Vertical Elliptic Operator for Wave Propagation in TTI Media

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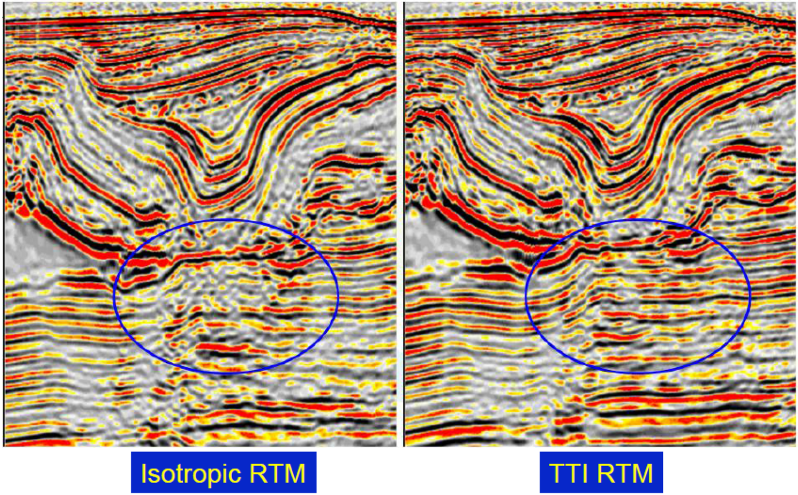
Motivation

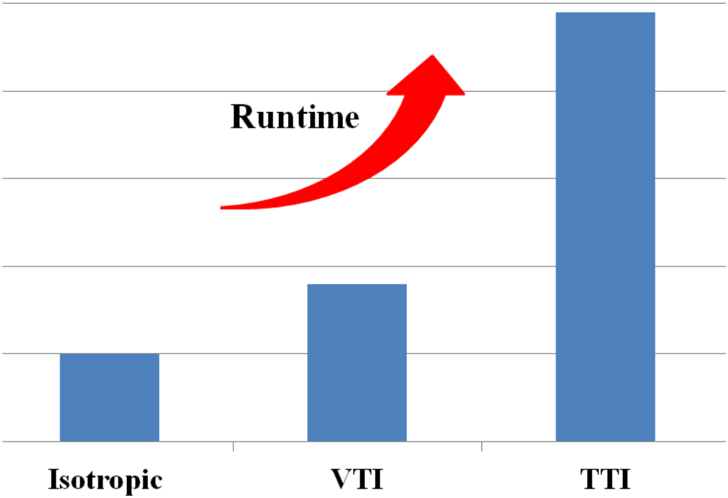
- **Two-way wave equation for depth imaging**
- **RTM is usually the preferred imaging tool**
- **Other applications: FWI, WEMVA**

TTI RTM

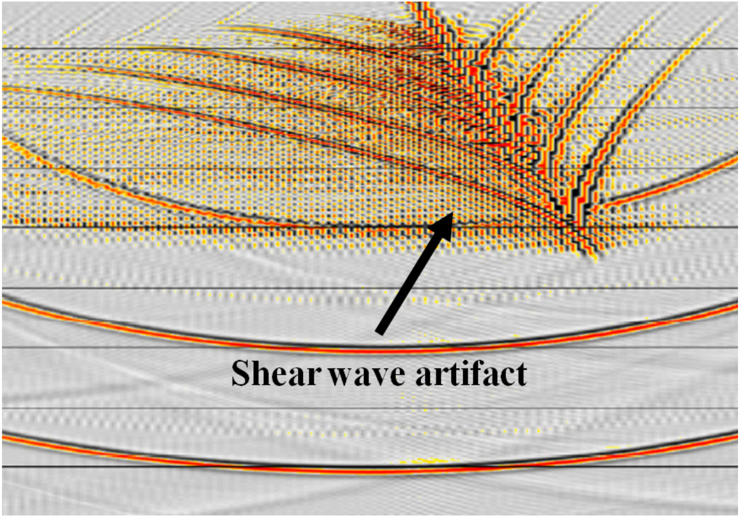


(Courtesy: Yu Zhang)

Motivation: Cost



Motivation: Shear wave artifact



(Zhang et al., 2009)

Single mode wave propagation

Single mode wave propagation

- **Wavefield separation method (Sun et al., 2004)**

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- **Acoustic anisotropic wave equation (Alkhalifah, 2000)**

