Determination of weak anisotropy parameters from the qP wave particle motion measurements [1]

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ABSTRACT

We propose an algorithm for evaluating weak anisotropy (WA) parameters from deviations of measured qP wave particle motions from the corresponding phase normal at individual receivers in a borehole in a multiple-source offset VSP measurement. The formulae are derived under the assumption of weak but arbitrary anisotropy and vertical inhomogeneity of the medium. The formulae represent, in several respects, generalization of an algorithm proposed by Gaiser (1990). In this contribution, only basic formulae are presented. Tests of the proposed algorithm with synthetic data are under preparation.

INTRODUCTION

Let us consider a multiple-source offset VSP experiment with three-component recordings not affected by a free surface. Let us further assume that the studied medium is weakly anisotropic (WA) with arbitrary symmetry and vertically inhomogeneous. For this medium, we find a background isotropic medium, from which the WA medium differs only slightly. In the background isotropic medium let us introduce three mutually perpendicular unit vectors $e_i^{(1)}$, $e_i^{(2)}$ and $e_i^{(3)}$ so that the vector $e_i^{(3)}$ is parallel to the P wave polarization vector. The vector $e_i^{(3)}$ is also parallel to the phase normal since in an isotropic medium the phase normal and polarization vector of a P wave are identical. The vectors $e_i^{(1)}$ and $e_i^{(2)}$ can be arbitrarily chosen in the plane perpendicular to n_i. The most practical choice of vectors $e_i^{(1)}$, $e_i^{(2)}$ expressed in terms of components of the vector $e_i^{(3)}$ seems to be the following one, see Pšenčík and Gajewski (1998):

$$\vec{e}^{(1)} = D^{-1}(n_1 n_3, n_2 n_3, n_3^2 - 1), \quad \vec{e}^{(2)} = D^{-1}(-n_2, n_1, 0), \quad \vec{e}^{(3)} = \vec{n} = (n_1, n_2, n_3), \tag{1}$$

where

$$D = (n_1^2 + n_2^2)^{1/2}, \quad n_1^2 + n_2^2 + n_3^2 = 1. \tag{2}$$

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Let us specify the phase normal as

$$\vec{n} = (\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta). \tag{3}$$

Here φ denotes an azimuthal angle, θ a polar angle, $(0 \leq \varphi \leq 2\pi,\ 0 \leq \theta \leq \pi)$. In this case we have $D = \sin\theta$ and vectors $\vec{e}^{(1)}$ and $\vec{e}(2)$ from Eq.(1) can be rewritten in the form

$$\vec{e}^{(1)} = (\cos\varphi\cos\theta, \sin\varphi\cos\theta, -\sin\theta), \quad \vec{e}^{(2)} = (-\sin\varphi, \cos\varphi, 0). \tag{4}$$

For the sake of simplicity, we use in most of the text the specification (1) in terms of the components of the phase normal.

PERTURBATION FORMULA FOR POLARIZATION

The correction Δg_i of the qP wave polarization vector from its orientation in a background isotropic medium, described by the P and S wave velocities α and β, is given by the formula, see Pšenčík (1998)

$$\Delta g_i = \alpha \frac{\partial \Delta\tau}{\partial q_I} e_i^{(I)} + \frac{B_{13} e_i^{(1)}}{\alpha^2 - \beta^2} + \frac{B_{23} e_i^{(2)}}{\alpha^2 - \beta^2}. \tag{5}$$

The symbols B_{mn} denote elements of the *weak anisotropy* matrix,

$$B_{mn} = \Delta a_{ijkl} e_i^{(m)} e_j^{(3)} e_l^{(3)} e_k^{(n)}. \tag{6}$$

The symbol Δa_{ijkl} in (6) denotes the deviation of WA parameters of the anisotropic medium from elastic parameters defining the isotropic background. Explicit expressions for B_{33}, B_{13} and B_{23} read, see Pšenčík & Gajewski (1998):

