Time-frequency Decomposition and Q-estimation using Complex Filters

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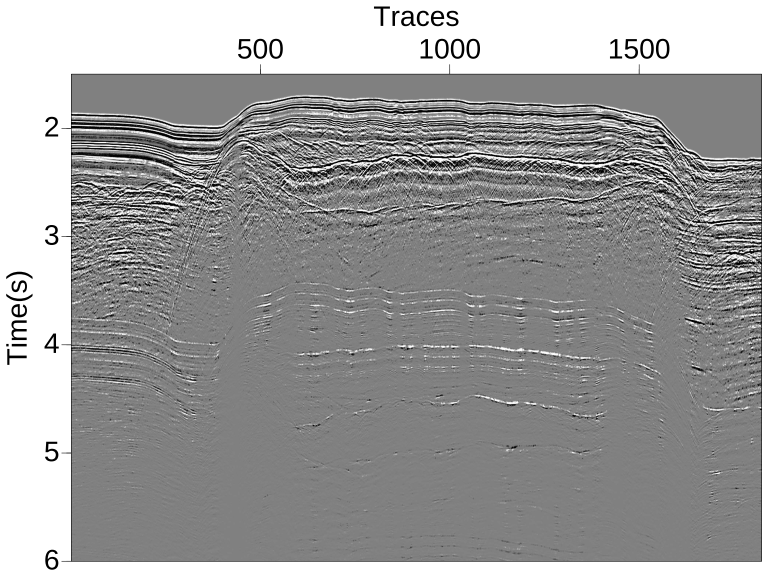
Motivation

- Knowledge of attenuation and dispersion may be used for:
  - reservoir characterization
  - resolution increase - inverse Q-based filtering.
- Attenuation and dispersion lead to loss of amplitude and broadening of the pulse - higher frequencies are lost.
- It would be nice to know quality factor Q along the trace.

Marine Acquisition - Gulf of Mexico



Introduction

- The main purpose of this talk is to estimate the quality factor associated with each data point on the trace, i.e.

$$d(t) \to Q(t)$$

- For this, two window-based methods are proposed:
  - Hight resolution complex minimum phase time-frequency decomposition (CMPD);
  - Q-estimator.

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Introduction - High Resolution Complex TFD

A time frequency decomposition is an analysis tools capable of parsing seismic energy into seismic pulse and the reflectivity model.

By high-resolution we follow Bouachache(1987);

> *... must provide an unbiased estimate of the instant frequency of the signal with best resolution and accuracy in TF domain ...*

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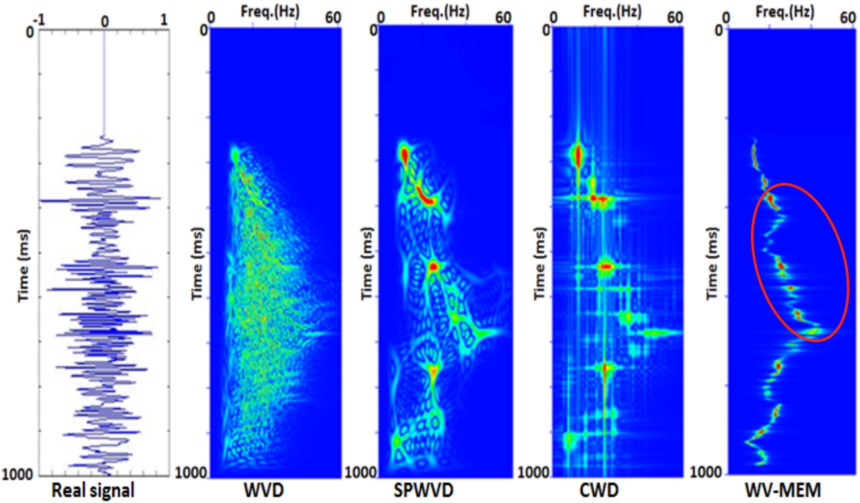
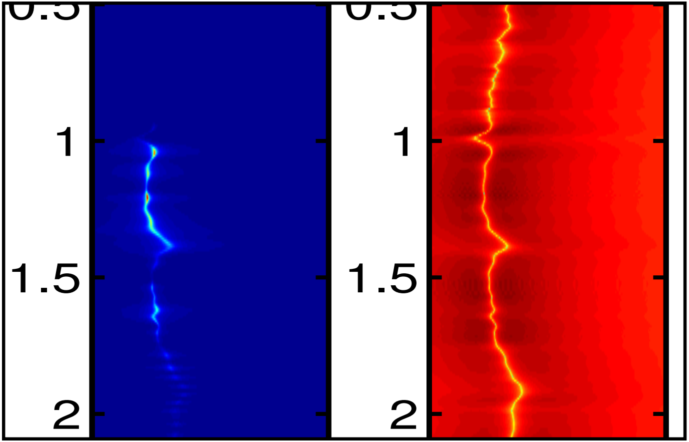
Different Methods - Zoukaneri(2013)