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- **Coupled 2×2 system of equations (Zhou et al., 2006)**

Single mode wave propagation

- **Wavefield separation method (Sun et al., 2004)**
- **Acoustic anisotropic wave equation (Alkhalifah, 2000)**
- **Coupled 2×2 system of equations (Zhou et al., 2006)**
- **Pseudospectral plus interpolation (Chu et al., 2013)**

Single mode wave propagation

- **Effective isotropic model (Alkhalifah et al., 2013)**

- **Obtained by embedding kinematic effects**

Single mode wave propagation

- **Effective isotropic model (Alkhalifah et al., 2013)**
- **Obtained by embedding kinematic effects**
- **Improved match using effective elliptic model (Waheed and Alkhalifah, 2014)**

Single mode wave propagation

- **Effective isotropic model (Alkhalifah et al., 2013)**
- **Obtained by embedding kinematic effects**
- **Improved match using effective elliptic model (Waheed and Alkhalifah, 2014)**
- **Limitation?**

Single mode wave propagation

- **Pure qP-wave equation (Xu and Zhou, 2014)**
- **Obtained by decomposition of the original wave equation**
- **Same dispersion relation as TTI wave equation**

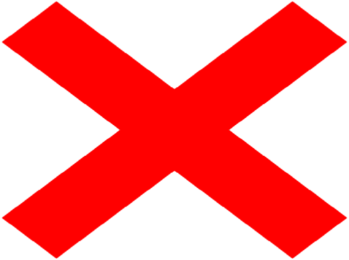
Single mode wave propagation

- **Better amplitude match using elliptic decomposition (Xu et al., 2015)**
- **Decomposition into a scalar operator and a TEA operator**
- **Higher cost due to presence of tilt**

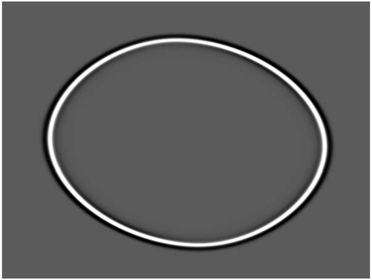
Single mode wave propagation

- **Better amplitude match using elliptic decomposition (Xu et al., 2015)**
- **Decomposition into a scalar operator and a TEA operator**
- **Higher cost due to presence of tilt**
- **True meaning of amplitude?**

Stability for $\epsilon < \delta$

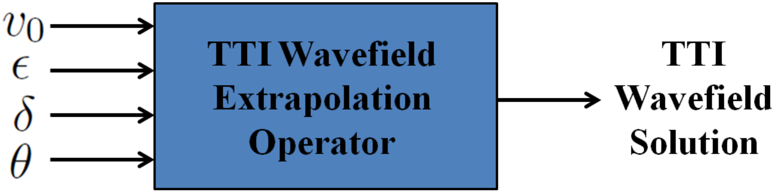


Conventional Finite-difference



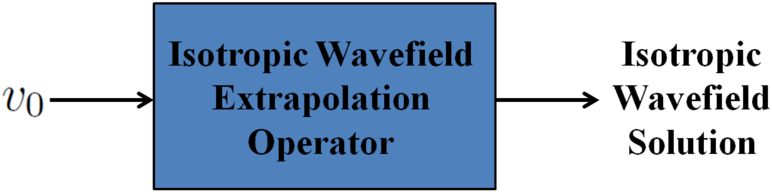
Proposed method

The Idea



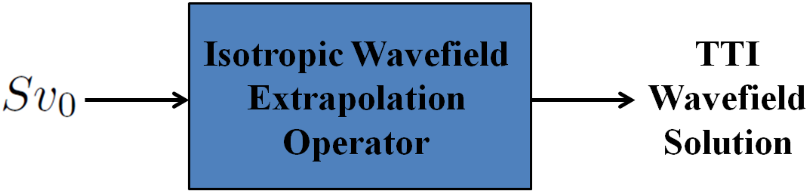
v_0
ϵ
δ
θ
TTI Wavefield Extrapolation Operator
TTI Wavefield Solution

