3D i-CRS stacking operator

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Wave Inversion Technology

Motivation

2D i-CRS stacking operator (Schwarz et al., 2014):

- Double-square-root expression
- Robust regarding heterogeneity
- Leads to good results for high reflector curvatures
- Provides higher accuracy than CRS and conventional MF

Requirements for 3D i-CRS stacking operator:

- Computationally efficient solution
- Expressed in CRS parameters

Outline

- Introduction
- Theory:
  - 2D + ? = 3D
  - Why, why, how?
  - Make it easier!
- Quantitative studies
- Synthetic data example
- Conclusions and outlook

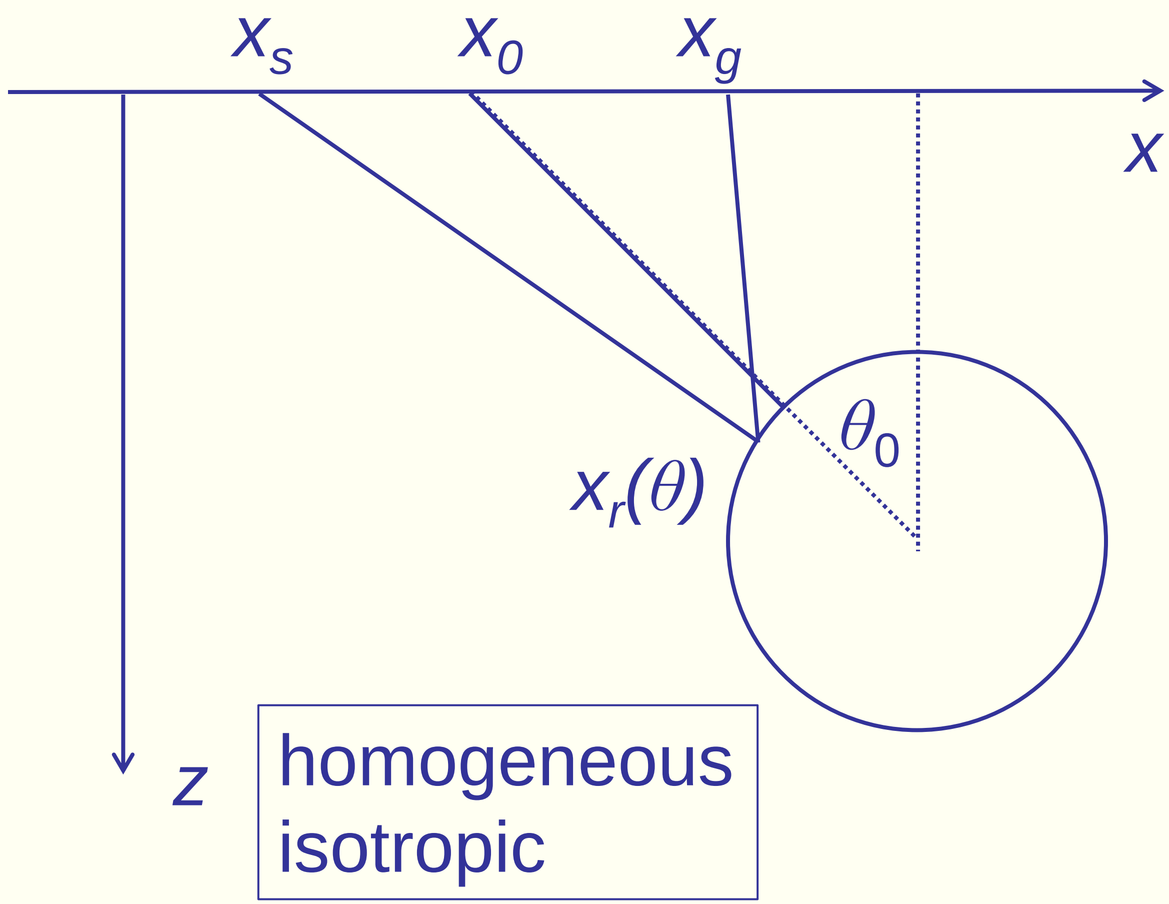
Introduction: 2D i-CRS

- CRS formula in 2D case reads:

$$t^2(\Delta x_m, h) = \left[t_0 + w\Delta x_m\right]^2 + N\Delta x_m^2 + Mh^2$$

- DSR expression for traveltime:

$$t = \sqrt{|\mathbf{x}_s - \mathbf{x}_r|^2}/V + \sqrt{|\mathbf{x}_g - \mathbf{x}_r|^2}/V$$



- Reflection point:

$$\frac{\partial t(\theta)}{\partial \theta} = 0,$$

$$\tan\theta = \tan\theta_0 + f(t_s, t_g)$$

Theory: 2D + ? = 3D

- CRS formula in 3D case reads:

$$t^2(\Delta\mathbf{x}_m, \mathbf{h}) = \left[t_0 + \mathbf{w}^{\mathbf{T}} \cdot \Delta\mathbf{x}_m\right]^2 + \Delta\mathbf{x}_m^{\mathbf{T}}\mathbf{N}\Delta\mathbf{x}_m + \mathbf{h}^{\mathbf{T}}\mathbf{M}\mathbf{h}$$

- DSR expression for traveltime:

$$t = \sqrt{|\mathbf{x}_s - \mathbf{x}_r|^2}/V_s + \sqrt{|\mathbf{x}_g - \mathbf{x}_r|^2}/V_g$$

- Reflection point:

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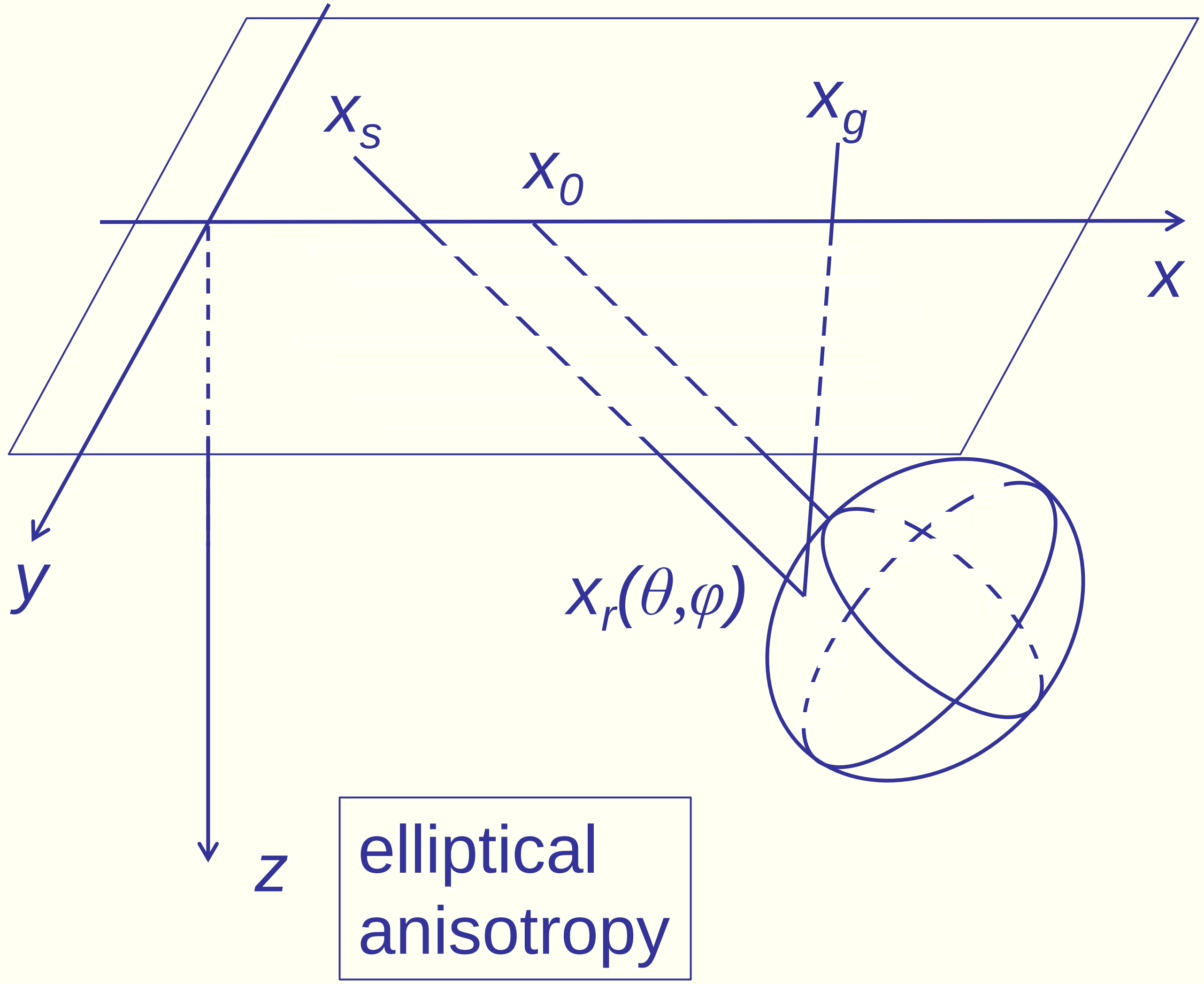
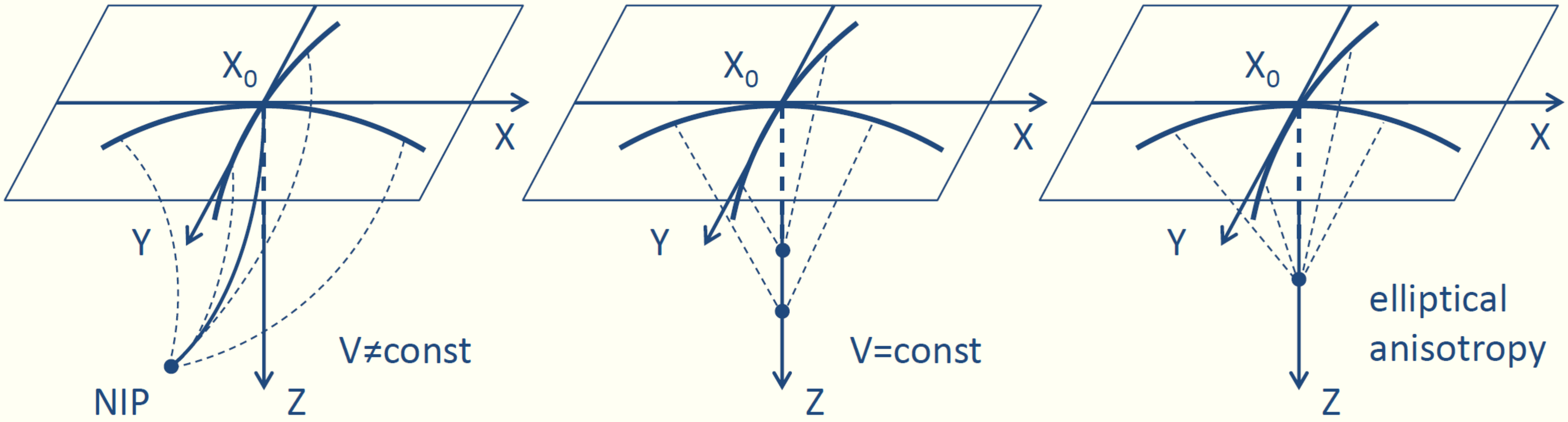


Why, why, how?

- Why is the auxiliary elliptical anisotropic medium required?
- Why is the ellipsoid considered as reflector?
- How does the presented algorithm work?

Why elliptical anisotropy?



- Elliptical anisotropic medium allows to focus wavefront with arbitrary curvature $\mathbf{K}_{NIP}$
- Simple relationship between anisotropic parameters and curvatures of wavefront:

$$A_{11} = \frac{2}{t_0}\frac{\zeta_0}{K_{NIP}^{11}}, \quad A_{22} = \frac{2}{t_0}\frac{\zeta_0}{K_{NIP}^{22}}, \quad A_{33} = \zeta_0^2$$

Why ellipsoid?