CMPD - Amplitude and Phase Spectrum

Amplitude and Phase Spectrum with Complex Minimum Phase Decomposition(CMPD) - Seismic Trace



Trace Model

We assume the seismic trace, $d(t)$ is given

$$d(t) = p(t) * r(t) + \underbrace{\eta(t)}_{noise}, \qquad d(t), p(t), r(t) \in \mathbb{C},$$

and

$$d(t) = d_o(t) + j\, H\,[\, d_o(t)\,], \tag{1}$$

where $H\,[\,\cdot\,]$ is the Hilbert transform.

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Trace Model: Non-stationary

Expressing the model in matrix form, we have

$$\begin{bmatrix} 1 & 0 & 0 & & 0 & 0 \\ p_{12} & 1 & 0 & & 0 & 0 \\ p_{13} & p_{22} & 1 & & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ p_{1,\ell} & p_{2,\ell-1} & p_{3,\ell-2} & & 0 & 0 \\ 0 & p_{2,\ell} & p_{3,\ell-1} & & 1 & 0 \\ 0 & 0 & p_{3,\ell} & & p_{N-1,2} & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_{N-2} \\ r_{N-1} \\ r_N \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{N-2} \\ d_{N-1} \\ d_N \end{bmatrix}$$

Trace Model: Non-stationary

Expressing the model in matrix form, we have

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & & 0 & 0 \\ p_{12} & 1 & 0 & & 0 & 0 \\ p_{13} & p_{22} & 1 & & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ p_{1,\ell} & p_{2,\ell-1} & p_{3,\ell-2} & & 0 & 0 \\ 0 & p_{2,\ell} & p_{3,\ell-1} & & 1 & 0 \\ 0 & 0 & p_{3,\ell} & & p_{N-1,2} & 1 \end{bmatrix}}_{N \text{ x } N} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_{N-2} \\ r_{N-1} \\ r_N \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{N-2} \\ d_{N-1} \\ d_N \end{bmatrix}$$

$$\mathbf{W}\mathbf{r} = \mathbf{d} \tag{2}$$

CMPD - Basic Idea

$$\Downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & & 0 & 0 \\ p_{12} & 1 & 0 & & 0 & 0 \\ p_{13} & p_{22} & 1 & & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ p_{1,\ell} & p_{2,\ell-1} & p_{3,\ell-2} & & 0 & 0 \\ 0 & p_{2,\ell} & p_{3,\ell-1} & & 1 & 0 \\ 0 & 0 & p_{3,\ell} & & p_{N-1,2} & 1 \end{bmatrix} \qquad \left.\begin{matrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{matrix}\right\} \begin{matrix} \\ d_6 \\ \vdots \end{matrix}$$

Estimated pulse $\leftrightarrow$ center of window

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CMPD - Basic Idea

$$
\begin{array}{cccccc}
r_1 & r_2 & r_3 & & r_{N-1} & r_N \\
\downarrow & \downarrow & \downarrow & & \downarrow & \downarrow
\end{array}
$$

$$
\underbrace{\begin{bmatrix}
1 & 0 & 0 & & 0 & 0 \\
p_{12} & 1 & 0 & & 0 & 0 \\
p_{13} & p_{22} & 1 & & 0 & 0 \\
\vdots & \vdots & \vdots & \ldots & \vdots & \vdots \\
p_{1,\ell} & p_{2,\ell-1} & p_{3,\ell-2} & & 0 & 0 \\
0 & p_{2,\ell} & p_{3,\ell-1} & & 1 & 0 \\
0 & 0 & p_{3,\ell} & & p_{N-1,2} & 1
\end{bmatrix}}_{\text{Weighed Pulse Matrix}}
\qquad
\begin{array}{c}
d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ \vdots
\end{array}
$$

Constructing pulses

The pulse is obtained assuming

$$g_k(t) * p_k(t) = \delta_k, \tag{3}$$

where $g_k(t)$ is the inverse of the pulse (CPEO).