The Idea

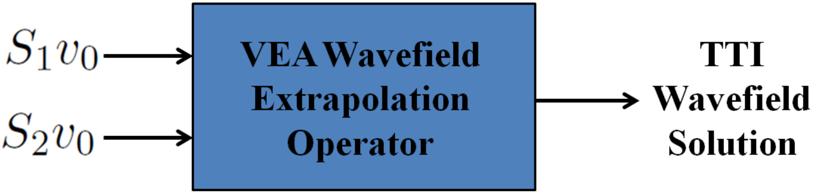


v_0
Isotropic Wavefield Extrapolation Operator
Isotropic Wavefield Solution

The Idea

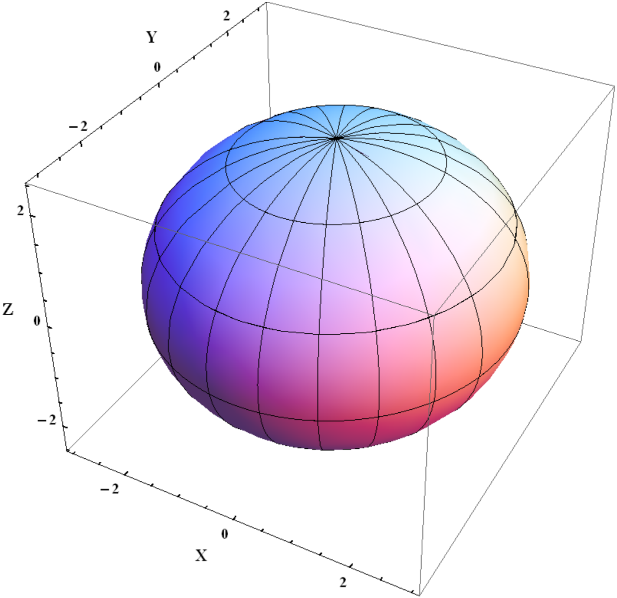

Sv_0
Isotropic Wavefield Extrapolation Operator
TTI Wavefield Solution

The Idea

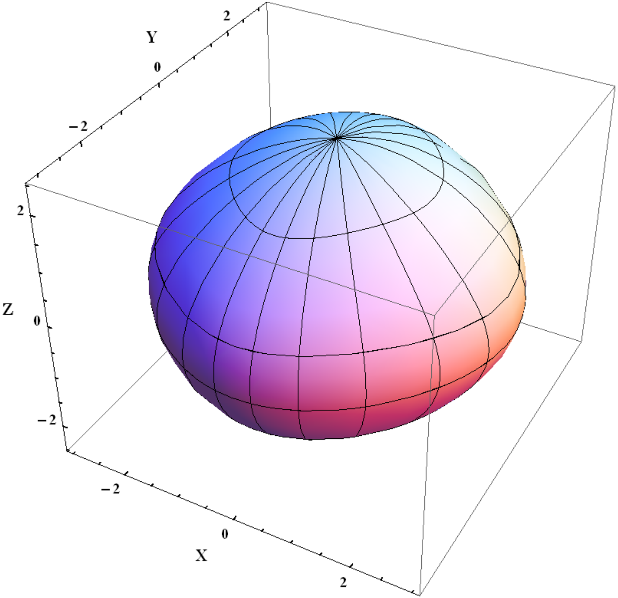


$S_1 v_0$
$S_2 v_0$
VEA Wavefield Extrapolation Operator
TTI Wavefield Solution

The Idea



The Idea

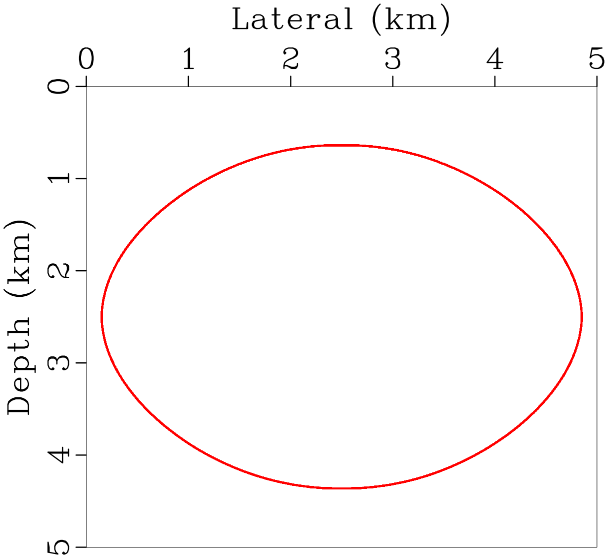


Homogeneous VTI Model

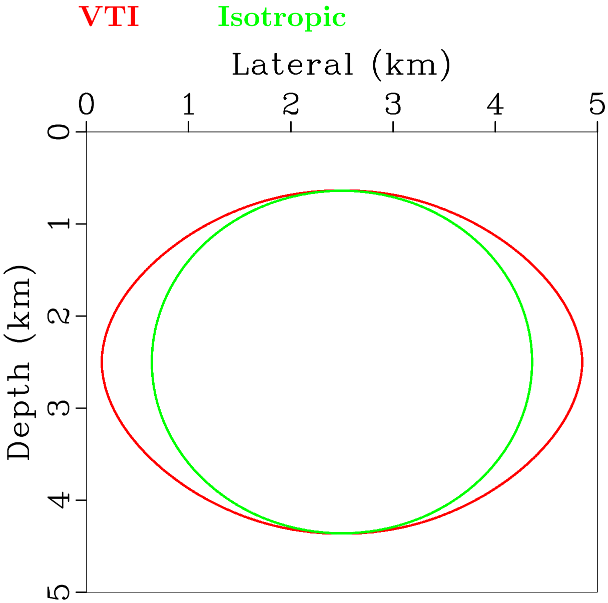
- **Dimensions: 5 km $\times$ 5 km**
- $\boldsymbol{v_0 = 2}$ **km/s**
- $\boldsymbol{\epsilon = 0.3}$
- $\boldsymbol{\delta = 0.15}$
- **Source location : (2.5 km, 2.5 km)**

Dispersion Curves

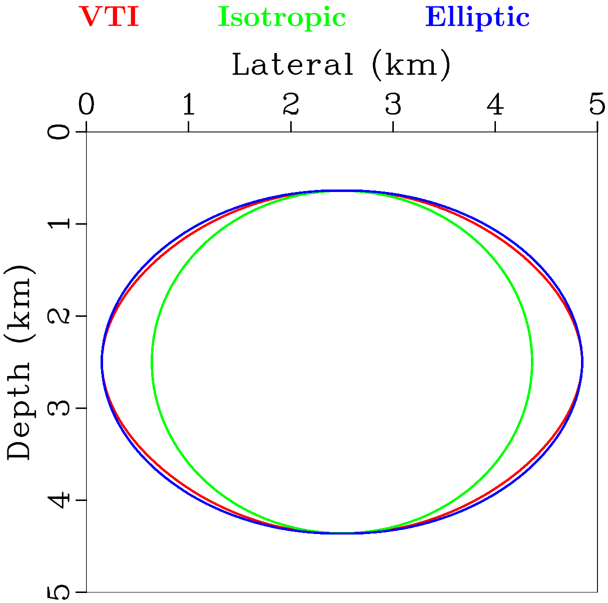
VTI



Dispersion Curves



Dispersion Curves



VTI Wave Equation

- **Acoustic wave equation for 2D VTI medium is:**

$$\left(\frac{\partial^2}{\partial t^2} - \frac{v_0^2}{2}\left(\frac{\partial^2}{\partial x^2}(1+2\epsilon) + \frac{\partial^2}{\partial z^2} + \sqrt{\left(\frac{\partial^2}{\partial x^2}(1+2\epsilon) + \frac{\partial^2}{\partial z^2}\right)^2 - 8(\epsilon-\delta)\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial z^2}}\right)\right) P = 0$$

v_0: **Vertical velocity**

ϵ, δ: **Thomsen parameters**

VTI Wave Equation

- **In f-k domain the VTI wave equation is:**

$$\left(\omega^2 - \frac{v_0^2}{2}\left(k_x^2(1+2\epsilon) + k_z^2 + \sqrt{\left(k_x^2(1+2\epsilon)+k_z^2\right)^2 - 8(\epsilon-\delta)\,k_x^2\,k_z^2}\right)\right)\widetilde{P} = 0$$

