First-arrival traveltime tomography in anisotropic media using the adjoint-state method

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Motivation

- **Near-surface model parameter estimation**
- **Initial models for PSDM and FWI**
- **Traditional methods combine eikonal solver with ray tracing**

Motivation

- **Current seismic surveys deploy thousands of sources and receivers**
- **Fréchet derivative matrix needs extremely large memory**
- **Solution: reduction in model size or data?**

Motivation

- **No explicit computation of Fréchet derivatives**
- **Cost = Two forward problems**
- **Gradient computation parallelizable**
- **Developed only for isotropic media (Leung and Qian, 2006; Taillandier et al., 2009)**

Parameterization

- **qP-wave traveltime in VTI medium needs three parameters**
- **We use v_{nmo}, η, and δ (Stopin and Plessix, 2014)**
- **δ is weakly resolvable from surface seismics alone (Plessix and Cao, 2011)**

Governing Equations

- **For a single shot, the misfit function is given as**

$$J(m) = \frac{1}{2}\int_{\partial\Omega} dr\,|T^{comp}(m) - T^{obs}|^2$$

- **The eikonal equation for VTI media is**

$$v_{nmo}^2(1+2\eta)\left(\frac{\partial T}{\partial x}\right)^2 + \frac{v_{nmo}^2}{1+2\delta}\left(\frac{\partial T}{\partial z}\right)^2\left(1 - 2\eta v_{nmo}^2\left(\frac{\partial T}{\partial x}\right)^2\right) = 1$$

$$T(s) = 0$$

Governing Equations

- **The adjoint variable $\boldsymbol{\lambda}$ satisfies**

$$\nabla.(L\lambda) = 0$$

with the boundary condition

$$(\mathrm{n}.L)\lambda = T^{obs} - T^{comp}$$

where

$$L = \begin{bmatrix} \left(v_{nmo}^2(1+2\eta) - 2\eta v_{nmo}^2 v_0^2 \left(\frac{\partial T}{\partial z}\right)^2\right)\left(\frac{\partial T}{\partial x}\right) \\ \left(v_0^2 - 2\eta v_{nmo}^2 v_0^2 \left(\frac{\partial T}{\partial x}\right)^2\right)\left(\frac{\partial T}{\partial z}\right) \end{bmatrix}$$

Governing Equations

- **The gradient of the misfit function is given as:**

$$\frac{\partial J}{\partial v_{nmo}} = \int_{\Omega} v_{nmo} \left((1+2\eta) \left(\frac{\partial T}{\partial x} \right)^2 + \frac{1}{1+2\delta} \left(\frac{\partial T}{\partial z} \right)^2 \right) \lambda dx$$

$$\frac{\partial J}{\partial \eta} = \int_{\Omega} v_{nmo}^2 \left(\frac{\partial T}{\partial x} \right)^2 \left(1 - \frac{v_{nmo}^2}{1+2\delta} \left(\frac{\partial T}{\partial z} \right)^2 \right) \lambda dx$$

- **The model parameters can be updated using:**

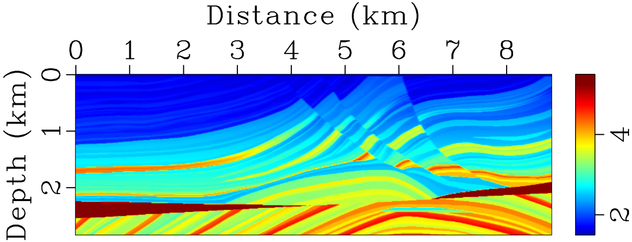
$$m^{k+1} = m^k - \alpha^k \nabla J(m^k)$$

Implementation Details

Forward and adjoint solvers

- **Eikonal equation: (Waheed et al., 2015)**
- **FS based solver for the adjoint-state equation**
- **Solving them gives us the adjoint-state variable $\boldsymbol{\lambda}$**