Reflector may be locally approximated in vicinity of NIP point:

$$z = z_0 - \frac{1}{2} \begin{pmatrix} x \\ y \end{pmatrix}^{\mathbf{T}} \mathbf{K} \begin{pmatrix} x \\ y \end{pmatrix}$$

Let us consider now ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{c^2} + \frac{z^2}{d^2} = 1, \quad \mathbf{K} = \begin{pmatrix} d/a^2 & d/c^2 \\ d/c^2 & d/b^2 \end{pmatrix}$$

Note! There are many ellipsoids with curvature $\mathbf{K}$. There is one free parameter: d

How? 3D i-CRS stacking operator

Traveltime of reflected wave is:

$$t(\theta,\phi) = \frac{\sqrt{X_s^2+Y_s^2+Z_s^2}}{\zeta(\theta_s,\phi_s)} + \frac{\sqrt{X_g^2+Y_g^2+Z_g^2}}{\zeta(\theta_g,\phi_g)}$$

where angles:

$$\tan\phi_i = \frac{Y_i}{X_i}, \qquad \tan\theta_i = \frac{\sqrt{X_i^2+Y_i^2}}{Z_i}, \qquad i = s, g$$

define group velocity:

$$\frac{1}{\zeta^2(\theta_i,\phi_i)} = \frac{\sin^2\theta_i\cos^2\phi_i}{A_{11}} + \frac{\sin^2\theta_i\sin^2\phi_i}{A_{22}} + \frac{\cos^2\theta_i}{A_{33}}$$

How? 3D i-CRS stacking operator

Traveltime of reflected wave is:

$$t(\theta,\phi) = \frac{\sqrt{\frac{A_{33}}{A_{11}}X_s^2 + \frac{A_{33}}{A_{22}}Y_s^2 + Z_s^2}}{\zeta_0} + \frac{\sqrt{\frac{A_{33}}{A_{11}}X_g^2 + \frac{A_{33}}{A_{22}}Y_g^2 + Z_g^2}}{\zeta_0}$$

To find the reflection point we should minimize the traveltime and therefore to calculate the derivatives:

$$\frac{\partial t(\theta,\phi)}{\partial \theta} = 0, \qquad \frac{\partial t(\theta,\phi)}{\partial \phi} = 0$$

How? Iterative scheme

Condition $\frac{\partial t}{\partial \phi} = 0$ leads to the implicit equation for ϕ:

$$\tan\phi = \frac{A_{11}}{A_{22}}\frac{b}{a}\frac{\left(\frac{Y_s}{R_s} + \frac{Y_g}{R_g}\right)}{\left(\frac{X_s}{R_s} + \frac{X_g}{R_g}\right)} \qquad (1)$$

Condition $\frac{\partial t}{\partial \theta} = 0$ leads to the implicit equation for θ:

$$\tan\theta = \frac{\left(\frac{\Lambda_s}{R_s} + \frac{\Lambda_g}{R_g}\right)}{\left(\frac{Z_s}{R_s} + \frac{Z_g}{R_g}\right)} \qquad (2)$$

where:

$$\Lambda_i = \frac{A_{33}}{A_{11}}\frac{a}{d}X_i\cos\phi + \frac{A_{33}}{A_{22}}\frac{b}{d}Y_i\sin\phi, \qquad i = s, g$$

How? Iterative scheme

- Zero iteration: $\theta^{(0)} = 0, \quad \phi^{(0)} = \arctan(y_m/x_m)$
  - With initial values of angles we compute:

$$X_i^{(0)}, \quad Y_i^{(0)}, \quad Z_i^{(0)}, \quad R_i^{(0)}, \quad i = s, g$$

  - Formula (1) updates $\phi^{(1)}$
  - With $\phi^{(1)}$ we compute $\Lambda_s^{(0)}, \Lambda_g^{(0)}$
  - Formula (2) updates $\theta^{(1)}$
- If required, we make next iteration
- Finally, after n iterations we obtain angles $\theta^{(n)}$ and $\phi^{(n)}$ and substitute them into DSR formula for traveltime

Make it easier!

- In case of diffracted waves ellipsoid collapses to the point

Make it easier!

- The hyperbolic CRS operator:

$$t^2_{hyp}(\Delta \mathbf{x}_m, \mathbf{h}) = \left[t_0 + \mathbf{w}^\mathbf{T} \cdot \Delta \mathbf{x}_m\right]^2 + \Delta \mathbf{x}_m^\mathbf{T} \mathbf{M} \Delta \mathbf{x}_m + \mathbf{h}^\mathbf{T} \mathbf{M} \mathbf{h}$$

- i-CRS stacking operator, case of diffracted waves, exact formula:

$$t_{dif}(\Delta \mathbf{x}_m, \mathbf{h}) = \frac{1}{2}\sqrt{\left[t_0 + \mathbf{w}^\mathbf{T} \cdot \Delta \mathbf{x}_S\right]^2 + \Delta \mathbf{x}_S^\mathbf{T} \mathbf{M} \Delta \mathbf{x}_S} + \frac{1}{2}\sqrt{\left[t_0 + \mathbf{w}^\mathbf{T} \cdot \Delta \mathbf{x}_G\right]^2 + \Delta \mathbf{x}_G^\mathbf{T} \mathbf{M} \Delta \mathbf{x}_G}$$

Make it easier!

- Reflection point may be approximated by:

$$\Delta \mathbf{x_r} \approx [\mathbf{I} - \mathbf{K}_{NIP}^{-1}\mathbf{K}_N]\mathbf{R^T}\Delta \mathbf{x}_m$$

  - $\mathbf{R}$ is a matrix that accounts for the transformation from the ray-centred to the general Cartesian coordinate system
  - $\mathbf{K}_{NIP}$ and $\mathbf{K}_N$ are curvatures of NIP and normal wave

Make it easier!

- The hyperbolic CRS operator:

$$t_{hyp}^2(\Delta\mathbf{x}_m, \mathbf{h}) = \left[t_0 + \mathbf{w}^{\mathbf{T}} \cdot \Delta\mathbf{x}_m\right]^2 + \Delta\mathbf{x}_m^{\mathbf{T}}\mathbf{N}\Delta\mathbf{x}_m + \mathbf{h}^{\mathbf{T}}\mathbf{M}\mathbf{h}$$