k_x: x-component of spatial wavenumber

k_z: z-component of spatial wavenumber

TTI Wave Equation

- **Wave equation for 2D TTI media in f-k domain:**

$$\left(\omega^2 - \frac{v_0^2}{2}\left(\hat{k}_x^2(1+2\epsilon) + \hat{k}_z^2 + \sqrt{\left(\hat{k}_x^2(1+2\epsilon)+\hat{k}_z^2\right)^2 - 8(\epsilon-\delta)\,\hat{k}_x^2\,\hat{k}_z^2}\right)\right)\widetilde{P} = 0$$

$$\hat{k}_x = k_x \cos\theta + k_z \sin\theta$$

$$\hat{k}_z = -k_x \sin\theta + k_z \cos\theta$$

Vertical Elliptic Decomposition

- **Re-write the equation as:**

$$\left(\omega^2 - \frac{v_0^2}{2}\left(\alpha_x\, k_x^2 + \alpha_z\, k_z^2\right)\left(1 + \frac{2\epsilon\, k_x\, k_z \sin 2\theta + \beta}{\alpha_x\, k_x^2 + \alpha_z\, k_z^2}\right)\right)\widetilde{P} = 0$$

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$$\beta = \sqrt{\left(\hat{k}_x^2(1+2\epsilon) + \hat{k}_z^2\right)^2 - 8(\epsilon - \delta)\,\hat{k}_x^2\,\hat{k}_z^2}$$

$$\alpha_x = (1+2\epsilon)\cos^2\theta + \sin^2\theta$$

$$\alpha_z = \cos^2\theta + (1+2\epsilon)\sin^2\theta$$

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$$\alpha_z = \cos^2\theta + (1+2\epsilon)\sin^2\theta$$

Vertical Elliptic Decomposition

- **Our VEA-like wave equation in space-time domain is:**

$$\frac{\partial^2 P}{\partial t^2} = \left(v_0^2\, \alpha_x \frac{\partial^2 P}{\partial x^2} + v_0^2\, \alpha_z \frac{\partial^2 P}{\partial z^2} \right) S_{\mathrm{e}}$$

Vertical Elliptic Decomposition

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$$S_{\mathrm{e}} = \left(1 + \frac{2\epsilon\, k_x\, k_z \sin 2\theta + \beta}{\alpha_x\, k_x^2 + \alpha_z\, k_z^2} \right)$$

Vertical Elliptic Decomposition

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$$S_{\mathrm{e}} = \left(1 + \frac{2\epsilon\, k_x\, k_z \sin 2\theta + \beta}{\alpha_x\, k_x^2 + \alpha_z\, k_z^2} \right)$$

- k_x **and** k_z **are computed using the** x**- and** z**-components of** ∇P

Algorithm

Solution of the TTI wave equation

1. **Compute the gradient of the wavefield $\boldsymbol{\nabla P}$**

2. **Compute the scalar operator $\boldsymbol{S_{\mathrm{e}}}$**

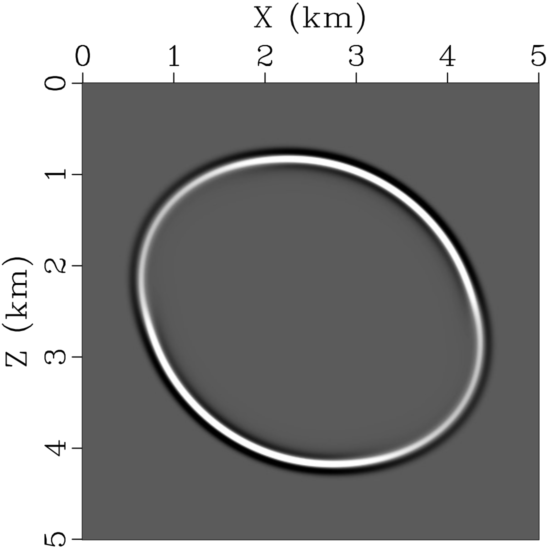
3. **Solve the new VEA-like wave equation**