Smoothing the gradient



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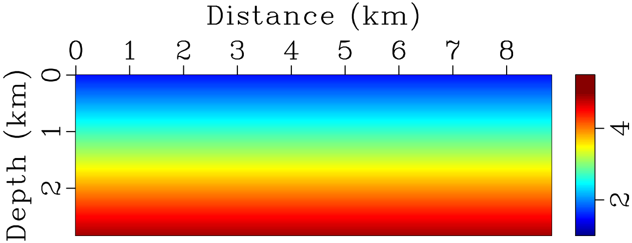
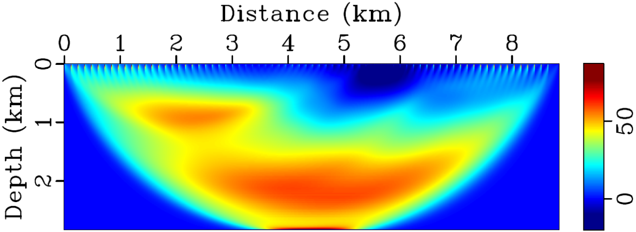


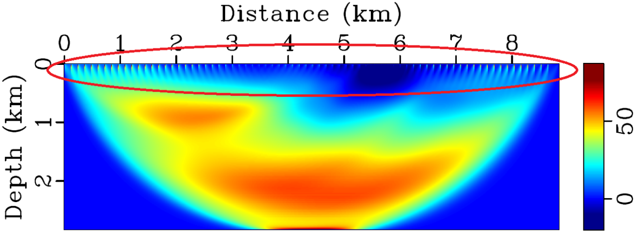
Initial

Smoothing the gradient



Before Smoothing

Smoothing the gradient



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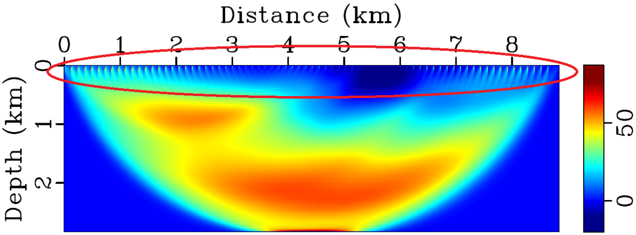

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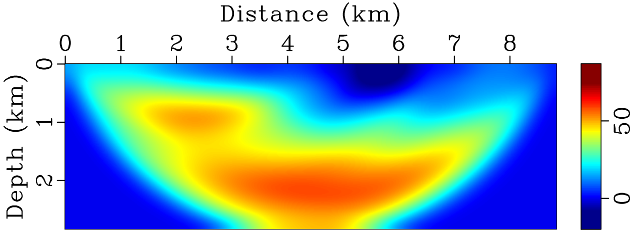

After Smoothing

Smoothing the gradient

- **Four stage adaptive smoothing**
- **Begins: 50 grid points window**
- **Ends: distance between two consecutive sources**

Inversion setup

- **Optimization algorithm: Nonlinear Conjugate Gradient**

  1. **Compute $\boldsymbol{\beta^k = \max(0, \beta^{PR})}$, where:**

$$\boldsymbol{\beta^{PR} = \frac{(\nabla J^k)^T(\nabla J^k - \nabla J^{k-1})}{(\nabla J^{k-1})^T(\nabla J^{k-1})}}$$

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  $$\boldsymbol{s^k = -\nabla J^k + \beta^k s^{k-1}}$$

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4. **Update the model:**

$$m^{k+1} = m^k + \alpha^k s^k$$

Inversion setup

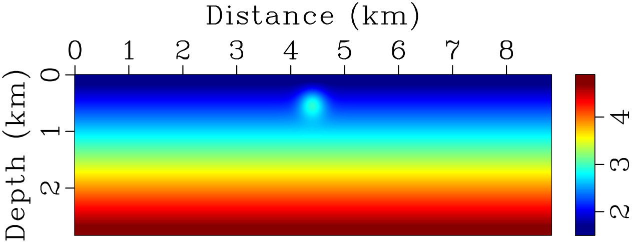
- **Line search: Satisfies strong Wolfe criteria**