- i-CRS stacking operator, general case, approximation:

$$t_{ref}(\Delta\mathbf{x}_m, \mathbf{h}) =$$

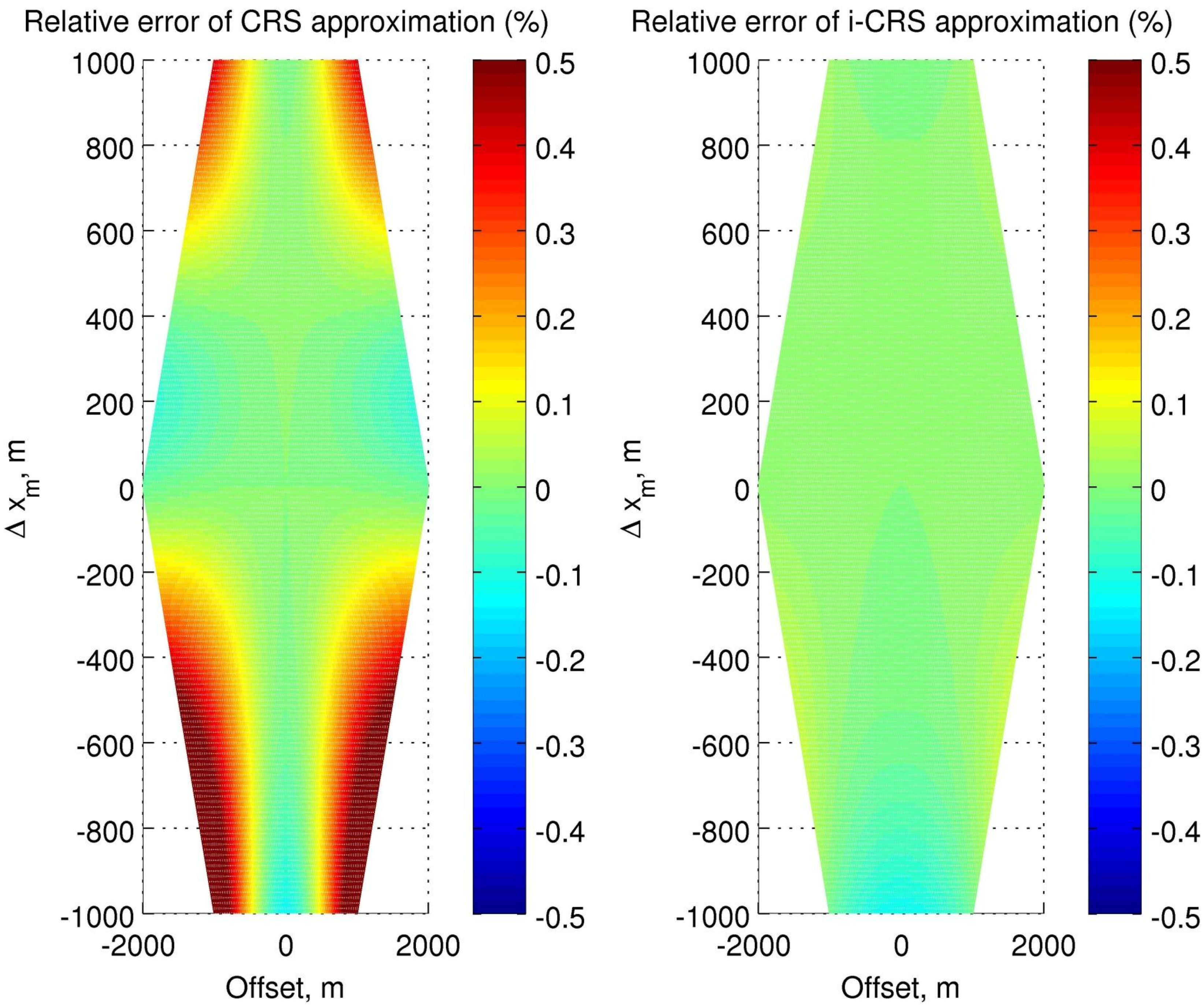
$$\frac{1}{2}\sqrt{\left[t_0 + \mathbf{w}^{\mathbf{T}} \cdot \Delta\mathbf{x}_S\right]^2 + \Delta\mathbf{x}_m^{\mathbf{T}}\mathbf{N}\Delta\mathbf{x}_m - 2\Delta\mathbf{x}_m^{\mathbf{T}}\mathbf{N}\mathbf{h} + \mathbf{h}^{\mathbf{T}}\mathbf{M}\mathbf{h}}$$

$$+\frac{1}{2}\sqrt{\left[t_0 + \mathbf{w}^{\mathbf{T}} \cdot \Delta\mathbf{x}_G\right]^2 + \Delta\mathbf{x}_m^{\mathbf{T}}\mathbf{N}\Delta\mathbf{x}_m + 2\Delta\mathbf{x}_m^{\mathbf{T}}\mathbf{N}\mathbf{h} + \mathbf{h}^{\mathbf{T}}\mathbf{M}\mathbf{h}}$$

Numerical example: traveltime

#	Reflector type	Radius	Medium	Depth
1	Sphere	1 km	Const	2 km
2	“Diffractor”	0.01 km	Const	2 km
3	“Plane”	10 km	Const	2 km
4	Ellipsoid	0.9, 2.8 km	Gradient	2 km