Numerical Tests

Homogeneous TTI Model

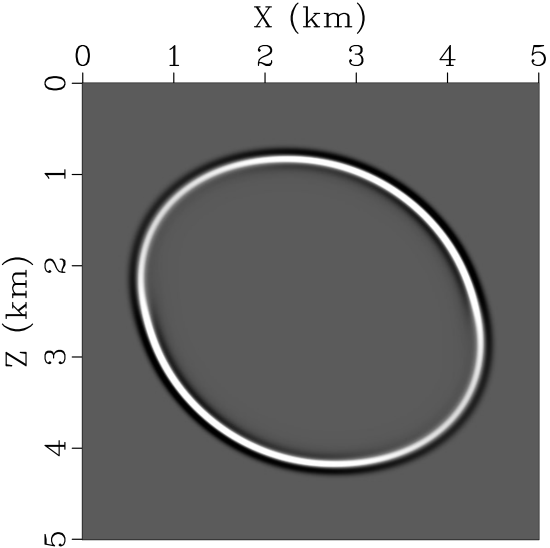
- **Dimensions: 5 km $\times$ 5 km**
- $\boldsymbol{v_0 = 1.8}$ **km/s**, $\boldsymbol{\epsilon = 0.2}$, $\boldsymbol{\delta = 0.1}$, $\boldsymbol{\theta = 30^\circ}$
- $\boldsymbol{\Delta x = \Delta z = 20}$ **m**
- **Source location: (2.5 km, 2.5 km)**
- **Peak frequency = 10 Hz**

