First-order reflection/transmission coefficients for unconverted plane P waves in weakly anisotropic media

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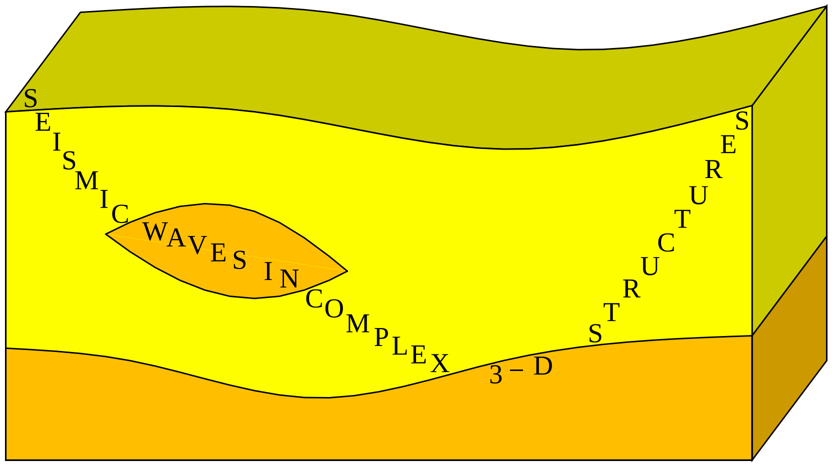


Outline:

Introduction

Displacement vector, traction

- Transformation of slowness vector
- Transformation of amplitudes

Numerical examples

Conclusions

Introduction

- generalization of FORT for layered media

  (first-order wrt deviations of anisotropy from isotropy)

- arbitrary-contrast interface
  separating arbitrary weakly anisotropic media

- first-order phase velocitites, slowness
  and polarization vectors

- P wave and one coupled S wave

Displacement vector, traction

Displacement vector

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{U}\mathrm{exp}[-\mathrm{i}\omega(t - \mathbf{p} \cdot \mathbf{x})]$$

$\mathbf{U}$ - first-order vectorial amplitude factor

$\mathbf{p}$ - first-order slowness vector, $\mathbf{p} = \mathbf{n}/c(\mathbf{n})$

c - first-order phase velocity

Displacement vector, traction

P wave

$$\mathbf{U} = \mathcal{C}\mathbf{f}^{[3]}$$

$\mathcal{C}$ - P-wave first-order scalar amplitude factor

$\mathbf{f}^{[3]}$ - P-wave first-order polarization vector (generally non-unit)

$$\mathbf{f}^{[3]}(\mathbf{p}^{[3]}) = \frac{B_{13}(\mathbf{p}^{[3]})\mathbf{e}^{[1]}(\mathbf{p}^{[3]}) + B_{23}(\mathbf{p}^{[3]})\mathbf{e}^{[2]}(\mathbf{p}^{[3]})}{1 - \frac{1}{2}[B_{11}(\mathbf{p}^{[3]}) + B_{22}(\mathbf{p}^{[3]})]} + \mathbf{e}^{[3]}(\mathbf{p}^{[3]})$$

Displacement vector, traction

S wave

$$\mathbf{U} = \mathcal{A}\mathbf{f}^{[1]} + \mathcal{B}\mathbf{f}^{[2]}$$

$\mathcal{A}$, $\mathcal{B}$ - S-wave first-order scalar amplitude factors

$\mathbf{f}^{[K]}$ - vectors defining S-wave first-order polarization plane (generally non-unit)

$$\mathbf{f}^{[K]}(\mathbf{p}^{[\mathcal{M}]}) = \mathbf{e}^{[K]}(\mathbf{p}^{[\mathcal{M}]}) + \frac{B_{K3}(\mathbf{p}^{[\mathcal{M}]})}{1-B_{33}(\mathbf{p}^{[\mathcal{M}]})}\mathbf{e}^{[3]}(\mathbf{p}^{[\mathcal{M}]})$$

Displacement vector, traction

Traction

$$T_i = \mathrm{i}\omega\rho a_{ijkl} N_j U_k p_l \exp[-\mathrm{i}\omega(t - \mathbf{p}\cdot\mathbf{x})]$$