Numerical example: traveltime



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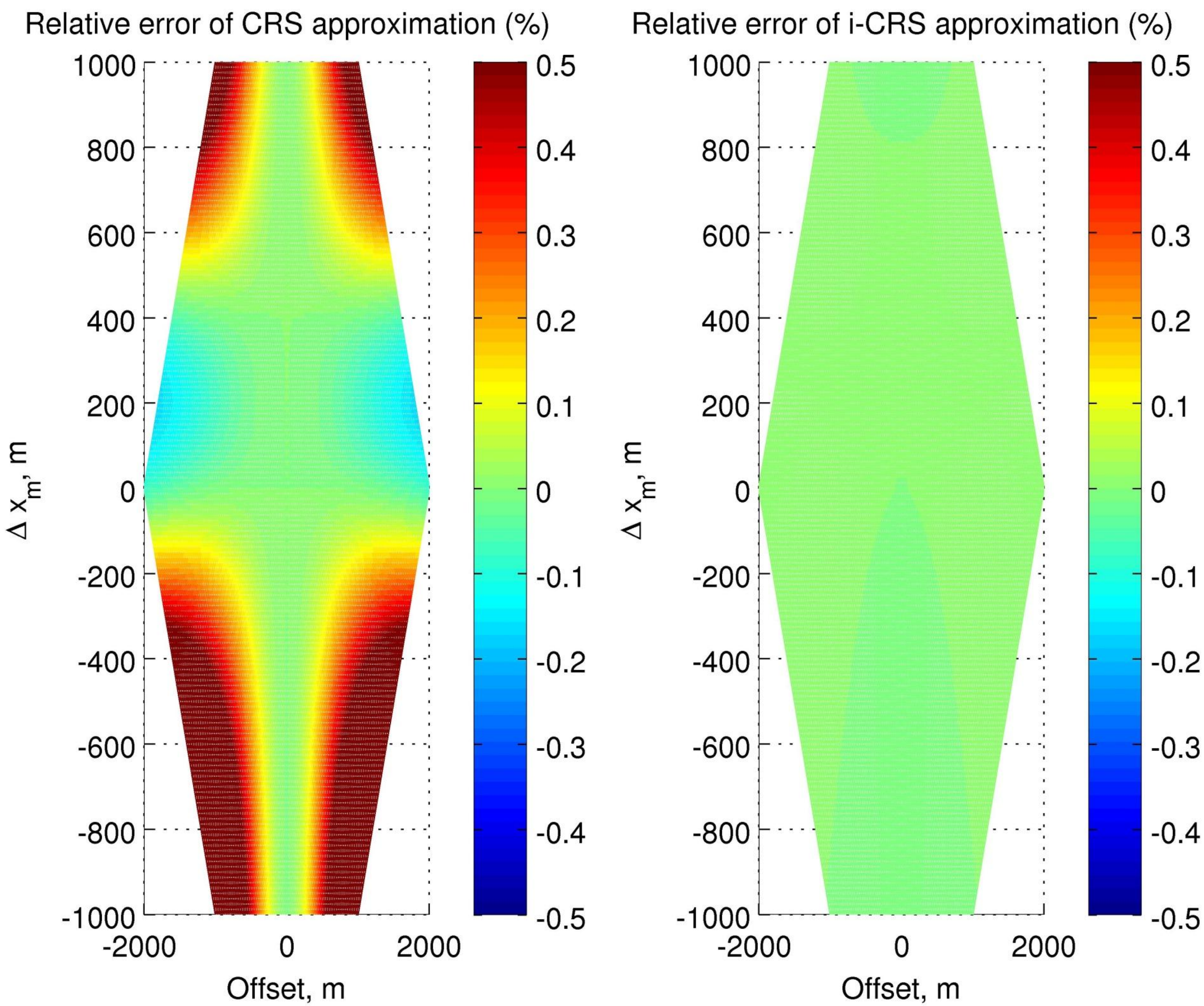

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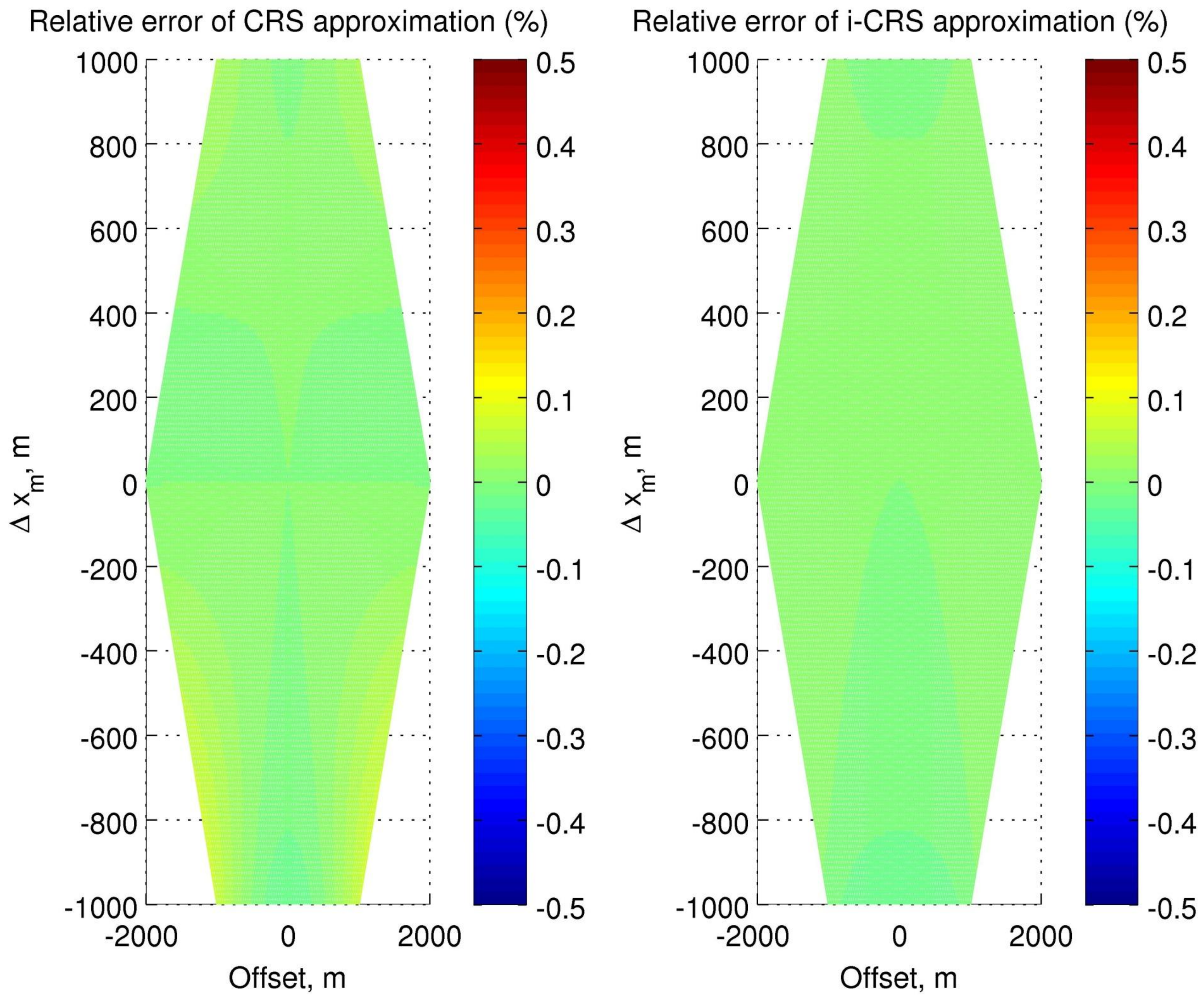

