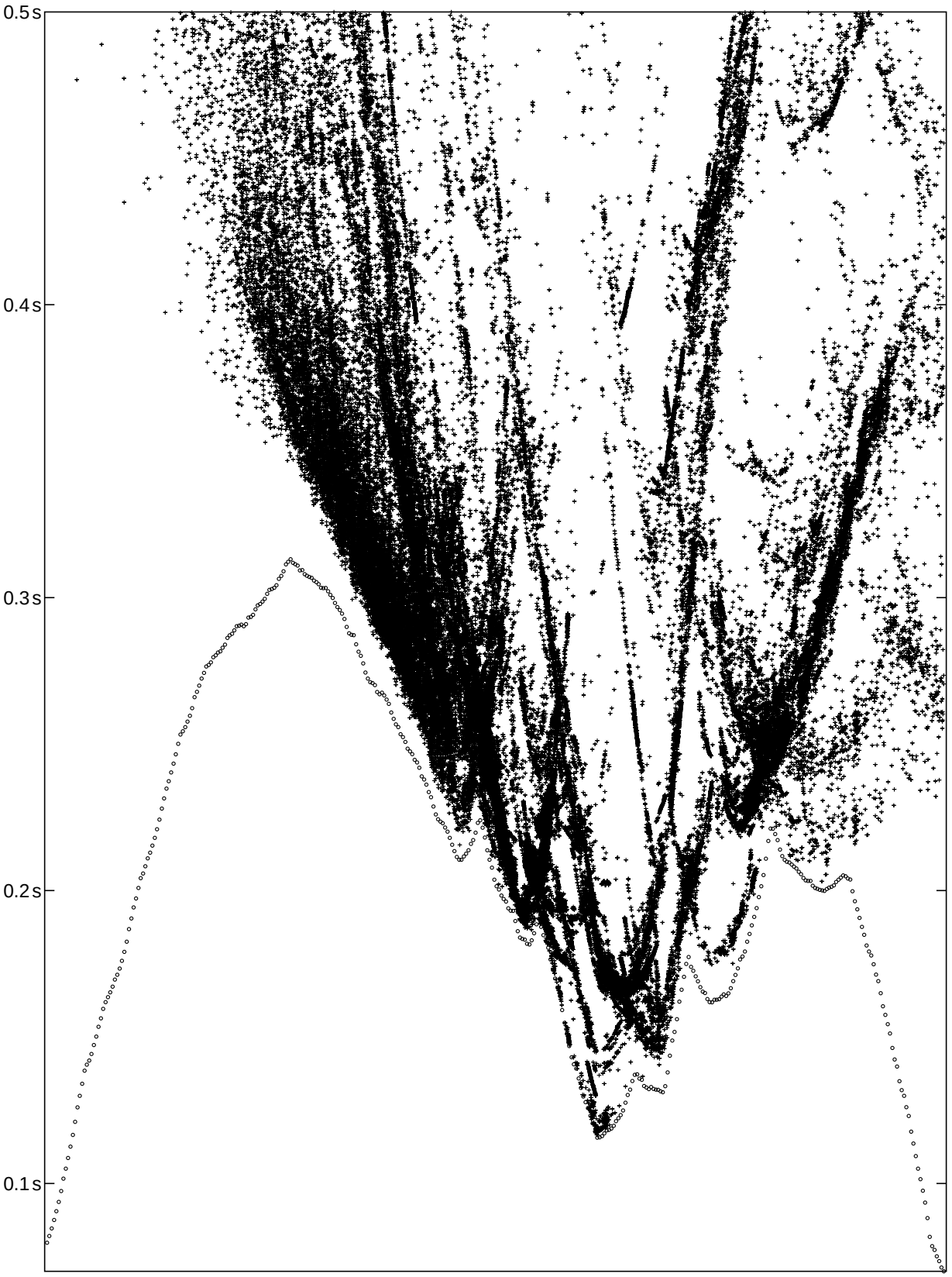

Figure 6. The first–arrival travel times (o) and the initial–value ray–theory travel times (+) in the "hard" INRIA bench–mark version of the 2-D Marmousi model. The travel times of Figure 3 are reduced here to velocity 3.2 km s^{-1} and are displayed in 10 times enlarged scale. Reduced travel times over 0.5 s are clipped off.