Determination of weak anisotropy parameters from the qP wave slowness and particle motion measurements in laterally inhomogeneous media [1]

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Abstract

We propose an algorithm for local evaluation of weak anisotropy (WA) parameters from mesurements of slowness vectors and/or of deviations of measured qP wave particle motions from the slowness vectors at individual receivers in a borehole in a multi-azimuthal multiple-source offset VSP experiment. The formulae are derived under assumption of weak but arbitrary anisotropy and lateral inhomogeneity of the medium. The formulae represent generalization of an algorithm proposed by Pšenčík and Zheng (1998) for vertically inhomogeneous media. In this contribution, only basic formulae are presented. Tests of the proposed algorithm with synthetic data are under preparation.

INTRODUCTION

The formulae presented in this contribution represent a generalization of formulae presented by Pšenčík and Zheng (1998) in the last year Report and tested by Zheng and Pšenčík (1999) in this Report.

Let us consider a multi-azimuthal multiple-source offset VSP experiment with three-component recordings not affected by a free surface. Studied medium can be weakly anisotropic (WA) with arbitrary symmetry and laterally inhomogeneous. In order to determine WA parameters in a vicinity of each receiver, we define a background isotropic medium, from which the WA medium differs only slightly. The background medium is specified by the P and S wave velocities α and β, which can differ from receiver to receiver. In the background isotropic medium let us introduce three mutually perpendicular unit vectors $e_i^{(1)}$, $e_i^{(2)}$ and $e_i^{(3)}$ so that the vector $e_i^{(3)}$ is parallel to the polarization vector of the P wave in the background isotropic medium. The vector $e_i^{(3)}$ is also parallel to the phase normal n_i since in an isotropic medium the phase normal and polarization vector of a P wave are identical, i.e. $e_i^{(3)} = n_i$. The vectors $e_i^{(1)}$ and $e_i^{(2)}$ can be chosen arbitrarily in the plane perpendicular to n_i. A practical choice of vectors $e_i^{(1)}$, $e_i^{(2)}$ expressed in terms

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of components of the vector $e_i^{(3)}$ seems to be the following one, see Pšenčík and Gajewski (1998):

$$\vec{e}^{(1)} = D^{-1}(n_1 n_3, n_2 n_3, n_3^2 - 1), \quad \vec{e}^{(2)} = D^{-1}(-n_2, n_1, 0), \quad \vec{e}^{(3)} = \vec{n} = (n_1, n_2, n_3), \tag{1}$$

where

$$D = (n_1^2 + n_2^2)^{1/2}, \quad n_1^2 + n_2^2 + n_3^2 = 1. \tag{2}$$

Let us specify the phase normal as

$$\vec{n} = (\cos\varphi \sin\theta, \sin\varphi \sin\theta, \cos\theta). \tag{3}$$

Here φ denotes an azimuthal angle, θ a polar angle, ($0 \leq \varphi \leq 2\pi$, $0 \leq \theta \leq \pi$). In this case we have $D = \sin\theta$ and vectors $\vec{e}^{(1)}$ and $\vec{e}^{(2)}$ from Eq.(1) can be rewritten in the form

$$\vec{e}^{(1)} = (\cos\varphi \cos\theta, \sin\varphi \cos\theta, -\sin\theta), \quad \vec{e}^{(2)} = (-\sin\varphi, \cos\varphi, 0). \tag{4}$$

For the sake of simplicity, we use in most of the text the specification (1) in terms of the components of the phase normal.

First, we present perturbation formulae for the qP wave polarization vector, then for its slowness vector. In the end we form equations for inversion from them.