N - unit normal to the interface

Displacement vector, traction

$$B_{jl}(\mathbf{p}) = \Gamma_{ik}(\mathbf{p}) e_i^{[j]} e_k^{[l]}$$

$\mathbf{\Gamma}(\mathbf{p})$ - generalized Christoffel matrix, $\Gamma_{ik} = a_{ijkl} p_j p_l$

a_{ijkl} - density-normalized elastic moduli

$\mathbf{e}^{[j]}$ - wavefront orthonormal basis

$\mathbf{e}^{[3]} = \mathbf{n}, \qquad \mathbf{e}^{[K]}$ chosen arbitrarily in the plane $\perp \mathbf{e}^{[3]}$

$G(\mathbf{p})$ - first-order eigenvalues of $\mathbf{\Gamma}$

$G(\mathbf{p}) = 1$ - first-order eikonal equation

$$(c^{[\mathcal{M}]})^2(\mathbf{n}) = \tfrac{1}{2}[G^{[1]}(\mathbf{n}) + G^{[2]}(\mathbf{n})], \qquad (c^{[3]})^2(\mathbf{n}) = G^{[3]}(\mathbf{n})$$

Transformation of slowness vector

$$p_i^G - (p_k^G N_k) N_i = p_i - (p_k N_k) N_i \qquad \text{(Snell law)}$$

N - unit normal to the interface

p - first-order slowness vector of incident wave

$\mathbf{p^G}$ - first-order slowness vector of generated wave

$$p_i^G = b_i + \xi^G N_i \qquad\qquad b_i = p_i - (p_k N_k) N_i$$

$G(b_i + \xi^G N_i) = 1$ - polynomial equation of 4th degree

Transformation of slowness vector

$G(b_i + \xi^G N_i) = 1$ - polynomial equation of 4th degree

a) numerical solution

b) iterative solution

Iterative solution

$$\mathbf{p}^{G\{j\}} = \mathbf{b} + \xi^{G\{j\}}\mathbf{N}$$

$$\xi^{G\{j\}} = \xi^{G\{j-1\}} - \frac{G(\mathbf{p}^{G\{j-1\}})-1}{N_k \partial G/\partial p_k(\mathbf{p}^{G\{j-1\}})}$$

Transformation of amplitudes

$$\mathcal{A}^R f_i^{[1]R} + \mathcal{B}^R f_i^{[2]R} + \mathcal{C}^R f_i^{[3]R} - \mathcal{A}^T f_i^{[1]T} - \mathcal{B}^T f_i^{[2]T} - \mathcal{C}^T f_i^{[3]T} = -U_i$$

$$\mathcal{A}^R X_i^{[1]R} + \mathcal{B}^R X_i^{[2]R} + \mathcal{C}^R X_i^{[3]R} - \mathcal{A}^T X_i^{[1]T} - \mathcal{B}^T X_i^{[2]T} - \mathcal{C}^T X_i^{[3]T} = -X_i$$

$$X_i = \rho^{(1)} a_{ijkl}^{(1)} N_j U_k p_l \quad ,$$

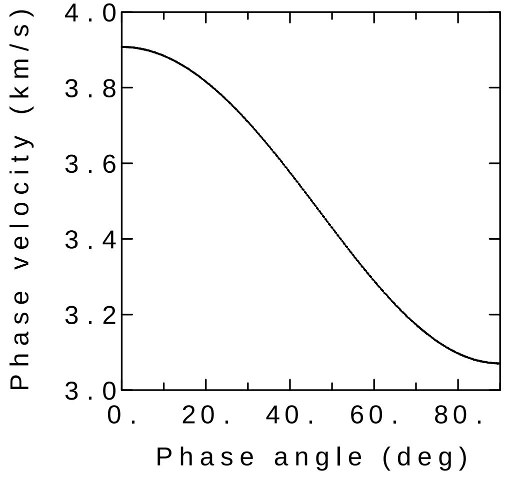
$$X_i^{[3]R} = \rho^{(1)} a_{ijkl}^{(1)} N_j f_k^{[3]R} p_l^{[3]R} \ , \quad X_i^{[3]T} = \rho^{(2)} a_{ijkl}^{(2)} N_j f_k^{[3]T} p_l^{[3]T} \ ,$$

$$X_i^{[N]R} = \rho^{(1)} a_{ijkl}^{(1)} N_j f_k^{[N]R} p_l^{[\mathcal{M}]R} \ , \quad X_i^{[N]T} = \rho^{(2)} a_{ijkl}^{(2)} N_j f_k^{[N]T} p_l^{[\mathcal{M}]T}$$