- **Gradient computed separately for each shot**

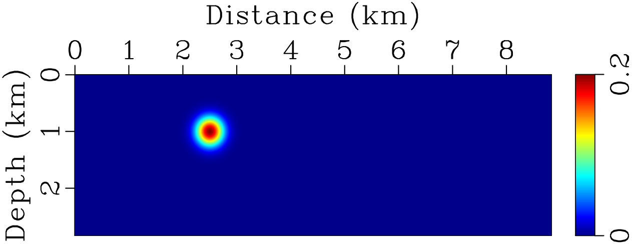
- **Parallelization: OpenMP**

Challenge: Multiparameter Inversion

Multiparameter Inversion: True Model

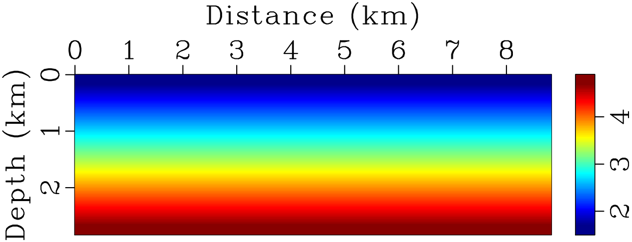


v_{nmo}

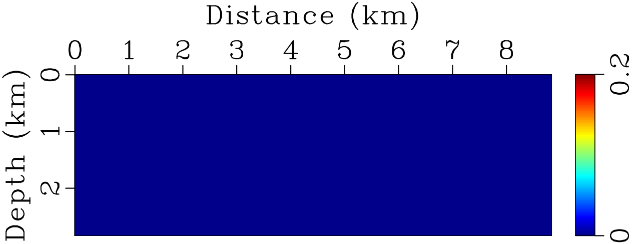


η

Multiparameter Inversion: Initial Model

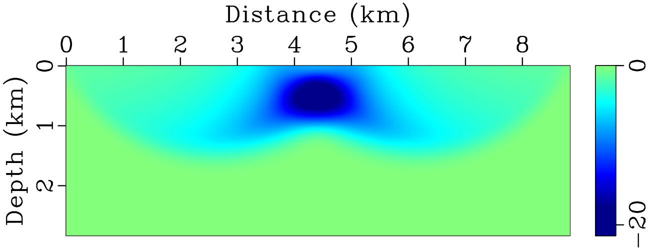


v_{nmo}

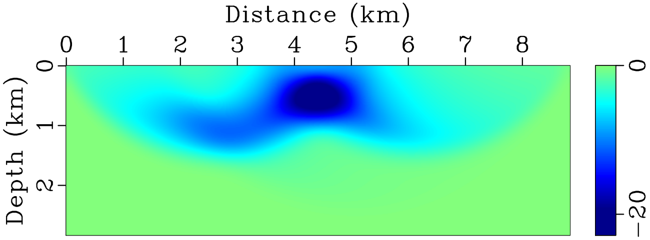


η

Multiparameter Inversion: Gradient

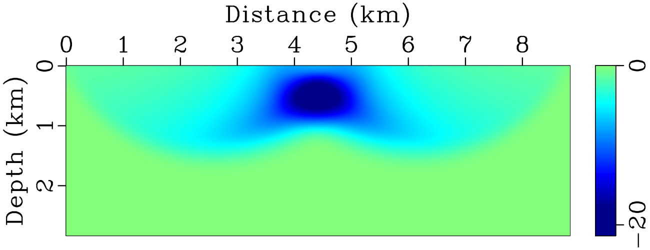


Isotropic

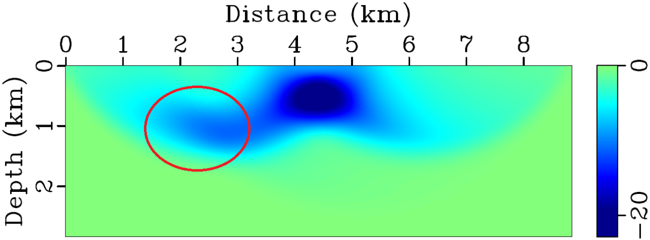


Anisotropic

Multiparameter Inversion: Gradient



Isotropic



Anisotropic

Multiparameter Inversion

- **Strategies tested:**

  1. **Simultaneous:** v_{nmo} **and** $v_h (v_h = v_{nmo}\sqrt{1+2\eta})$

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3. **Sequential: Stage 1 -** v_{nmo}
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     Stage 2 - v_{nmo} **and** η
  3. **Sequential: Stage 1 -** v_{nmo}
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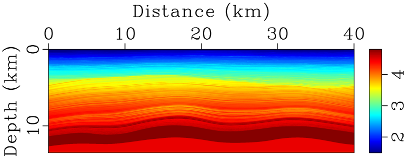
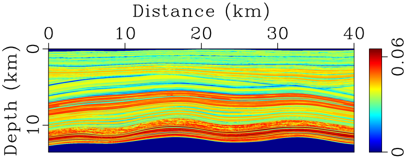
- **When to switch from Stage 1 to Stage 2?**