For each window k:

- Estimate CPEO using;
  - Wiener-Levinson method;
  - Burg algorithm.
- Determine pulse eq. (3).

Complex Prediction Error Operator

CPEO is obtained through

$$\begin{bmatrix} r_0 & r_{-1} & r_{-2} & & r_N \\ r_1 & r_0 & r_{-1} & \cdots & r_{N-1} \\ r_2 & r_1 & r_0 & & \vdots \\ & \vdots & & \ddots & r_{-1} \\ r_N & r_{N-1} & \cdots & r_1 & r_0 \end{bmatrix} \begin{bmatrix} 1 \\ c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix} = \begin{bmatrix} E_{c,N} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (4)$$

- $r_0, r_1, r_2, \ldots, r_N$ are the autocorrelation coefficients.
- CPEO is given by $[1, c_1, c_2, \ldots, c_N]^T$ and is minimum phase!

Solving for Reflectivity - Going back to $Wr = d$

Once pulses are placed in columns, we solve for reflectivity.

$$\begin{bmatrix} 1 & 0 & 0 & & 0 & 0 \\ p_{12} & 1 & 0 & & 0 & 0 \\ p_{13} & p_{22} & 1 & & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ p_{1,\ell} & p_{2,\ell-1} & p_{3,\ell-2} & & 0 & 0 \\ 0 & p_{2,\ell} & p_{3,\ell-1} & & 1 & 0 \\ 0 & 0 & p_{3,\ell} & & p_{N-1,2} & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_{N-2} \\ r_{N-1} \\ r_N \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{N-2} \\ d_{N-1} \\ d_N \end{bmatrix}$$

CMPD: DFT by column

$$W' = \overbrace{\left[\begin{array}{ccccc} r_1 & r_2 & r_3 & & r_N \\ r_1p_{11} & r_2p_{21} & r_3p_{31} & \cdots & r_Np_{N1} \\ r_1p_{12} & r_2p_{22} & r_3p_{32} & & r_Np_{N2} \\ \vdots & \vdots & \vdots & & \vdots \end{array}\right]}^{t \longrightarrow} \Bigg\} \tau$$

$$\Updownarrow \quad \Updownarrow \quad \Updownarrow \qquad\qquad \Updownarrow \qquad\qquad \leftarrow \text{DFT}$$

$$\left[\quad \text{Amplitude Spectrum} \quad\right] \Bigg\} freq.$$

$$\left[\quad \text{Phase Spectrum} \quad\right] \Bigg\} freq.$$

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CMPD Algorithm

CMPD Basic Steps

- Hilbert transform trace.
- For a sliding window;
  - Obtain CPEO (Wiener-Levinson or Burg);
  - Obtain pulse.
- Arrange pulses in column in a lower triangular matrix.
- Solve for reflectivity.
- Align pulses in columns weighed by reflectivity.
- DFT columns for amplitude and phase spectrum.