PERTURBATION FORMULA FOR POLARIZATION

The deviation Δg_i of the polarization vector of a qP wave in an anisotropic medium from the orientation of the polarization vector of the corresponding P wave in a background isotropic medium, specified by the P and S wave velocities α and β, is given by the formula, see Pšenčík (1998)

$$\Delta g_i = \alpha \frac{\partial \Delta\tau}{\partial q_I} e_i^{(I)} + \frac{B_{13} e_i^{(1)}}{\alpha^2 - \beta^2} + \frac{B_{23} e_i^{(2)}}{\alpha^2 - \beta^2}. \tag{5}$$

The symbols B_{mn} denote elements of the *weak anisotropy* matrix,

$$B_{mn} = \Delta a_{ijkl} e_i^{(m)} e_j^{(3)} e_l^{(3)} e_k^{(n)}. \tag{6}$$

The symbol Δa_{ijkl} in (6) denotes the deviation of elastic parameters of the anisotropic medium from elastic parameters defining the isotropic background. Explicite expressions for B_{33}, B_{13} and B_{23} read, see Pšenčík & Gajewski (1998):

$$\begin{aligned} B_{33} = 2\alpha^2 [&\epsilon_z n_3^4 + 2n_3^3(\epsilon_{34} n_2 + \epsilon_{35} n_1) + n_3^2(\delta_x n_1^2 + \delta_y n_2^2 + 2\chi_z n_1 n_2) \\ &+ 2n_3(\chi_x n_1^2 n_2 + \chi_y n_1 n_2^2 + \epsilon_{15} n_1^3 + \epsilon_{24} n_2^3) + \epsilon_x n_1^4 + \delta_z n_1^2 n_2^2 \\ &+ \epsilon_y n_2^4 + 2\epsilon_{16} n_1^3 n_2 + 2\epsilon_{26} n_1 n_2^3], \end{aligned} \tag{7a}$$

$$B_{13} = \alpha^2 D^{-1} \Big[2\epsilon_z n_3^5 + n_3^4(\epsilon_{34} n_2 + \epsilon_{35} n_1) + n_3^3(\delta_x n_1^2 + \delta_y n_2^2 + 2\chi_z n_1 n_2 - 2\epsilon_z)$$

$$+n_3^2[(4\chi_x-3\epsilon_{34})n_1^2n_2+(4\chi_y-3\epsilon_{35})n_1n_2^2+(4\epsilon_{15}-3\epsilon_{35})n_1^3+(4\epsilon_{24}-3\epsilon_{34})n_2^3]$$
$$+n_3[(2\delta_z-\delta_x-\delta_y)n_1^2n_2^2+2(2\epsilon_{16}-\chi_z)n_1^3n_2+2(2\epsilon_{26}-\chi_z)n_1n_2^3+(2\epsilon_x-\delta_x)n_1^4+(2\epsilon_y-\delta_y)n_2^4]$$
$$-\chi_xn_1^2n_2-\chi_yn_1n_2^2-\epsilon_{15}n_1^3-\epsilon_{24}n_2^3\Big] \tag{7b}$$

and

$$B_{23}=\alpha^2D^{-1}\Big[n_3^3(\epsilon_{34}n_1-\epsilon_{35}n_2)+n_3^2[(\delta_y-\delta_x)n_1n_2+\chi_zn_1^2-\chi_zn_2^2]$$
$$+n_3[(2\chi_y-3\epsilon_{15})n_1^2n_2-(2\chi_x-3\epsilon_{24})n_1n_2^2+\chi_xn_1^3-\chi_yn_2^3]$$
$$+(\delta_z-2\epsilon_x)n_1^3n_2+(2\epsilon_y-\delta_z)n_1n_2^3+3(\epsilon_{26}-\epsilon_{16})n_1^2n_2^2+\epsilon_{16}n_1^4-\epsilon_{26}n_2^4\Big]. \tag{7c}$$

The quantity D is defined in Eq.(2) and the WA parameters appearing in (7) are defined as follows