Numerical Tests

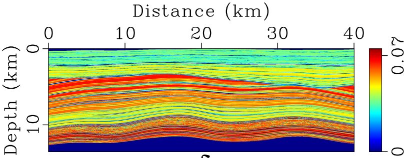
VTI SEAM Model



$\boldsymbol{v_{nmo}}$



$\boldsymbol{\eta}$

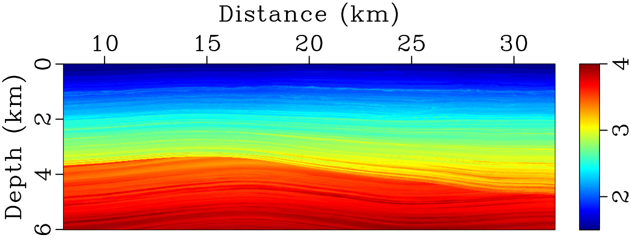


$\boldsymbol{\delta}$

VTI SEAM Model Test

- $\boldsymbol{nx = 2001, nz = 676}$
- $\boldsymbol{dx = dz = 20\,\mathrm{m}}$
- **Number of shots = 101**
- **Receiver spacing = 20 m**

NMO Velocity



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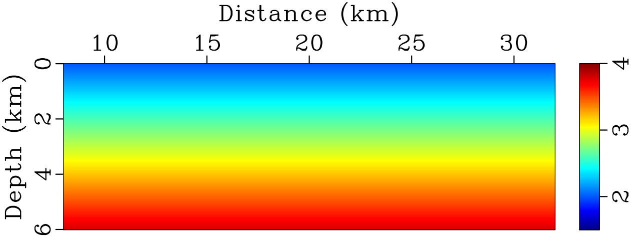
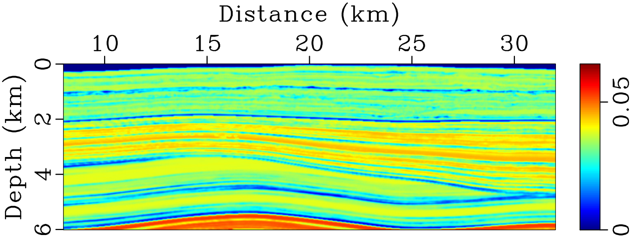


Initial

η parameter



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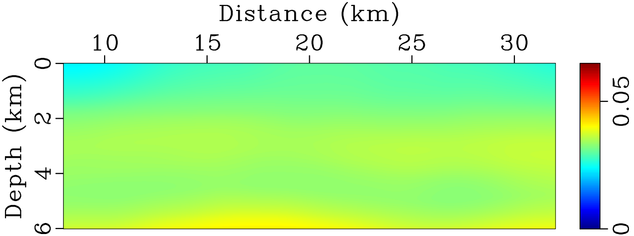
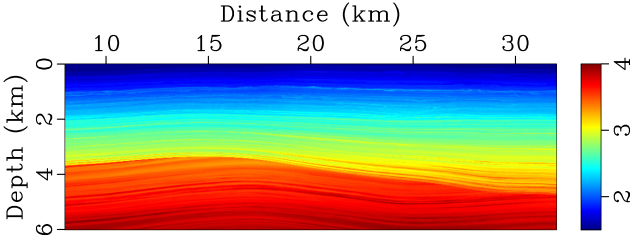