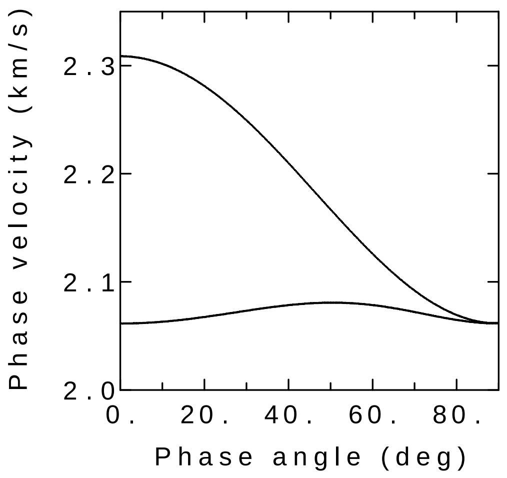
MODEL: ISO/HTI

ISO: α=3.0 km/s, β=1.73 km/s, ρ=2.2 g/cm^3

HTI:



P WAVE



S WAVES

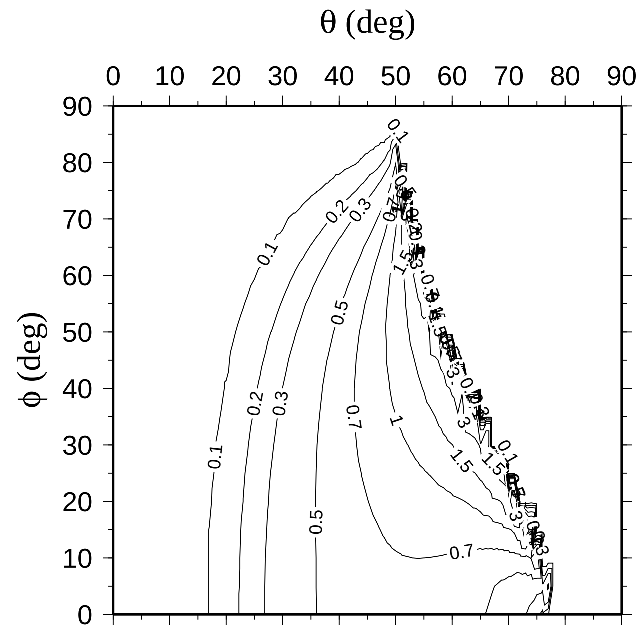
Anisotropy: P wave $\sim$ 24%, S1 wave $\sim$ 11%

Contrast (normal inc. $\rightarrow$ tangent. inc.):

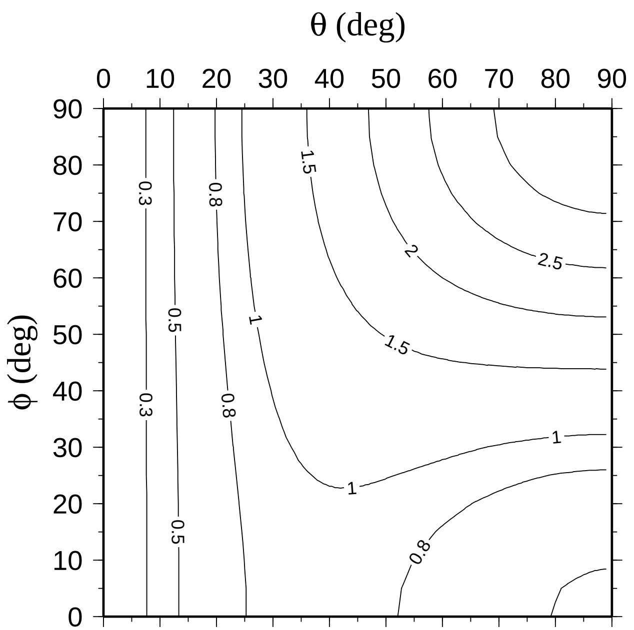
P wave - 26% $\rightarrow$ 2%, S1 wave - 29% $\rightarrow$ 17%, S2 wave $\sim$ 17%

