NONLINEAR HYPOCENTRE DETERMINATION

Petr Bulant & *Luděk Klimeš*

Department of Geophysics, Faculty of Mathematics and Physics
Charles University in Prague
Ke Karlovu 3, 121 16 Praha 2, Czech Republic, http://sw3d.cz

The parameter space of the kinematic hypocentre determination consists of three hypocentral coordinates x_i and hypocentral time x_4. The data space consists of N measured arrival times corresponding to P waves or S waves at various receivers. The probability of measured arrival times is proportional to the nonnormalized data density function. We assume that the data density function is Gaussian, and is specified in terms of N mean arrival times and the $N \times N$ data covariance matrix corresponding to the measured arrival times. The data covariance matrix is usually diagonal, composed of the squares of the standard deviations of arrival times. We need at least 4 measured arrival times, but the more, the better.

The relation between the data and parameters is described by the nonnormalized theoretical density function. The theoretical density function describes the relation between the measured arrival times and the inaccurate theoretical travel times calculated in the inaccurate velocity model. We assume that the theoretical density function is Gaussian in each data subspace corresponding to particular hypocentral coordinates. The Gaussian theoretical density function is specified in terms of N mean theoretical travel times calculated in the mean velocity model, and the $N \times N$ matrix of geometrical covariances of theoretical travel times, which describes the inaccuracy of the velocity model.

In Cartesian coordinates, null information density function is unit, and parameter a priori density function is unit. Since both the data density function and the theoretical density function are Gaussian in each data subspace, the resulting a posteriori density function (Tarantola & Valette, 1982) is also Gaussian in each data subspace. We thus integrate the a posteriori density function with respect to data represented by measured arrival times, and obtain the resulting marginal a posteriori density function of hypocentral coordinates and hypocentral time.

For each hypocentral coordinates, the resulting marginal a posteriori density function is Gaussian with respect to hypocentral time, and is thus described by its maximum value with respect to hypocentral time, the corresponding mean hypocentral time and the standard deviation of hypocentral time. These three quantities are functions of hypocentral coordinates, and are discretized at the nodes of a sufficiently dense rectangular grid of points. The spatial dependence $\sigma_{\mathrm{P3}}(x_i)$ of the maximum value of the resulting marginal a posteriori density function describes the relative probability of hypocentral position. Its maximum value $\sigma_{\mathrm{P3}}^{\max}$ depends primarily on the relation between the estimated inaccuracy of the velocity model and the actual inaccuracy of the velocity model.

For 4 measured arrival times, the maximum value $\sigma_{\mathrm{P3}}^{\max}$ of the resulting marginal a posteriori density function is unit. For more measured arrival times, the maximum value $\sigma_{\mathrm{P3}}^{\max}$ of the resulting marginal a posteriori density function is smaller and we may use it for the estimation of the inaccuracy of the velocity model.

If both the data covariance matrix and the matrix of geometrical covariances of theoretical travel times are correctly estimated, the mean value $\langle y \rangle$ of arrival–time misfit $y = -2\ \ln(\sigma_{\mathrm{P3}}^{\max})$ should be $\langle y \rangle = N - 4$.

For example, we may assume that the covariance functions describing the relative inaccuracies of the P–wave and S–wave velocity models are power–law functions specified in terms of their Hurst exponents and their amplitudes. For given Hurst exponents, we may then use the mean value $\langle y \rangle$ of arrival–time misfit obtained from many events for estimating the amplitudes of the velocity–model covariance functions.

Reference

Tarantola, A. & Valette, B. (1982): Inverse problems = quest for information. *J. Geophys.*, **50**, 159–170.