Initial

NMO Velocity



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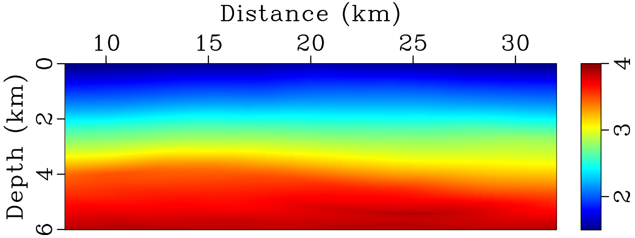
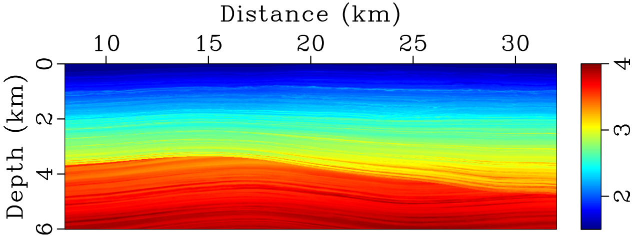


Inverted

NMO Velocity



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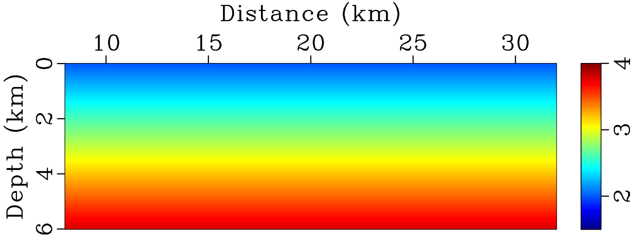
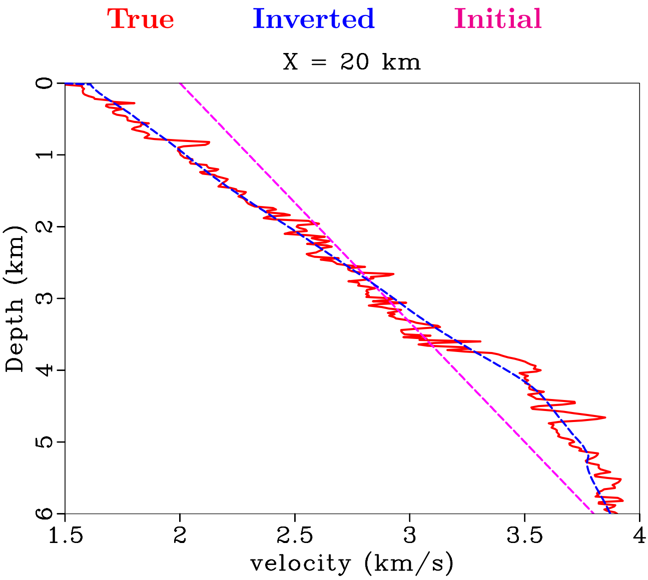


Initial

NMO Velocity



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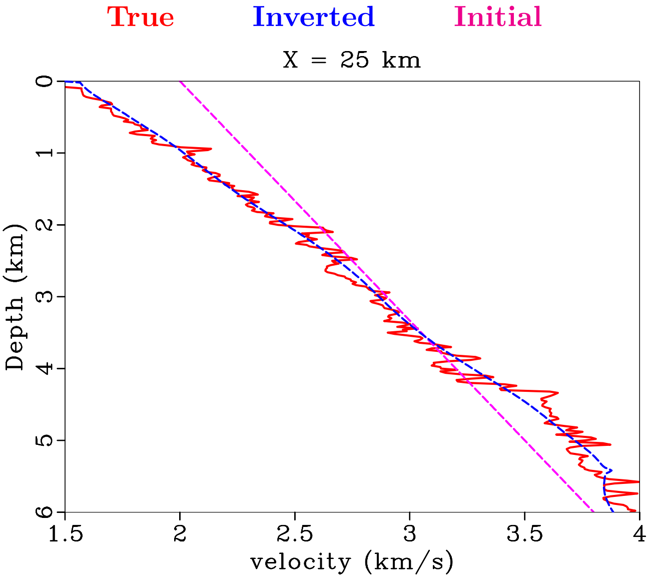
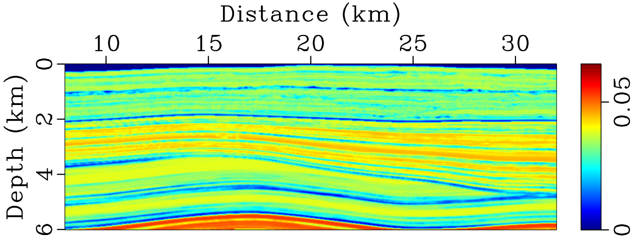