$$\epsilon_x=\frac{A_{11}-\alpha^2}{2\alpha^2},\quad \epsilon_y=\frac{A_{22}-\alpha^2}{2\alpha^2},\quad \epsilon_z=\frac{A_{33}-\alpha^2}{2\alpha^2},$$
$$\delta_x=\frac{A_{13}+2A_{55}-\alpha^2}{\alpha^2},\quad \delta_y=\frac{A_{23}+2A_{44}-\alpha^2}{\alpha^2},\quad \delta_z=\frac{A_{12}+2A_{66}-\alpha^2}{\alpha^2},$$
$$\chi_x=\frac{A_{14}+2A_{56}}{\alpha^2},\quad \chi_y=\frac{A_{25}+2A_{46}}{\alpha^2},\quad \chi_z=\frac{A_{36}+2A_{45}}{\alpha^2},$$
$$\epsilon_{15}=\frac{A_{15}}{\alpha^2},\quad \epsilon_{16}=\frac{A_{16}}{\alpha^2},\quad \epsilon_{24}=\frac{A_{24}}{\alpha^2},\quad \epsilon_{26}=\frac{A_{26}}{\alpha^2},\quad \epsilon_{34}=\frac{A_{34}}{\alpha^2},\quad \epsilon_{35}=\frac{A_{35}}{\alpha^2}. \tag{8}$$

The first term on the RHS of Eq.(5) represents a correction of the slowness vector at a receiver from its orientation in the background isotropic medium. This correction is due to the perturbation of the phase front, which, in turn, is due to the perturbation Δa_{ijkl}. This term can be determined by integration along the ray in the background isotropic medium, see, e.g., Farra & Le Bégat (1995). Here, we propose an alternative way of its determination requiring no integration, see below.

The second and third terms on the RHS of Eq.(5) represent deviations of the polarization vector of the qP wave from its slowness vector. They can also be obtained by the use of the perturbation method, see Jech & Pšenčík (1989), Pšenčík & Gajewski (1998).

All terms in (5) lead to the deviation of the polarization vector from the direction $e_i^{(3)}=n_i$ of the polarization vector in the background isotropic medium but none of the terms changes linearity of the polarization. The terms in Eq.(5) remain non-zero even in homogeneous media.

Using (5), we can write the first-order expression for the polarization vector in an arbitrary WA medium in the following form

$$g_i\sim n_i+\Delta g_i=n_i+\alpha\frac{\partial\Delta\tau}{\partial q_I}e_i^{(I)}+\frac{B_{13}e_i^{(1)}}{\alpha^2-\beta^2}+\frac{B_{23}e_i^{(2)}}{\alpha^2-\beta^2}. \tag{9}$$

Multiplying Eq.(9) by $e_i^{(K)}$, we get the following equation

$$\alpha\frac{\partial\Delta\tau}{\partial q_K}+\frac{B_{K3}}{\alpha^2-\beta^2}=g_ie_i^{(K)}. \tag{10}$$

The RHS of Eq.(10) can be determined from observations. The second term on the LHS of Eq.(10) is related to sought WA parameters through Eqs.(7b) and (7c). The partial derivative in the first term on the LHS of Eq.(10) can be rewritten as follows:

$$\frac{\partial \Delta\tau}{\partial q_K} = \frac{\partial(\tau - \tau^0)}{\partial q_K} = \frac{\partial \tau}{\partial q_K} - \frac{\partial \tau^0}{\partial q_K} = \frac{\partial \tau}{\partial x_i}\frac{\partial x_i}{\partial q_K} - \frac{\partial \tau^0}{\partial x_i}\frac{\partial x_i}{\partial q_K} = p_i e_i^{(K)} - p_i^0 e_i^{(K)} = p_i e_i^{(K)}. \quad (11)$$

Here p_i is the lsowness vector of the qP wave in the anisotropic medium, p_i^0 is the slowness vector of the corresponding P wave in the background isotropic medium. Eq.(10) can be now rewritten into the following form

$$\alpha p_i e_i^{(K)} + \frac{B_{K3}}{\alpha^2 - \beta^2} = g_i e_i^{(K)}. \quad (12)$$

PERTURBATION FORMULA FOR SLOWNESS VECTOR

The slowness vector at a receiver can be expressed in the following way:

$$p_i = p_i^0 + \Delta\xi i_i + \Delta\zeta j_i + \Delta\eta k_i = (\xi + \Delta\xi)i_i + (\zeta + \Delta\zeta)j_i + (\eta + \Delta\eta)k_i. \quad (13)$$

The symbols i_i, j_i and k_i represent unit vectors along the $x-$, $y-$ and $z-$axis, respectively. The symbols ξ, ζ and η denote sizes of projections of the slowness vector p_i^0 into i_i, j_i and k_i. $\eta_i + \Delta\eta_i$ is the $z-$component of the slowness vector p_i, which is assumed to be known. It can be determined from measurements in the way described by Gaiser (1990). $\Delta\xi$ and $\Delta\zeta$ are unknown perturbations of ξ and ζ caused by the deviation Δa_{ijkl} of the anisotropic medium from the isotropic background.