Isotropic Decomposition



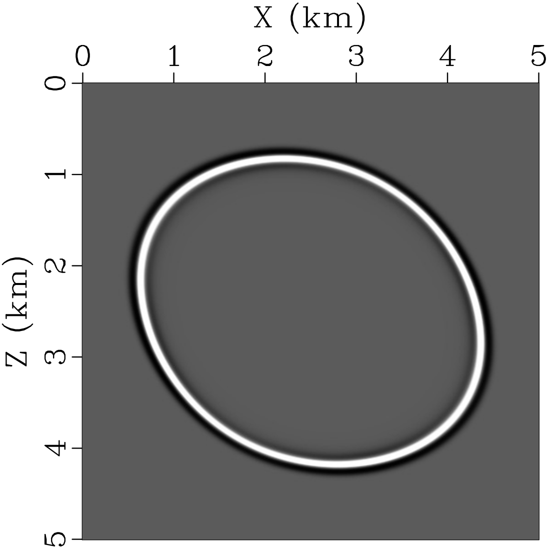
t = 0.9 s

Vertical Elliptic Decomposition



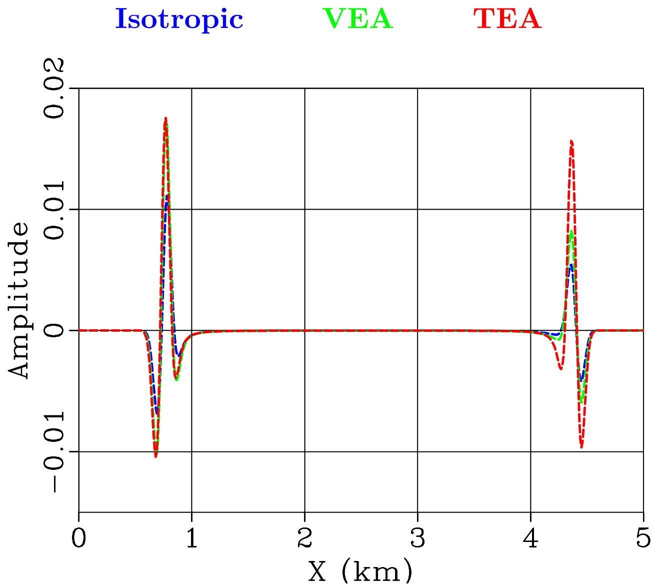
t = 0.9 s

Tilted Elliptic Decomposition

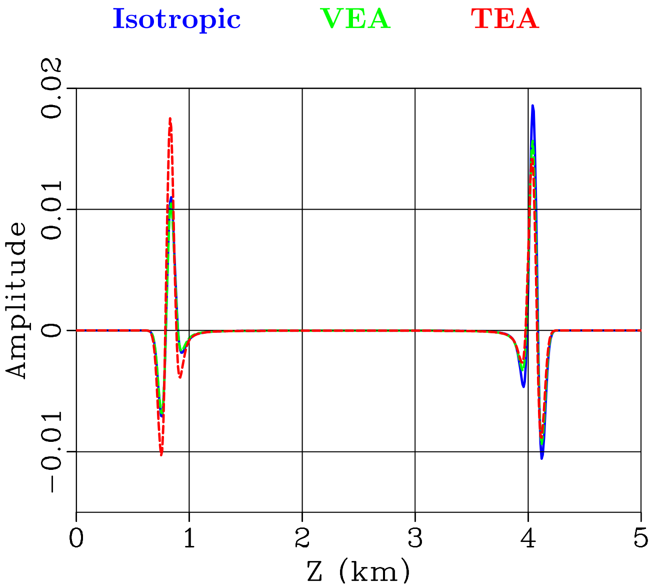


t = 0.9 s

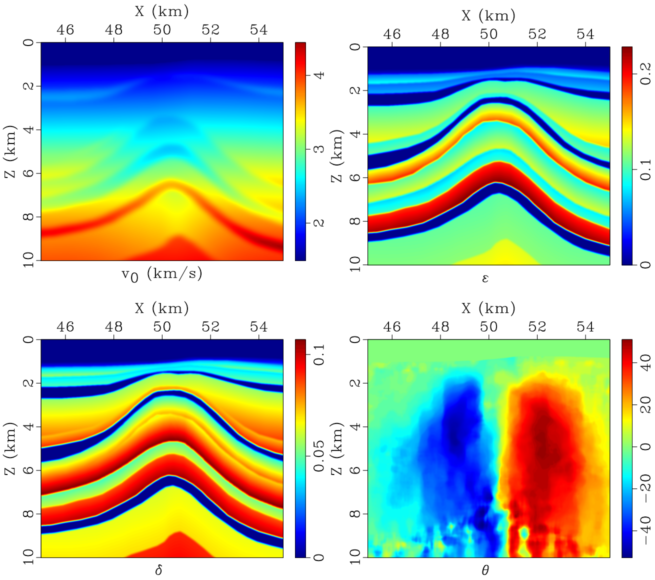
Wavefield profile: Z = 2 km, t = 0.9 s



Wavefield profile: X = 2.8 km, t = 0.9 s

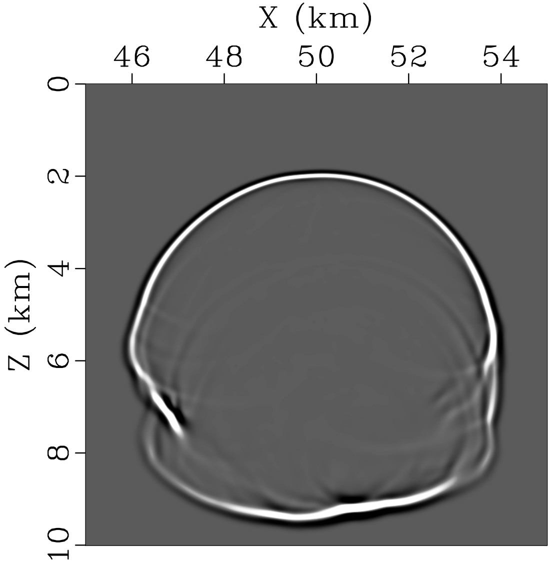


BP TTI Model



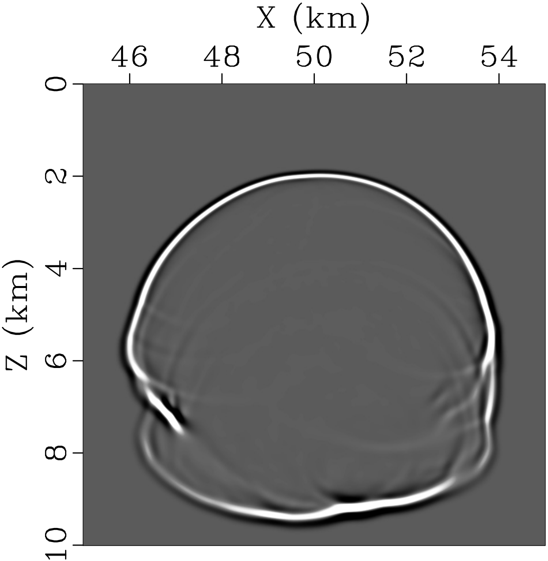
X (km)
46
48
50
52
54
0
2
4
6
8
10
Z (km)
v_0 (km/s)
2
3
4
ε
0
0.1
0.2
δ
0
0.05
0.1
θ
−40
−20
0
20
40