η parameter



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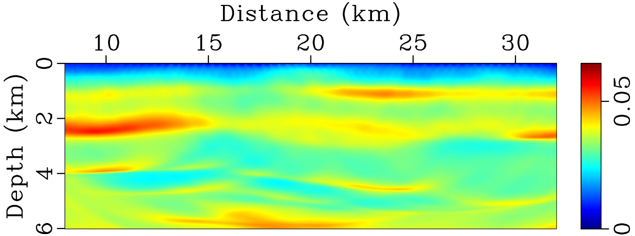
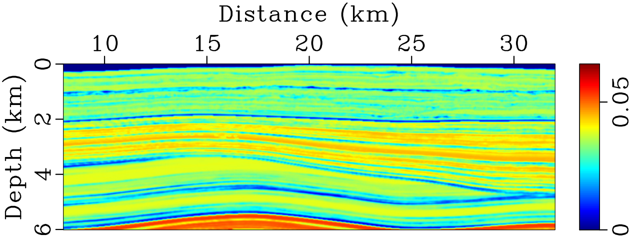


Inverted

η parameter



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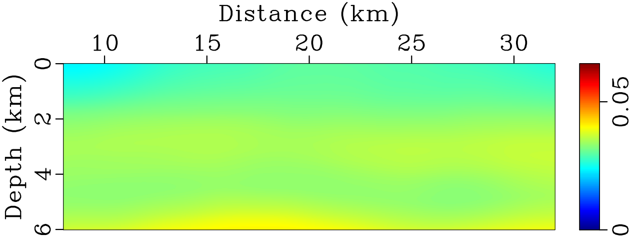
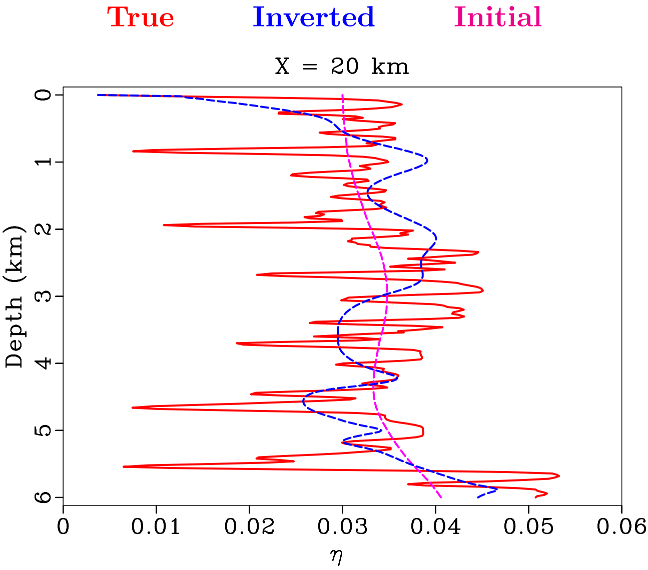
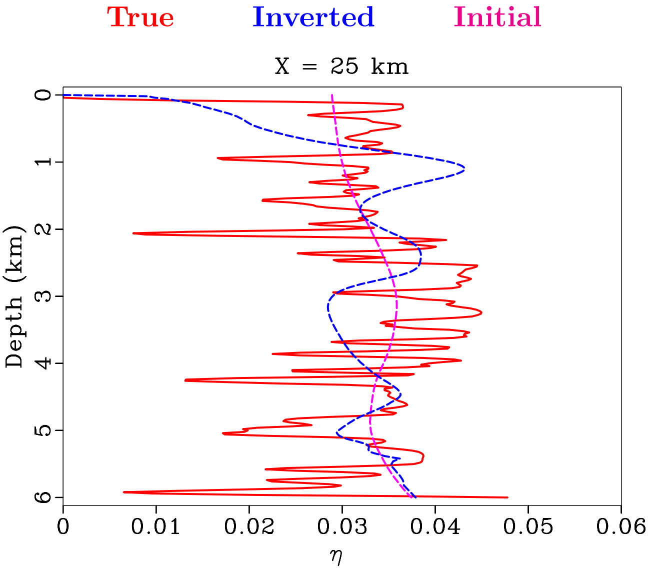