$$\begin{aligned} B_{33} = 2\alpha^2 [&\epsilon_z n_3^4 + 2n_3^3(\epsilon_{34} n_2 + \epsilon_{35} n_1) + n_3^2(\delta_x n_1^2 + \delta_y n_2^2 + 2\chi_z n_1 n_2) \\ &+ 2n_3(\chi_x n_1^2 n_2 + \chi_y n_1 n_2^2 + \epsilon_{15} n_1^3 + \epsilon_{24} n_2^3) + \epsilon_x n_1^4 + \delta_z n_1^2 n_2^2 \\ &+ \epsilon_y n_2^4 + 2\epsilon_{16} n_1^3 n_2 + 2\epsilon_{26} n_1 n_2^3], \end{aligned} \tag{7a}$$

$$\begin{aligned} B_{13} = \alpha^2 D^{-1} \Big[&2\epsilon_z n_3^5 + n_3^4(\epsilon_{34} n_2 + \epsilon_{35} n_1) + n_3^3(\delta_x n_1^2 + \delta_y n_2^2 + 2\chi_z n_1 n_2 - 2\epsilon_z) \\ &+ n_3^2[(4\chi_x - 3\epsilon_{34}) n_1^2 n_2 + (4\chi_y - 3\epsilon_{35}) n_1 n_2^2 + (4\epsilon_{15} - 3\epsilon_{35}) n_1^3 + (4\epsilon_{24} - 3\epsilon_{34}) n_2^3] \\ &+ n_3[(2\delta_z - \delta_x - \delta_y) n_1^2 n_2^2 + 2(2\epsilon_{16} - \chi_z) n_1^3 n_2 + 2(2\epsilon_{26} - \chi_z) n_1 n_2^3 + (2\epsilon_x - \delta_x) n_1^4 + (2\epsilon_y - \delta_y) n_2^4] \\ &- \chi_x n_1^2 n_2 - \chi_y n_1 n_2^2 - \epsilon_{15} n_1^3 - \epsilon_{24} n_2^3 \Big] \end{aligned} \tag{7b}$$

and

$$B_{23} = \alpha^2 D^{-1}\Big[n_3^3(\epsilon_{34}n_1 - \epsilon_{35}n_2) + n_3^2[(\delta_y - \delta_x)n_1n_2 + \chi_z n_1^2 - \chi_z n_2^2]$$
$$+n_3[(2\chi_y - 3\epsilon_{15})n_1^2n_2 - (2\chi_x - 3\epsilon_{24})n_1n_2^2 + \chi_x n_1^3 - \chi_y n_2^3]$$
$$+(\delta_z - 2\epsilon_x)n_1^3n_2 + (2\epsilon_y - \delta_z)n_1n_2^3 + 3(\epsilon_{26} - \epsilon_{16})n_1^2n_2^2 + \epsilon_{16}n_1^4 - \epsilon_{26}n_2^4\Big]. \qquad (7c)$$

The quantity D is defined in Eq.(2) and the WA parameters are defined as follows

$$\epsilon_x = \frac{A_{11}-\alpha^2}{2\alpha^2}, \quad \epsilon_y = \frac{A_{22}-\alpha^2}{2\alpha^2}, \quad \epsilon_z = \frac{A_{33}-\alpha^2}{2\alpha^2},$$
$$\delta_x = \frac{A_{13}+2A_{55}-\alpha^2}{\alpha^2}, \quad \delta_y = \frac{A_{23}+2A_{44}-\alpha^2}{\alpha^2}, \quad \delta_z = \frac{A_{12}+2A_{66}-\alpha^2}{\alpha^2},$$
$$\chi_x = \frac{A_{14}+2A_{56}}{\alpha^2}, \quad \chi_y = \frac{A_{25}+2A_{46}}{\alpha^2}, \quad \chi_z = \frac{A_{36}+2A_{45}}{\alpha^2},$$
$$\epsilon_{15} = \frac{A_{15}}{\alpha^2}, \quad \epsilon_{16} = \frac{A_{16}}{\alpha^2}, \quad \epsilon_{24} = \frac{A_{24}}{\alpha^2}, \quad \epsilon_{26} = \frac{A_{26}}{\alpha^2}, \quad \epsilon_{34} = \frac{A_{34}}{\alpha^2}, \quad \epsilon_{35} = \frac{A_{35}}{\alpha^2}. \qquad (8)$$

The first term on the RHS of Eq.(5) represents a correction of the slowness vector at a receiver from its orientation in the background isotropic medium. This correction is due to the perturbation of the phase front, which, in turn, is due to the perturbation Δa_{ijkl}. This term can be determined by integration along the ray in the background isotropic medium, see, e.g., Farra & Le Bégat (1995). Here, we propose an alternative way of its determination, see below.