The slowness vector p_i^0 in the background isotropic medium reads

$$p_i^0 = \alpha^{-1} n_i. \quad (14)$$

This means that ξ, ζ and η_i in Eq.(13) read

$$\xi = \frac{n_1}{\alpha}, \quad \zeta = \frac{n_2}{\alpha}, \quad \eta_i = \frac{n_3}{\alpha}. \quad (15)$$

The P wave velocity in the isotropic backround can be chosen as an average of the values

$$\alpha_m^2 = \frac{(n_3^{(m)})^2}{\eta^2} \quad (16)$$

obtained at a given receiver for the ray from the m-th source.

Let us now insert the expression for the slowness vector (13) into the eikonal equation for anisotropic media

$$a_{ijkl} p_i p_l g_j g_k = 1. \quad (17)$$

In weakly anisotropic media the eikonal equation can be expanded as follows

$$a^0_{ijkl}p^0_i p^0_l g^0_j g^0_k + \Delta a_{ijkl}p^0_i p^0_l g^0_j g^0_k + 2a^0_{1jkl}\Delta\xi p^0_l g^0_j g^0_k + 2a^0_{2jkl}\Delta\zeta p^0_l g^0_j g^0_k + 2a^0_{3jkl}\Delta\eta p^0_l g^0_j g^0_k = 1. \quad (18)$$

Here a^0_{ijkl} denotes a tensor of elastic parameters of the background isotropic medium

$$a^0_{ijkl} = (\alpha^2 - 2\beta^2)\delta_{ij}\delta_{kl} + \beta^2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}). \quad (19)$$

and g^0_i are the polarization vectors in the background isotropic medium. Since we consider only qP waves, we have $g^0_i = e^{(3)}_i = n_i$. Taking into account that

$$a^0_{ijkl}p^0_i p^0_l g^0_j g^0_k = 1, \quad (20)$$

Eq.(18) can be rewritten into the final form

$$B_{33} + 2\alpha^4\xi\Delta\xi + 2\alpha^4\zeta\Delta\zeta = -2\alpha^4\eta\Delta\eta. \quad (21)$$

EQUATIONS FOR INVERSION - GENERAL WEAK ANISOTROPY

Eq.(21) represents a set of equations for the WA parameters and for perturbations of the slowness vector, $\Delta\xi$ and $\Delta\zeta$ for each source-receiver pair. We have clearly more unknowns than equations. Additional equations can be obtained from formulae for polarization.

Inserting Eq.(13) into Eq.(12), we get the following set of equations

$$\frac{B_{K3}}{\alpha^2 - \beta^2} + \alpha\Delta\xi e^{(K)}_1 + \alpha\Delta\zeta e^{(K)}_2 = g_i e^{(K)}_i - \alpha\Delta\eta e^{(K)}_3. \quad (22)$$

Equations (21) and (22) represent a set of 3xN_S linear equations for 15 WA parameters (8) and for 2xN_S unknown perturbations of the slowness vector, $\Delta\xi$ and $\Delta\zeta$. Here N_S is a number of sources generating waves recorded at a given receiver. The WA parameters enter in Eqs.(21) and (22) through the elements of the matrix B_{mn} given in Eqs.(7). We can see that in order to make the system of Eqs.(21), (22) overdetermined, we need N_S to be larger than 15.

CONCLUSIONS

Generalization of an algorithm proposed by Pšenčík and Zheng (1998) for laterally varying weakly anisotropic structures was proposed. In contrast to the algorithm for laterally homogeneous media, in which slowness or polarization information could be used independently in order to retrieve values of the WA parameters at individual receivers, in the present case information on slowness vectors and polarization must be used together. Tests of the proposed algorithm on synthetic data are under preparation.

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