Initial

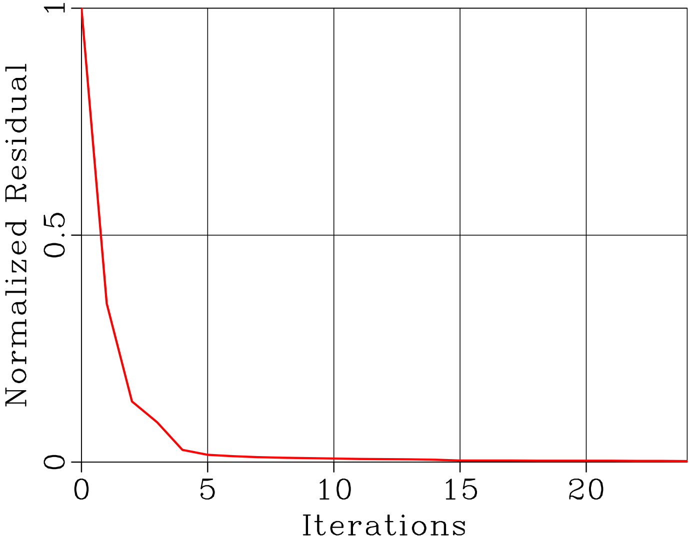
η parameter



η parameter



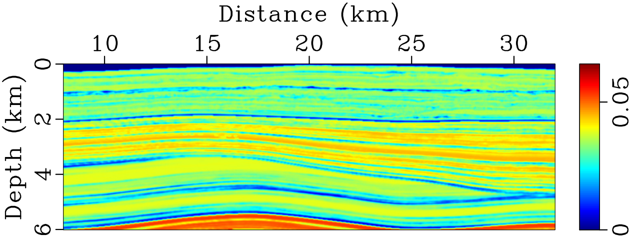
Normalized Residual



Switching between parameters

- **Stage 1: reduce the residual up to 10% of its initial value**
- **Stage 2: force the remaining residual into η**
- **Assumption: knowledge of upper bound on η**

Switching between parameters



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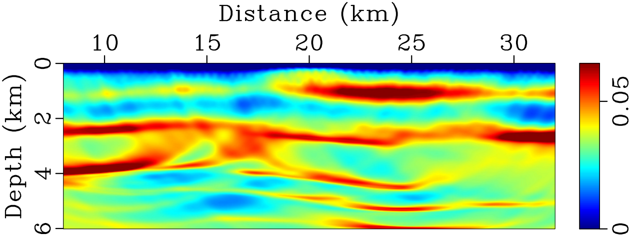
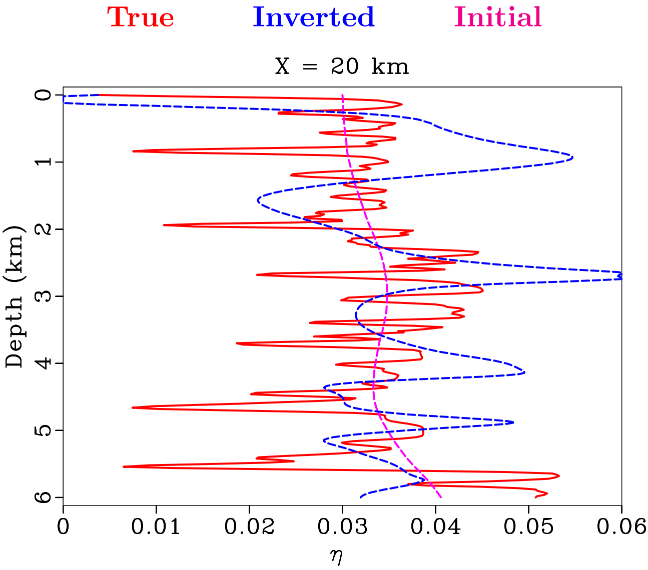