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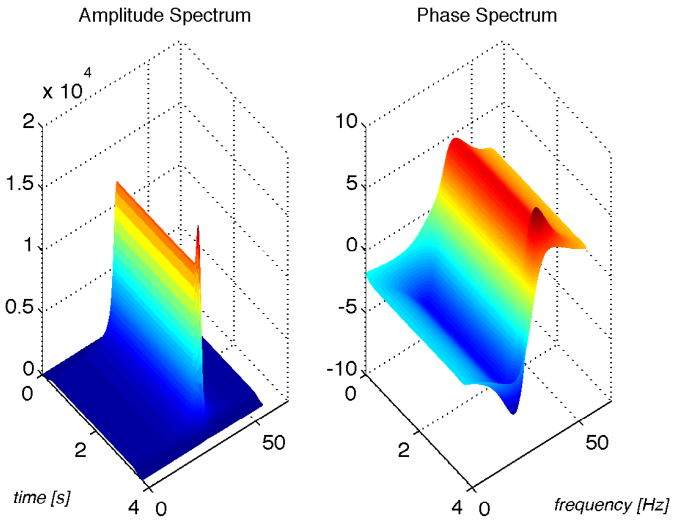
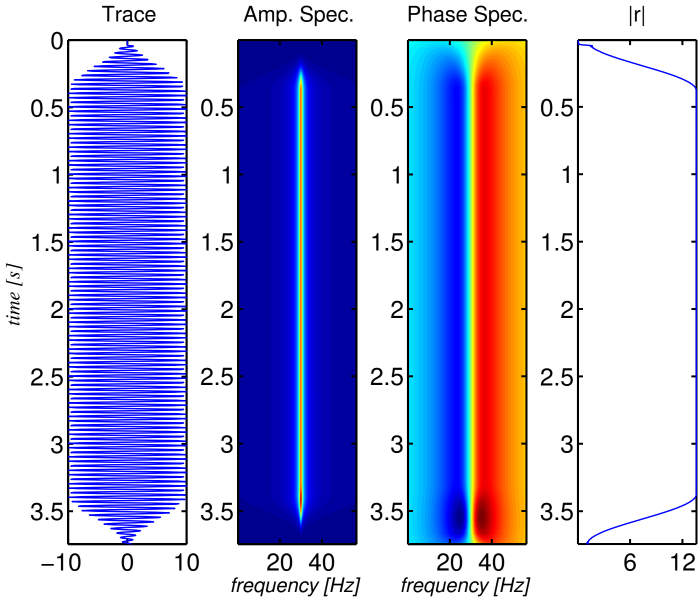
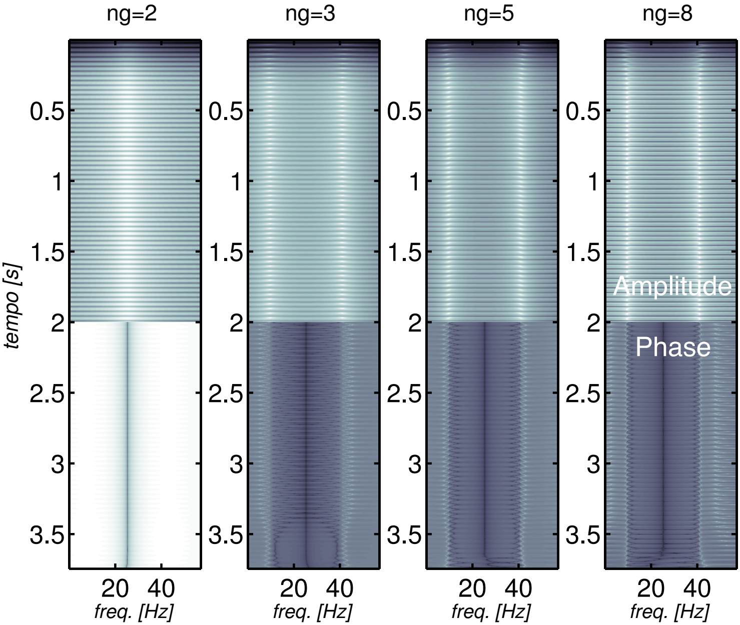
CMPD: Single Frequency



CMPD: Single Frequency



CMPD: Two Frequencies



The Q-Estimator Method

Recall our goal: to estimate $Q(t)$ for each $d(t)$.

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ \vdots \end{bmatrix} \qquad \begin{matrix} \left.\begin{matrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{matrix}\right\} \\ d_6 \\ d_7 \\ d_8 \\ \vdots \end{matrix}$$

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Q-Estimator Method

Recall our goal: to estimate $Q(t)$ for each $d(t)$.

$$
\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ \vdots \end{bmatrix}
\quad
\begin{bmatrix} \\ \text{Amplitude} \\ \text{Spectrum} \\ \\ \\ \\ \\ \\ \vdots \end{bmatrix}
\quad
\begin{bmatrix} \\ \text{Phase} \\ \text{Spectrum} \\ \\ \\ \\ \\ \\ \vdots \end{bmatrix}
\quad
\begin{matrix} \left.\begin{matrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{matrix}\right\} \\ d_6 \\ d_7 \\ d_8 \\ \vdots \end{matrix}
$$