Isotropic Decomposition



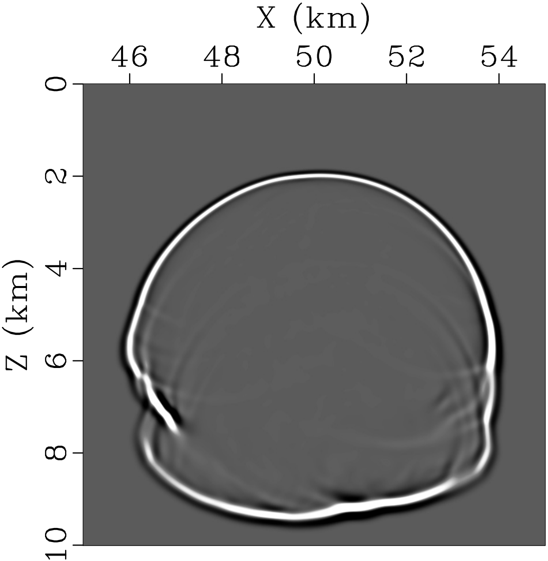
t = 1.3 s

Vertical Elliptic Decomposition



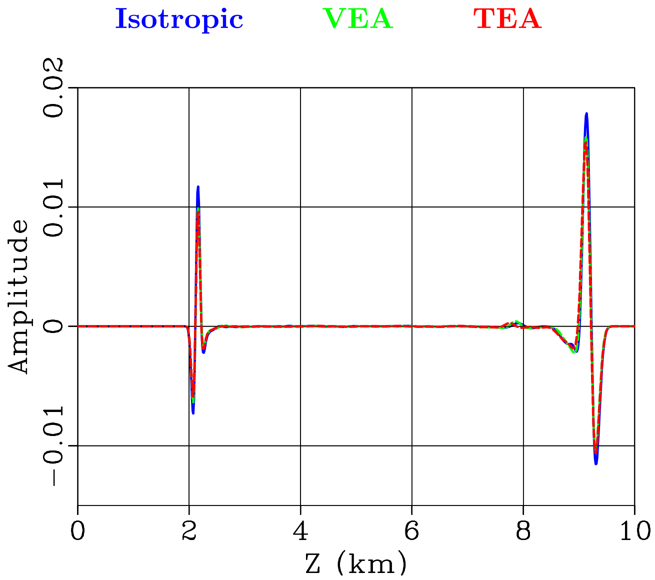
t = 1.3 s

Tilted Elliptic Decomposition

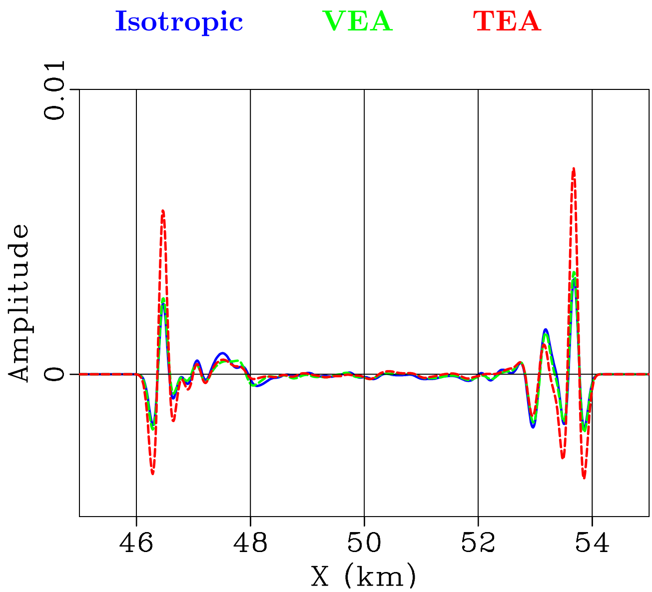


t = 1.3 s

Wavefield profile: X = 51.5 km, t = 1.3 s



Wavefield profile: Z = 53 km, t = 1.3 s



Summary

- **Decomposition of the TTI wave equation**
- **Vertical elliptic decomposition yields better cost versus accuracy tradeoff**
- **No shear wave artifact**
- **Extendable to lower anisotropic symmetries**

Acknowledgments

- **KAUST for financial support**
- **Sheng Xu for useful discussions**