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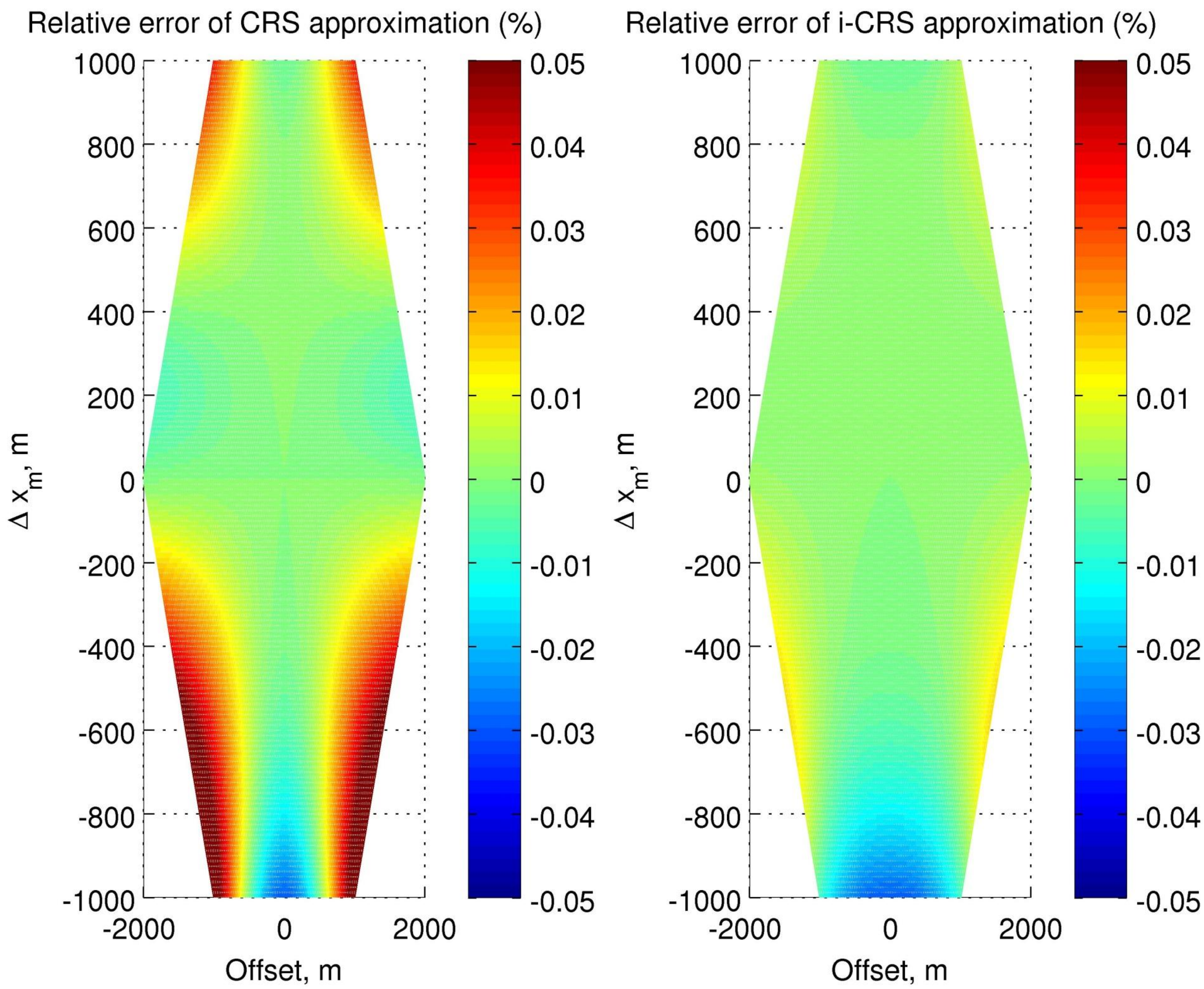