SLOWNESS VECTORS

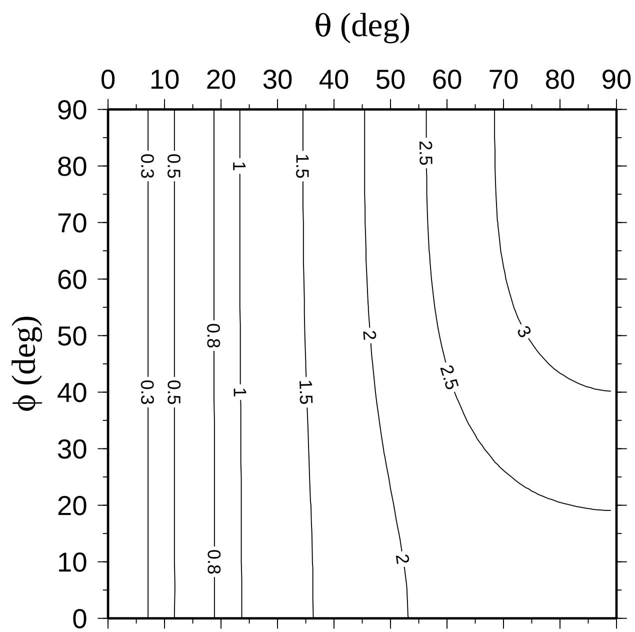
P wave



S1 wave

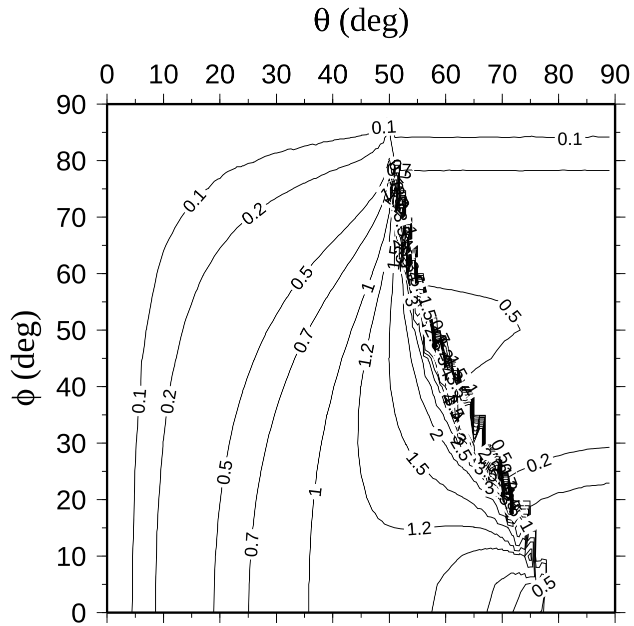


S2 wave

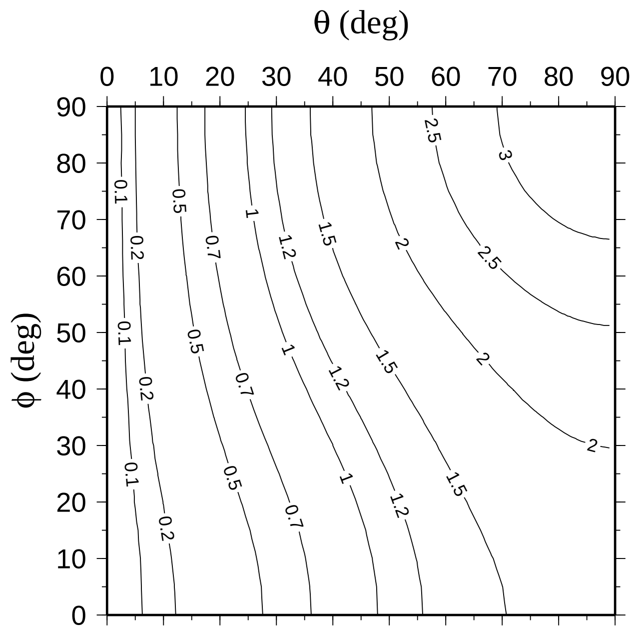


POLARIZATION VECTORS

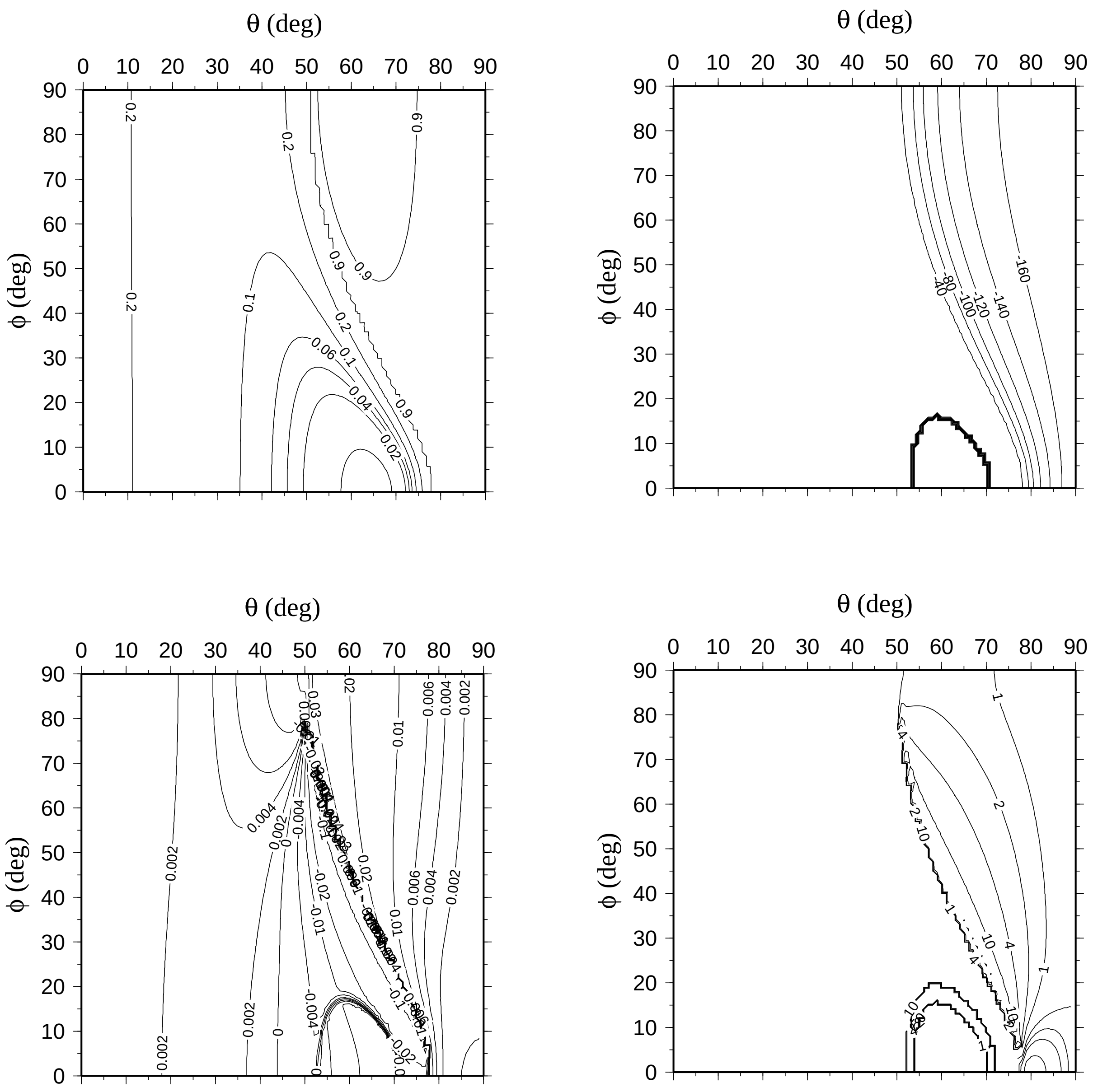