The second and third terms on the RHS of Eq.(5) represent corrections of the polarization vector due to the perturbation Δa_{ijkl} with respect to the perturbed slowness vector. They can also be obtained by the use of the perturbation method, see Jech & Pšenčík (1989), Pšenčík & Gajewski (1998).

All terms in (5) lead to the deviation of the polarization vector from the direction $e_i^{(3)} = n_i$ of the polarization vector in the background isotropic medium but none of the terms changes linearity of the polarization. The terms in Eq.(5) remain non-zero even in homogeneous media.

Using (5), we can write the first-order expression for the polarization vector in an arbitrary WA medium in the following form

$$g_i \sim n_i + \Delta g_i = n_i + \alpha\frac{\partial \Delta\tau}{\partial q_I}e_i^{(I)} + \frac{B_{13}e_i^{(1)}}{\alpha^2-\beta^2} + \frac{B_{23}e_i^{(2)}}{\alpha^2-\beta^2}. \qquad (9)$$

Multiplying Eq.(9) by $e_i^{(K)}$, we get the following equation

$$\alpha\frac{\partial\Delta\tau}{\partial q_K} + \frac{B_{K3}}{\alpha^2-\beta^2} = g_i e_i^{(K)}. \qquad (10)$$

The RHS of Eq.(10) can be determined from observations. The second term on the LHS of Eq.(10) is related to sought WA parameters through Eqs.(7b) and (7c). The partial derivative in the first term on the LHS of Eq.(10) can be rewritten as follows:

$$\frac{\partial \Delta\tau}{\partial q_K} = \frac{\partial(\tau - \tau^0)}{\partial q_K} = \frac{\partial \tau}{\partial q_K} - \frac{\partial \tau^0}{\partial q_K} = \frac{\partial \tau}{\partial x_i}\frac{\partial x_i}{\partial q_K} - \frac{\partial \tau^0}{\partial x_i}\frac{\partial x_i}{\partial q_K} = p_i e_i^{(K)} - p_i^0 e_i^{(K)} = p_i e_i^{(K)}. \quad (11)$$

Thus the problem of determination of the first term on the LHS of Eq.(10) reduces to the determination of the slowness vector p_i in the WA medium.

DETERMINATION OF THE SLOWNESS VECTOR

The most straightforward way of the determination of the slowness vector in the anisotropic (perturbed) medium is to follow Farra & Le Bégat (1995). We propose an alternative way, which represents a generalization of the approach used by Gaiser (1990). For the determination of elastic parameters of a VTI medium from all three waves, Gaiser used slowness vector estimates. We could do the same for qP waves, and we would, in this way, generalize Gaiser's approach to arbitrary but weakly anisotropic media, see the Appendix. Instead of seeking the WA parameters from measured slowness vectors, we propose to seek the WA parameters from the measured qP wave polarizations.

As Gaiser (1990), we assume that the medium is *laterally homogeneous*. In contrast to him, however, we consider arbitrary anisotropy but weak. The reason for limiting ourselves to the case of laterally homogeneous media is the same as in Gaiser's case: In the VSP configuration, "*only the vertical in situ component of slowness* (vector) *can be estimated from multiple offset VSP travel times, observed in a "single" borehole. The horizontal component must be obtained from travel time differences between adjacent sources. It is important to realize that these components are related to elastic properties at the source and not to rock properties between receivers at depth. To consider them appropriate at the receiver requires the assumption of lateral homogeneity where horizontal* (component of) *slowness* (vector) *is independent of depth (Snell's law)*".