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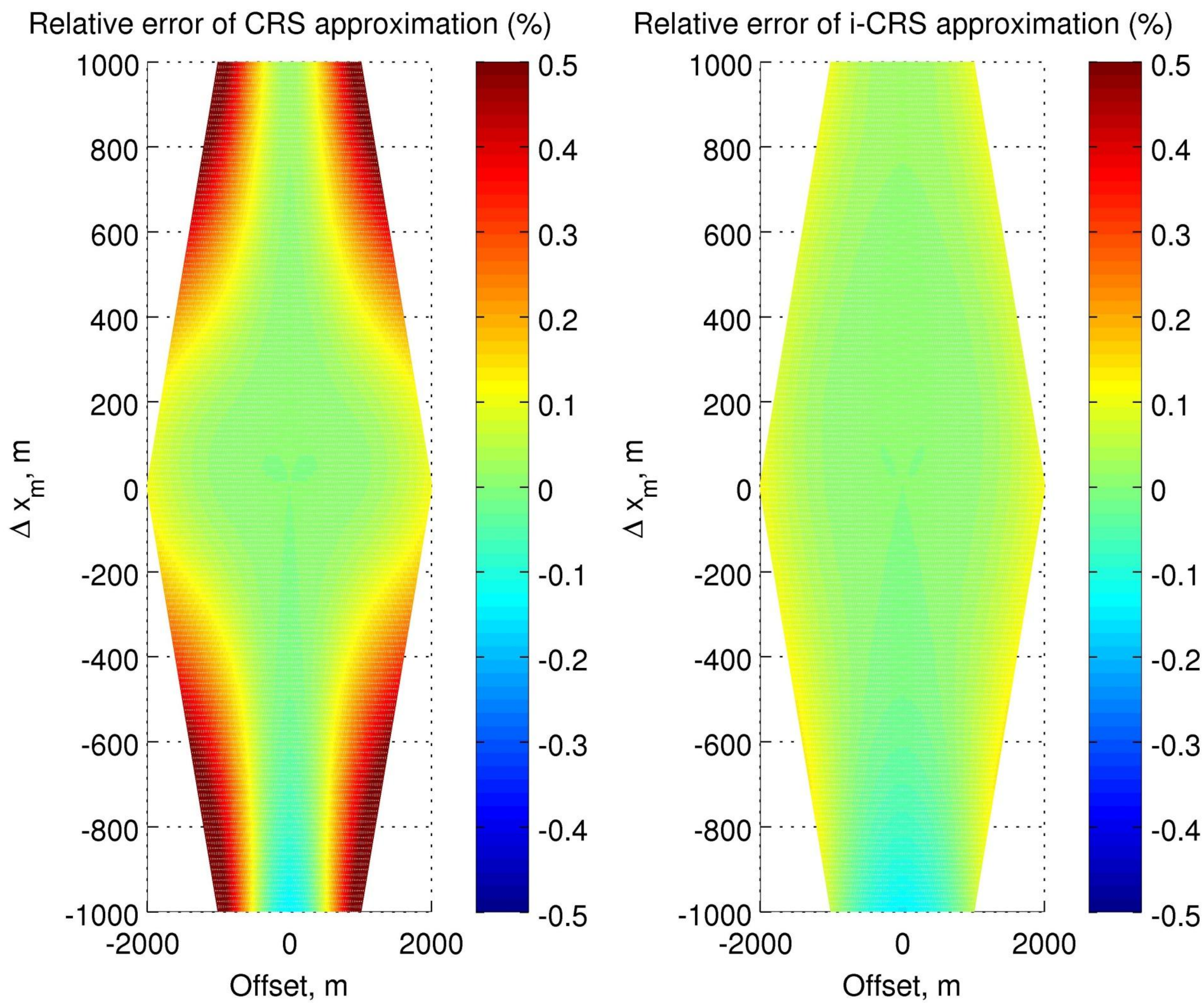
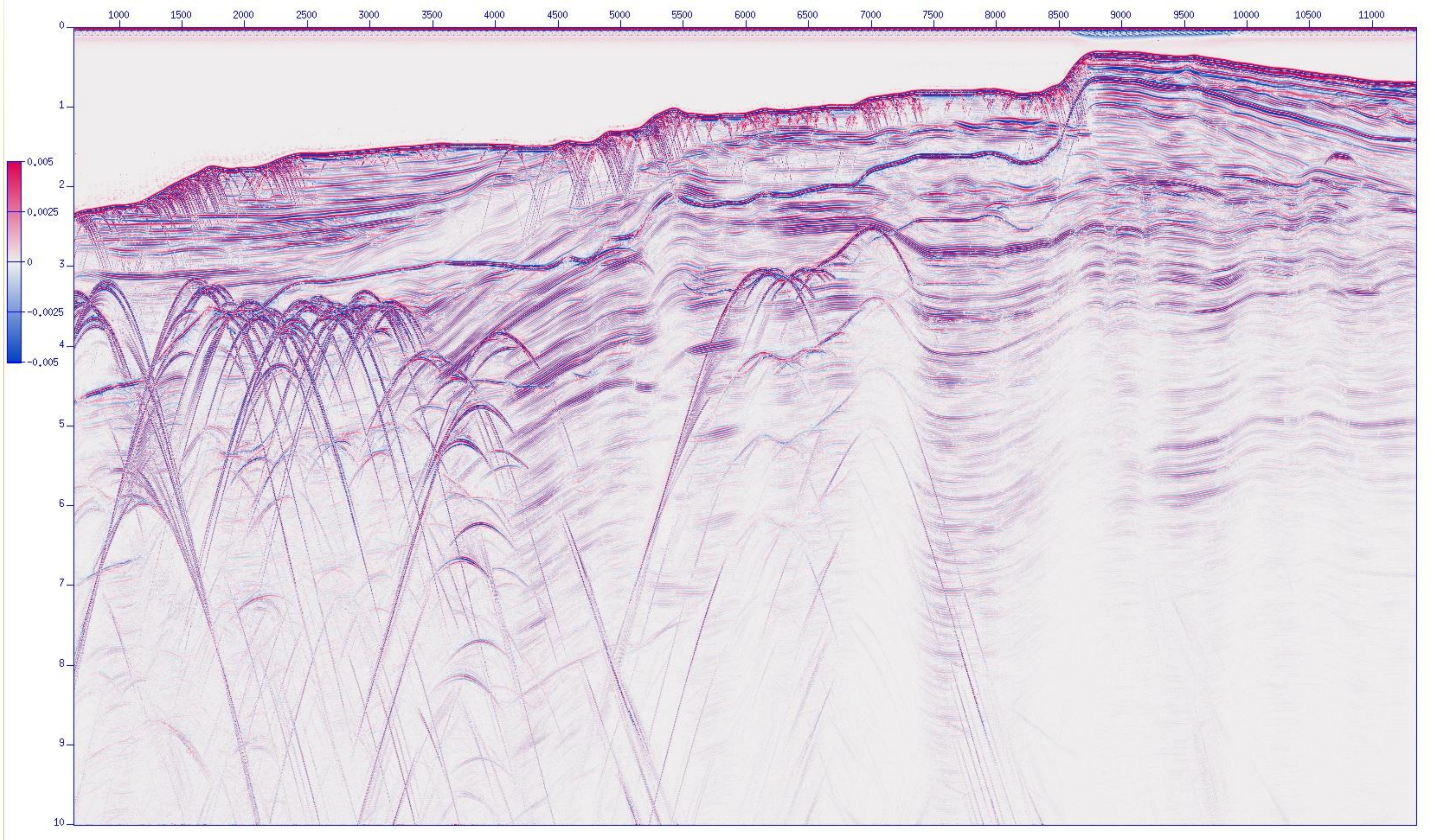