P wave



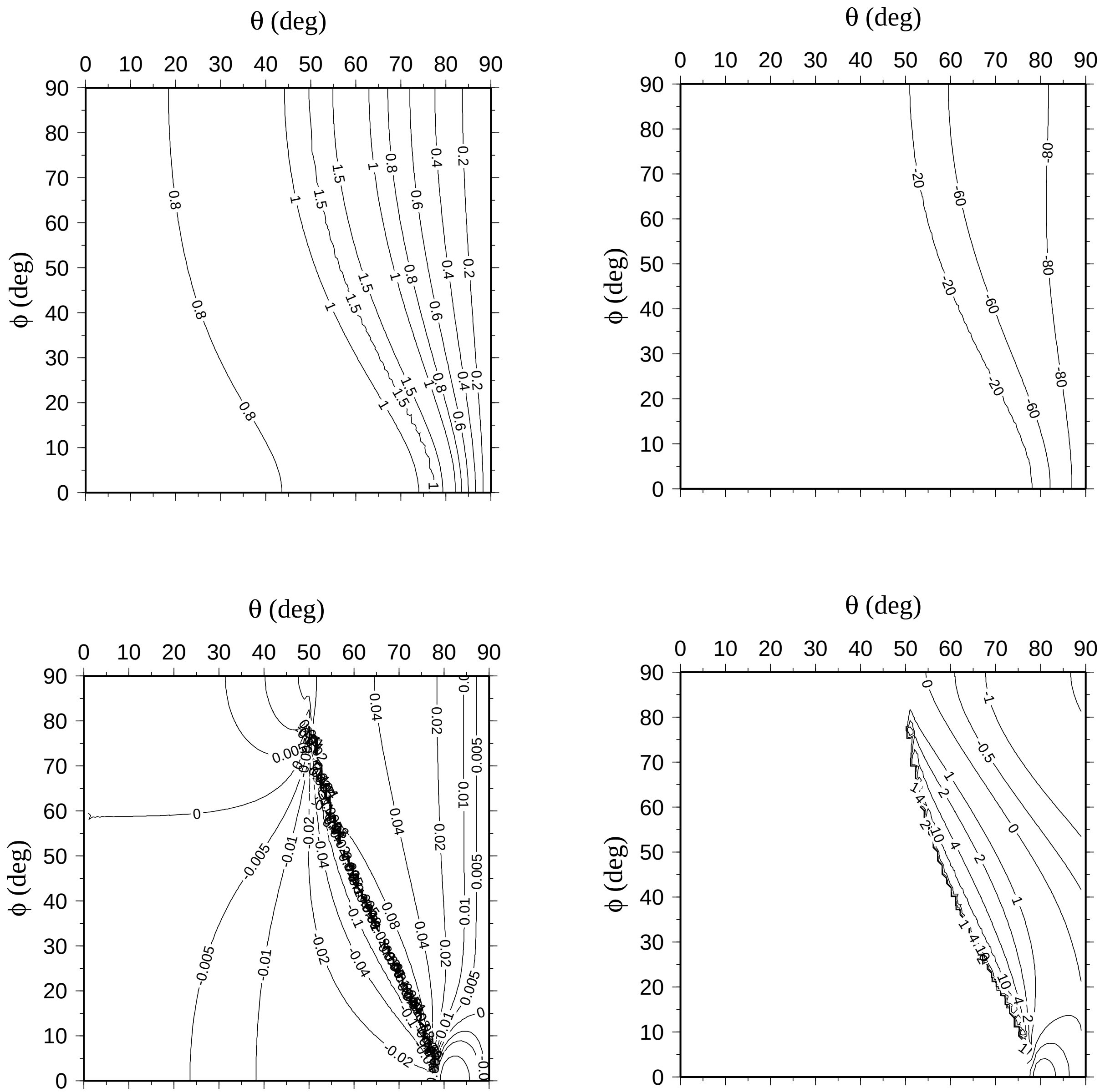
S wave



RPP COEFFICIENT



TPP COEFFICIENT



Conclusions

- applicability to interfaces of arbitrary contrast
- applicability to arbitrary incidence angles and azimuths
- coupled S waves considered as one wave $\Rightarrow$
  no failures in directions close to singular
- separate kinematics for P and S waves
- applicability to weakly anisotropic media only
- lower accuracy in regions of critical or Brewster angles