Inverted

Switching between parameters



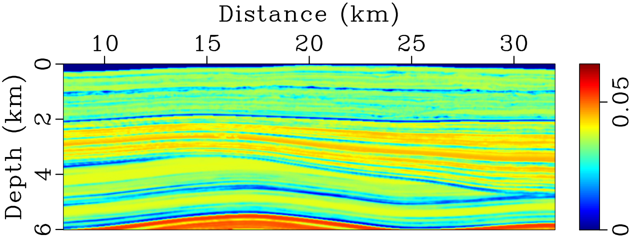
Switching between parameters

- **Stage 1: reduce the residual up to 5% of its initial value**

- **Stage 2: force the remaining residual into $\boldsymbol{\eta}$**

- **Assumption: knowledge of upper bound on $\boldsymbol{\eta}$**

Switching between parameters



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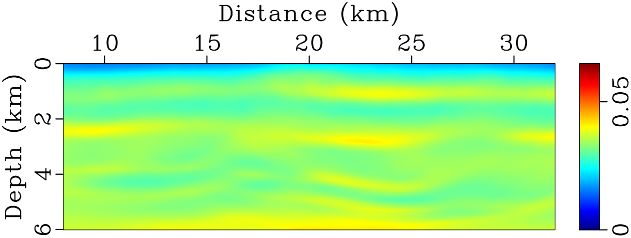

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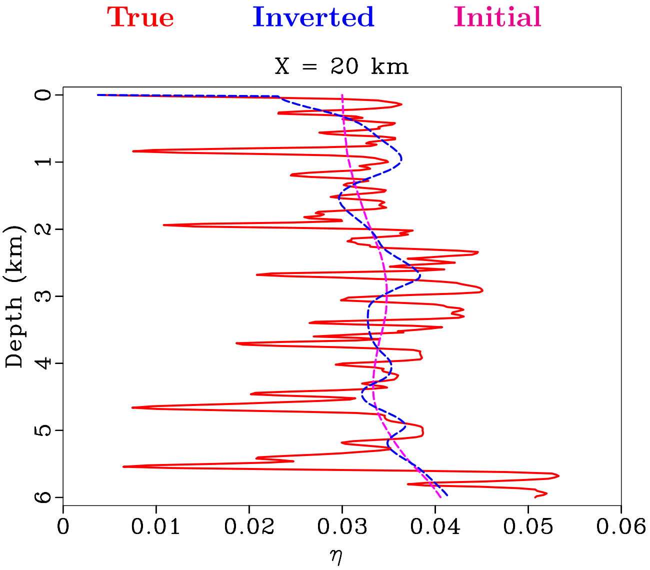

Inversion for δ

- **Additional source of information, such as receivers in borehole**

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- **The adjoint variable λ satisfies**

$$\nabla.(L\lambda) = 0$$

with the boundary condition

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Inversion for δ

- **Additional source of information, such as receivers in borehole**

- **The adjoint variable $\boldsymbol{\lambda}$ satisfies**

$$\boldsymbol{\nabla}.(L\boldsymbol{\lambda}) = 0$$

with the boundary condition

$$(\mathrm{n}.L)\boldsymbol{\lambda} = T^{obs} - T^{comp}$$

- **For receivers inside the medium, $\boldsymbol{\lambda}$ satisfies**

$$\boldsymbol{\nabla}.(L\boldsymbol{\lambda}) = T^{obs} - T^{comp}$$

Conclusions

- **FAT tomography algorithm using the adjoint-state method**
- **Candidate for first round on board model building**
- **Multi-parameter inversion**

Conclusions

- **Extendable to lower anisotropic symmetries**

- **Need a strategy to decouple additional parameters**

- **Borehole measurements can help resolve $\boldsymbol{\delta}$**

Acknowledgments

- **Schlumberger for financial support and permission to share results**

- **Marvin Decker, Xin Cheng, Marta Woodward, James Rickett, Chris Chapman, and Michael Williams for useful discussions**

Strong Wolfe Criteria

- **Sufficient decrease condition:**

$$\boldsymbol{f(x + \alpha p) \leq f(x) + c_1 \alpha_k \nabla f(x).p}$$

- **Curvature condition:**

$$\boldsymbol{|\nabla f(x + \alpha p).p| \leq c_2 |\nabla f(x).p|}$$