BP test model



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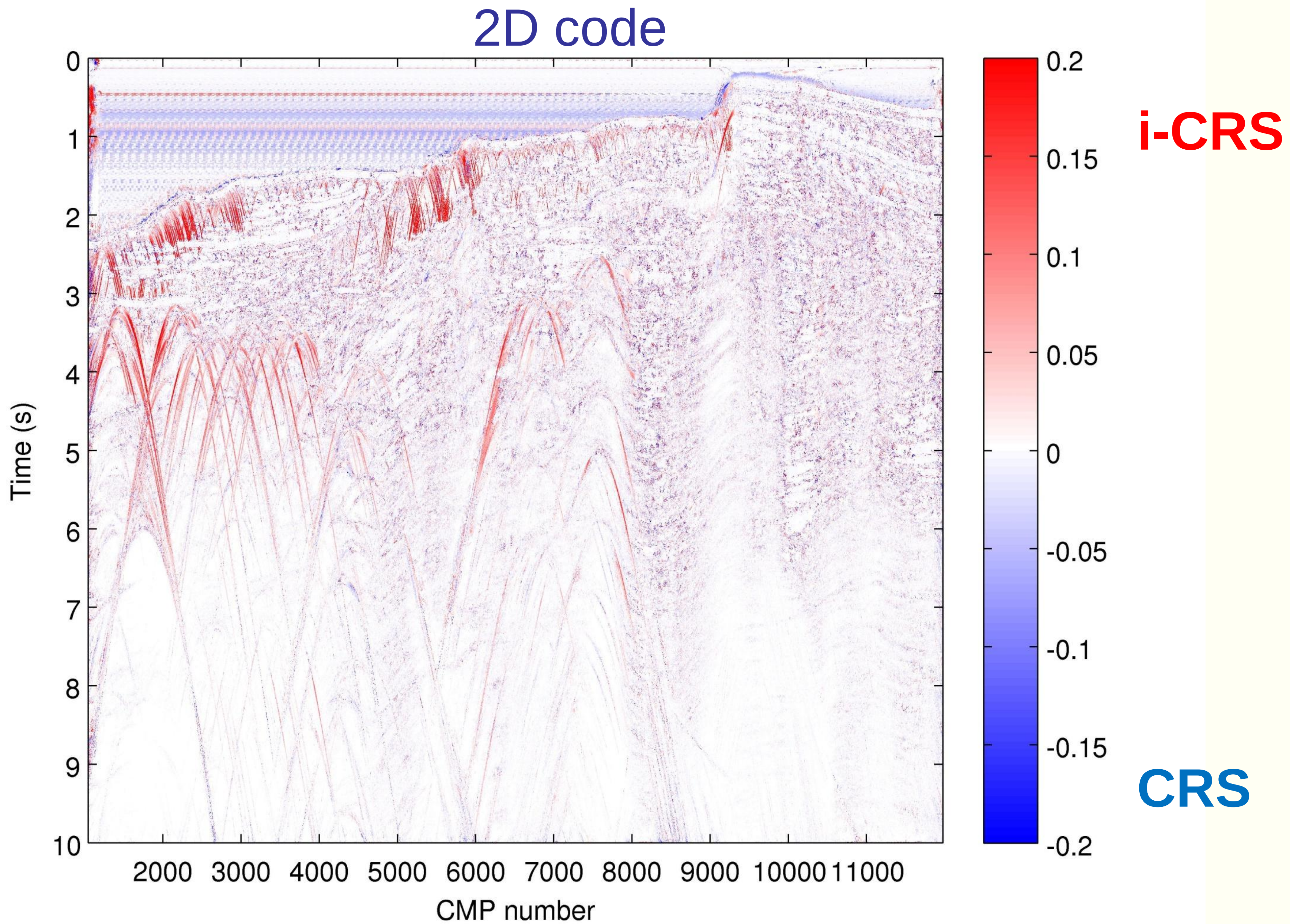

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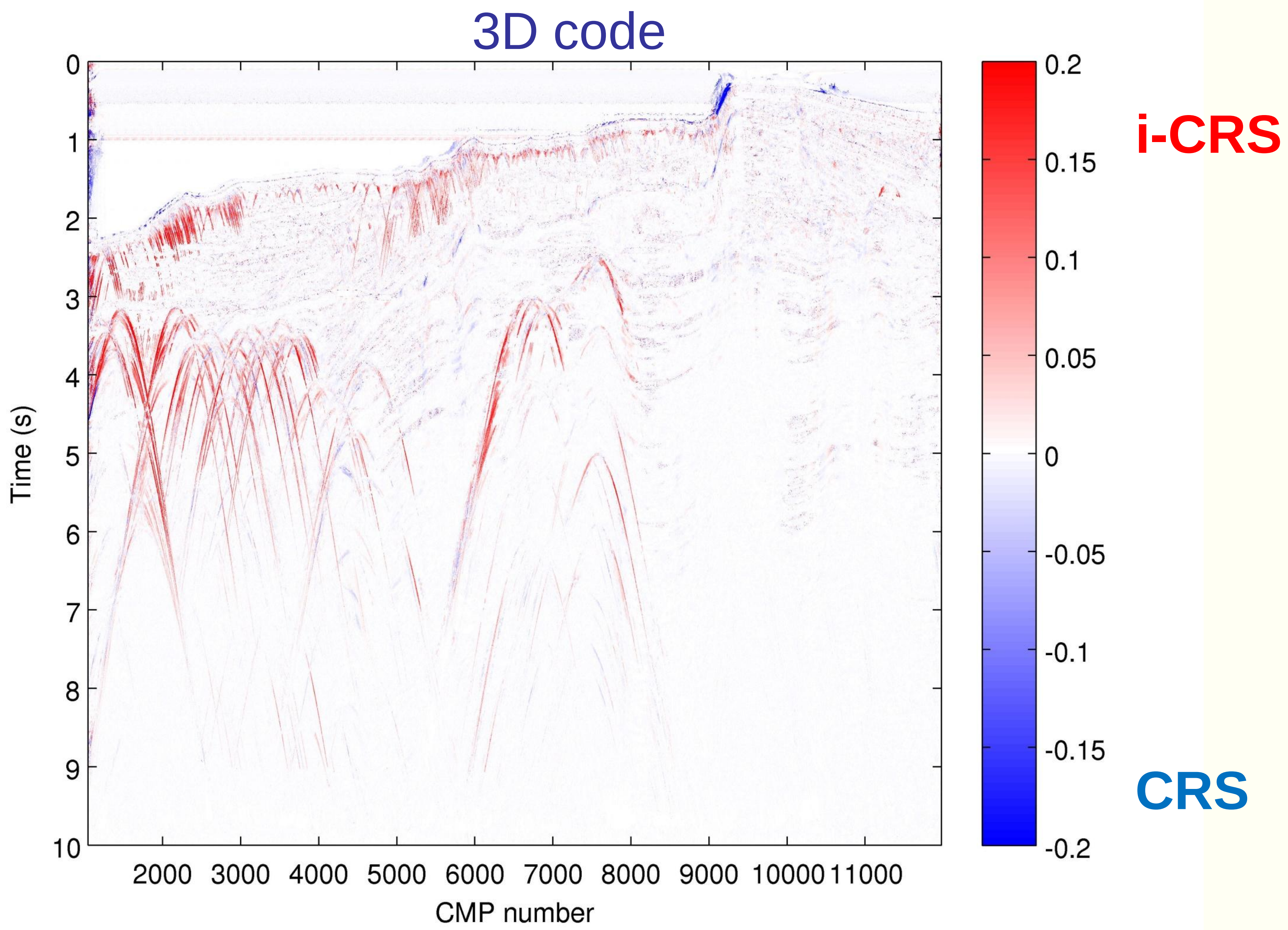

Conclusions

New 3D i-CRS stacking operator:

- intuitive DSR expression
- perfectly fits scattered waves
- easy to implement in the standard CRS workflow
- reasonable price for improved accuracy as compared with standard CRS operator

Effective anisotropic medium and CO geometry:

- key point for understanding traveltime operators in anisotropic medium

Outlook

- Extension to common offset case
- Anisotropy and converted waves
- Improved migration algorithms

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