Fast Sweeping Methods for Factored TTI Eikonal Equation

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Traveltime computation is a key tool for processing and interpretation of seismic data, especially in the presence of seismic anisotropy. An eikonal equation describes the traveltime of a propagating wave. Ray tracing and finite-difference methods are widely used to numerically solve the eikonal equation. Finite-difference based methods are computationally more efficient than two-point ray tracing and do not require interpolation of traveltimes on regular grids, as needed for imaging applications. Moreover, eikonal solvers rarely produce shadow zones which are common in ray tracing methods. Although limited to computing first arrival traveltimes, eikonal solvers can be used to image multiple arrivals.

However, finite difference based eikonal solution has upwind source singularity. This renders all finite difference solvers, even high-order ones, to exhibit at most polluted first-order convergence, since the initial error at the source can spread out to the whole domain. To combat this, one technique is to fix a neighborhood of the source so that one can carry out mesh refinement in other regions to quantify error behaviors (Cecil et al., 2006). Another approach is to use adaptive gridding near the source (Qian and Symes, 2002). However, these methods are ad hoc in nature and their accuracy and computational load is closely linked to complexity of the model. A better approach to tackle the problem of source singularity has been proposed by Fomel et al. (2009) for the isotropic case. They demonstrate clean first-order convergence by using the idea of factorization. The unknown traveltime function is factored into two multiplicative functions. One of these two factors is specified to capture source singularity, such that the other factor is differentiable in a neighborhood of the source. Later, this scheme was extended to handle elliptical anisotropy by Luo and Qian (2012).

Anisotropy deviating from elliptical anisotropy introduces higher-order nonlinearity into the eikonal equation, which makes it challenging to solve. Recently, a fast sweeping based method has been proposed to solve the eikonal equation for transversely isotropic (TI) media by iteratively solving a sequence of a much simpler elliptic eikonal equation (Waheed et al., 2014). The method relies on computing traveltime derivatives after each iteration. The upwind source singularity makes these derivatives inaccurate around the source region. Therefore, the final solution has much worse accuracy. We extend the idea of factorization to eikonal equations for TI media that significantly improves accuracy of the iterative scheme. Extensive numerical tests along with a discussion on the associated computational cost of the method will be presented.

References

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