In contrast to Gaiser's laterally homogeneous VTI medium, in which the slowness vector always remains confined to the vertical plane containing the source and the borehole, the slowness vector in our model can deviate from that plane. We must, therefore, determine the third component of the slowness vector too. It can be determined from the eikonal equation

$$a_{ijkl} p_i p_l g_j g_k = 1. \quad (12)$$

The slowness vector at a receiver can be expressed as follows

$$p_i = p_i^0 + \Delta\xi\nu_i = b_i + \xi\nu_i + \Delta\xi\nu_i, \quad (13)$$

where b_i is the part of the slowness vector, which is assumed to be known. It can be determined from measurements in the way described by Gaiser (1990). The vector b_i is confined to the vertical plane containing the source and the bowehole. For simplicity, we shall identify this plane as the (x, z) plane. The symbol ν_i denotes a unit vector in the direction, in which the projection of p_i is unknown, in our case in the y direction. We choose the vector ν_i so that it points into the positive direction of axis y. With the symbol p_i^0 we denote an estimate of the slowness vector in the background isotropic medium. The symbol ξ denotes the size of the projection of the slowness vector p_i^0 into ν_i, $\Delta\xi$ is its perturbation due to the deviation Δa_{ijkl} of the anisotropic medium from the isotropic background.

We choose the vector p_i^0 so that it is parallel with the observed polarization vector g_i. This means that

$$\alpha p_i^0 = g_i \quad \Rightarrow \quad \alpha\xi = g_i\nu_i \tag{14}$$

Eq.(14) contains two unknowns, the P wave velocity of the isotropic background α and the factor ξ. They can be determined from Eq.(14) and from the eikonal equation for the isotropic background

$$p_i^0 p_i^0 = \alpha^{-2}. \tag{15}$$

We can find that

$$\xi = \frac{g_i\nu_i}{\alpha}, \quad \alpha^2 = \frac{1-(g_i\nu_i)^2}{b_i b_i}. \tag{16}$$

In a weakly anisotropic medium, taking into account Eq.(13), Eq. (12) can be expanded as follows

$$a_{ijkl}^0 p_i^0 p_l^0 g_j^0 g_k^0 + \Delta a_{ijkl} p_i^0 p_l^0 g_j^0 g_k^0 + 2a_{ijkl}^0 \Delta\xi \nu_i p_l^0 g_j^0 g_k^0 = 1. \tag{17}$$

Here a_{ijkl}^0 denotes a tensor of elastic parameters of the background isotropic medium

$$a_{ijkl}^0 = (\alpha^2 - 2\beta^2)\delta_{ij}\delta_{kl} + \beta^2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}). \tag{18}$$

and g_i^0 are the polarization vectors in the background isotropic medium. Since we consider only qP waves, we have $g_i^0 = e_i^{(3)}$. Taking into account that

$$a_{ijkl}^0 p_i^0 p_l^0 g_j^0 g_k^0 = 1, \tag{19}$$

Eq.(17) can be solved for $\Delta\xi$. It yields

$$\Delta\xi = -\frac{B_{33}}{2\alpha^4\xi}. \tag{20}$$

Note that equation (20) cannot be used when $\xi = 0$, i.e. when the observed polarization is confined to the plane (x, z). We therefore consider separately the cases $\xi = 0$ and $\xi \neq 0$. In the case $\xi \neq 0$, the slowness vector p_i has a form

$$p_i = b_i + \left(\frac{g_k\nu_k}{\alpha} - \frac{B_{33}}{2\alpha^4\xi}\right)\nu_i. \tag{21}$$

In the case $\xi = 0$, the slowness vector p_i is confined to the plane (x, z) and has a simple form

$$p_i = b_i. \tag{22}$$

EQUATIONS FOR INVERSION - GENERAL WEAK ANISOTROPY

We can now rewrite Eq.(10) in its final form. First, let us write it for the case $\xi \neq 0$. We get

$$\frac{B_{K3}}{\alpha^2-\beta^2}-\frac{B_{33}}{\alpha^2}\frac{\nu_i e_i^{(K)}}{2\alpha\xi}=[g_i-(g_k\nu_k)\nu_i]e_i^{(K)}-\alpha b_i e_i^{(K)}. \tag{23}$$

The elements of the matrix B_{mn} are given by Eqs.(7). They are lineraly related to the sought WA parameters (8).