Q-Estimator Method

$$
\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ \vdots \end{bmatrix}
\quad
\begin{bmatrix} A_{11} & A_{12} & & A_{1N} \\ A_{21} & A_{22} & & A_{2N} \\ A_{31} & A_{32} & \cdots & A_{3N} \\ A_{41} & A_{42} & & A_{4N} \\ A_{51} & A_{52} & & A_{5N} \\ A_{61} & A_{62} & & A_{6N} \\ A_{71} & A_{72} & & A_{7N} \\ A_{81} & A_{82} & & A_{8N} \\ \vdots & \vdots & & \vdots \end{bmatrix}
\begin{bmatrix} \phi_{11} & \phi_{12} & & \phi_{1N} \\ \phi_{21} & \phi_{22} & & \phi_{2N} \\ \phi_{31} & \phi_{32} & \cdots & \phi_{3N} \\ \phi_{41} & \phi_{42} & & \phi_{4N} \\ \phi_{51} & \phi_{52} & & \phi_{5N} \\ \phi_{61} & \phi_{62} & & \phi_{6N} \\ \phi_{71} & \phi_{72} & & \phi_{7N} \\ \phi_{81} & \phi_{82} & & \phi_{8N} \\ \vdots & \vdots & & \vdots \end{bmatrix}
\quad
\begin{matrix} \left.\begin{matrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{matrix}\right\} \\ d_6 \\ d_7 \\ d_8 \\ \vdots \end{matrix}
$$

The KF-Propagator

The Kolsky-Futterman propagator is given by

$$P(\Delta t, \omega; Q, \omega_r) = exp\left[\underbrace{i\,\omega\Delta t}_{\text{(A)}} + \underbrace{i\,\frac{\omega\Delta t}{\pi Q}\log\left|\frac{\omega_r}{\omega}\right|}_{\text{(B)}} - \underbrace{\frac{|\omega|\Delta t}{2Q}}_{\text{(C)}}\right]$$

(A) Time Translation

(B) Dispersion

(C) Attenuation

Q-Estimator

Data Window:

$$\begin{bmatrix} \\ \\ Q_3 \\ \\ \\ \end{bmatrix} \quad \begin{bmatrix} A_{11} & A_{12} & & A_{1N} \\ A_{21} & A_{22} & & A_{2N} \\ A_{31} & A_{32} & \cdots & A_{3N} \\ A_{41} & A_{42} & & A_{4N} \\ A_{51} & A_{52} & & A_{5N} \end{bmatrix} \begin{bmatrix} \phi_{11} & \phi_{12} & & \phi_{1N} \\ \phi_{21} & \phi_{22} & & \phi_{2N} \\ \phi_{31} & \phi_{32} & \cdots & \phi_{3N} \\ \phi_{41} & \phi_{42} & & \phi_{4N} \\ \phi_{51} & \phi_{52} & & \phi_{5N} \end{bmatrix}$$

Simulated Window:

$$\begin{bmatrix} \\ \\ Q'_3 \\ \\ \\ \end{bmatrix} \quad \begin{bmatrix} \\ \\ A_{31} & A_{32} & \cdots & A_{3N} \\ \\ \\ \end{bmatrix} \begin{bmatrix} \\ \\ \phi_{31} & \phi_{32} & \cdots & \phi_{3N} \\ \\ \\ \end{bmatrix}$$

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Q-Estimator

Data Window:

$$\begin{bmatrix} \\ \\ Q_3 \\ \\ \\ \end{bmatrix} \quad \begin{bmatrix} A_{11} & A_{12} & & A_{1N} \\ A_{21} & A_{22} & & A_{2N} \\ A_{31} & A_{32} & \cdots & A_{3N} \\ A_{41} & A_{42} & & A_{4N} \\ A_{51} & A_{52} & & A_{5N} \end{bmatrix} \begin{bmatrix} \phi_{11} & \phi_{12} & & \phi_{1N} \\ \phi_{21} & \phi_{22} & & \phi_{2N} \\ \phi_{31} & \phi_{32} & \cdots & \phi_{3N} \\ \phi_{41} & \phi_{42} & & \phi_{4N} \\ \phi_{51} & \phi_{52} & & \phi_{5N} \end{bmatrix}$$