For the case $\xi=0$ Eq.(10) yields

$$B_{K3}=(\alpha^2-\beta^2)(g_i-\alpha b_i)e_i^{(K)}. \tag{24}$$

In the considered (x,z) plane, Eqs.(1) reduces to

$$\vec{e}^{(1)}=(n_3,0,-n_1),\quad \vec{e}^{(2)}=(0,1,0), \tag{25}$$

Combining (23) and (24), we get

$$B_{13}=(\alpha^2-\beta^2)(g_i-\alpha b_i)e_i^{(1)},\quad B_{23}=0. \tag{26}$$

EQUATIONS FOR INVERSION - WEAK VTI

In VTI anisotropy the polarization of qP wave is cofined in the vertical plane containing the axis of symmetry, which means that in this case $\xi=0$. The WA parameters satisfy the following relations in this case:

$$\delta_y=\delta_x,\quad \delta_z=2\epsilon_x,\quad \epsilon_y=\epsilon_x,\quad \gamma_y=\gamma_x,$$

$$\chi_x=\chi_y=\chi_z=\epsilon_{15}=\epsilon_{16}=\epsilon_{24}=\epsilon_{26}=\epsilon_{34}=\epsilon_{35}=0 \tag{27}$$

If we use (27) in (26), we get simple equation for three WA parameters,

$$\delta_x\cos 2\theta-2\epsilon_z\cos^2\theta+2\epsilon_x\sin^2\theta=\frac{\alpha^2-\beta^2}{\alpha^2}\Big(\frac{g_1-\alpha b_1}{\sin\theta}-\frac{g_3-\alpha b_3}{\cos\theta}\Big). \tag{28}$$

If there are observations from several sources at a receiver, we can form a system of equations from Eq.(28) for ϵ_x, ϵ_z and δ_x.

CONCLUSIONS

A procedure for the determination of elastic parameters of a weak anisotropic medium of arbitrary symmetry at receivers situated in a borehole is suggested. It is proposed to determine the elastic parameters from the deviations of the particle motions from the direction of the corresponding phase normal. The medium is assumed to be weakly but arbitrarily anisotropic and vertically inhomogeneous. It is assumed that the effects of the free surface at considered receivers can be neglected. The proposed procedure is going to be tested on synthetic data. Any suggestion concerning practical applicability of the proposed scheme will be appreciated.

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APPENDIX

Let us assume that we found the slowness vector p_i by the method described above, see Eqs.(21), (22). This slowness vector can be expressed as follows

$$p_i = \frac{t_i}{c} \qquad (A.1)$$

where t_i is a unit vector specifying the direction of the slowness vector p_i. According to Pšenčík and Gajewski (1998) the phase velocity c for the direction t_i can be expressed in terms of WA parameters so that (A.1) reads

$$p_i = \frac{t_i}{c} \sim \frac{t_i}{\alpha} - \frac{t_i}{\alpha^2}\Delta c = \frac{t_i}{\alpha}(1 - \frac{B_{33}}{2\alpha^2}), \qquad (A.2)$$

where Δc denotes phase velocity deviation from a background P wave velocity α. Combination of (A.2) with (21) or (22) leads to an equation, which can be used for the determination of WA parameters contained in the term B_{33}.

In the special case of VTI anisotropy specified by the conditions (27), combination of (A.2) and (22) yields

$$c^{-1} = \alpha^{-1}(1 - \epsilon_z \cos^4\theta + \delta_x \sin^2\theta \cos^2\theta + \epsilon_x \sin^4\theta). \qquad (A.3)$$

If there are observations from several sources at a receiver, we can form a system of equations from Eq.(A.3) for ϵ_x, ϵ_z and δ_x.