Simulated Window:

$$\begin{bmatrix} \\ \\ Q'_3 \\ \\ \\ \end{bmatrix} \quad \begin{bmatrix} A'_{11} & A'_{12} & & A'_{1N} \\ A'_{21} & A'_{22} & & A'_{2N} \\ A_{31} & A_{32} & \cdots & A_{3N} \\ A'_{41} & A'_{42} & & A'_{4N} \\ A'_{51} & A'_{52} & & A'_{5N} \end{bmatrix} \begin{bmatrix} \phi'_{11} & \phi'_{12} & & \phi'_{1N} \\ \phi'_{21} & \phi'_{22} & & \phi'_{2N} \\ \phi_{31} & \phi_{32} & \cdots & \phi_{3N} \\ \phi'_{41} & \phi'_{42} & & \phi'_{4N} \\ \phi'_{51} & \phi'_{52} & & \phi'_{5N} \end{bmatrix}$$

Target Function

We minimize

$$F(Q) = \sum_t \sum_f |W(t,f) - P(\Delta t, f; Q) W(t_0, f)|^2$$

- The problem is non-linear.
- Newton's method is used.

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Q-Estimator Algorithm

Q-Estimator Basic Steps

- Set initial Q_k.
- For each window (k):
    - * Choose central amplitude and phase distribution.
    - * Use KF-Propagator to create simulated window with Q'_k.
    - * Evaluate target Function.
    - * Adjust Q'_k to minimize target function using Newton's Method.
    - * Save optimal Q'_k.

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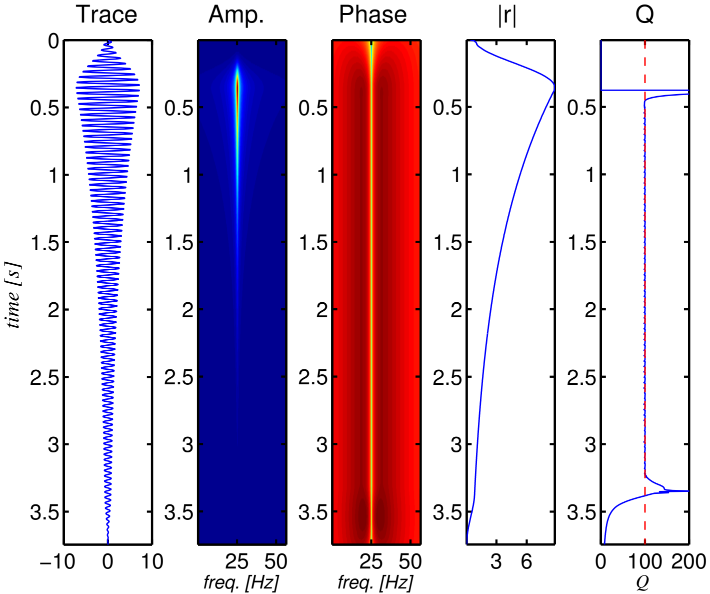
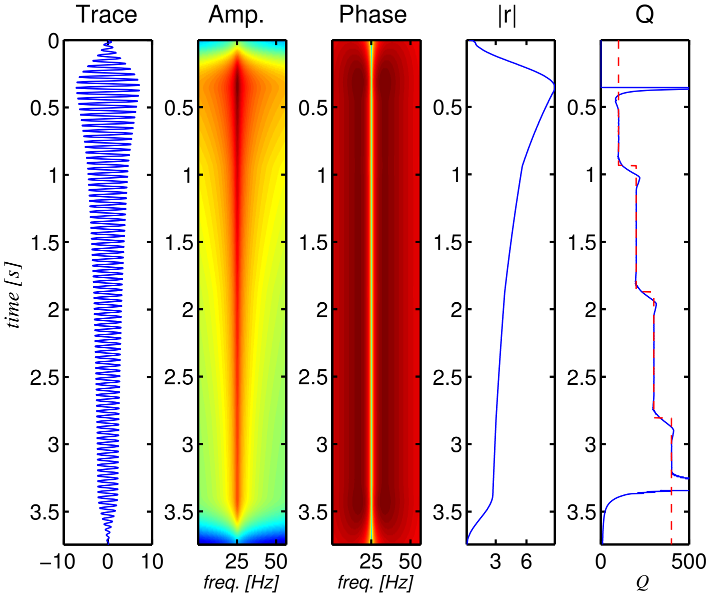
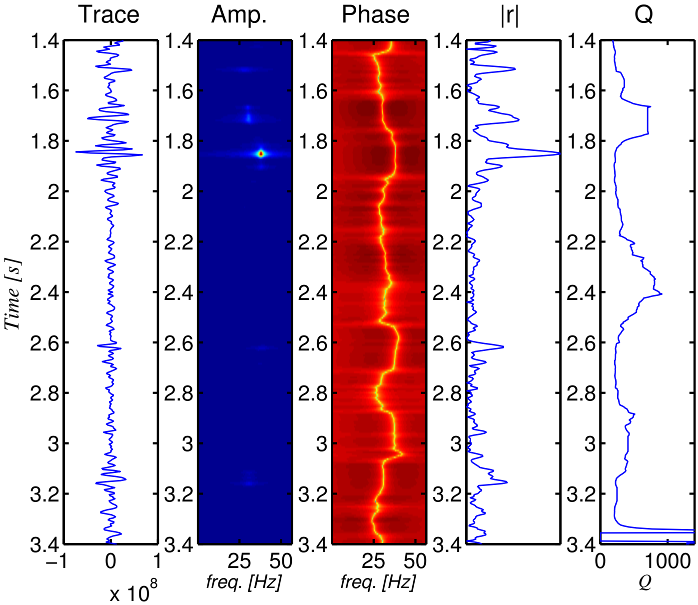
Results - Single Frequency with Attenuation



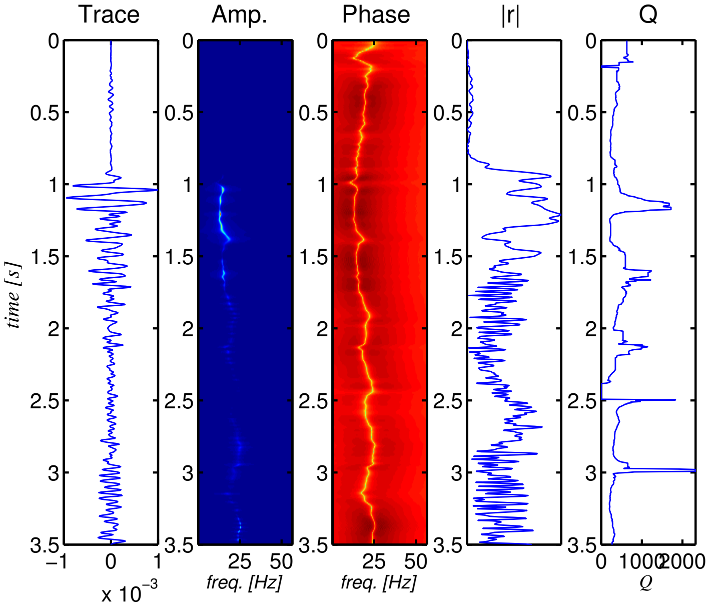
Results - Single Frequency, 4-Layers



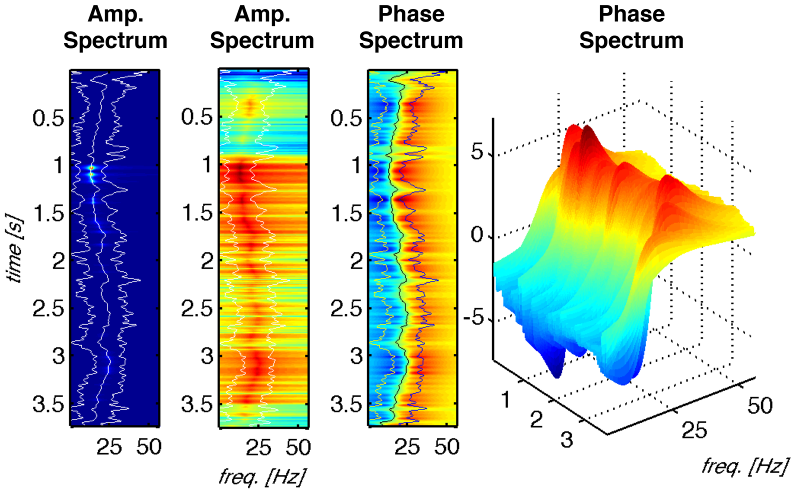
Results - Marine Acquisition Trace



Results - Land Acquisition Trace



Potential Attributes



Conclusions and Recomendations

- CMPD produces a high-resolution time frequency decomposition with a phase spectrum obtained directly from data;
- CMPD is very precise in determining average instantaneous frequency;
- $Q(t) \leftarrow d(t)$ (high resolution Q estimate),
- Trace-by-trace analysis,
- Various attributes for seismic interpretation.

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For Further Reading

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