Paraxial Super-Gaussian beams

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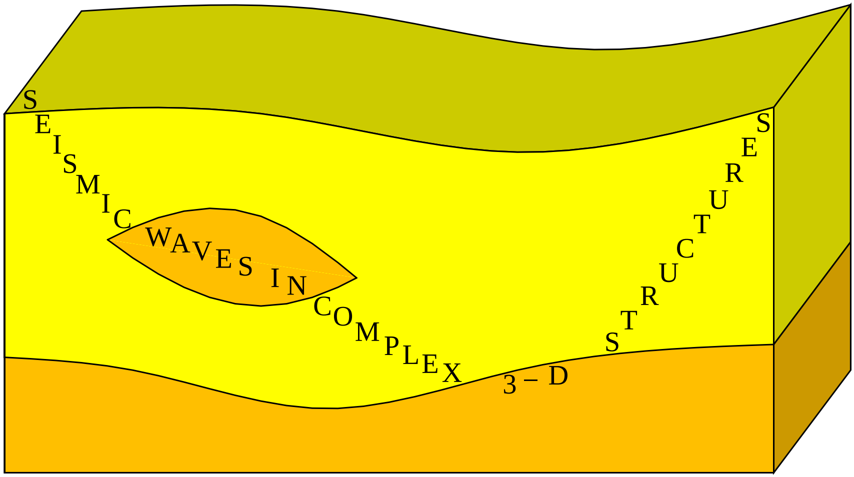

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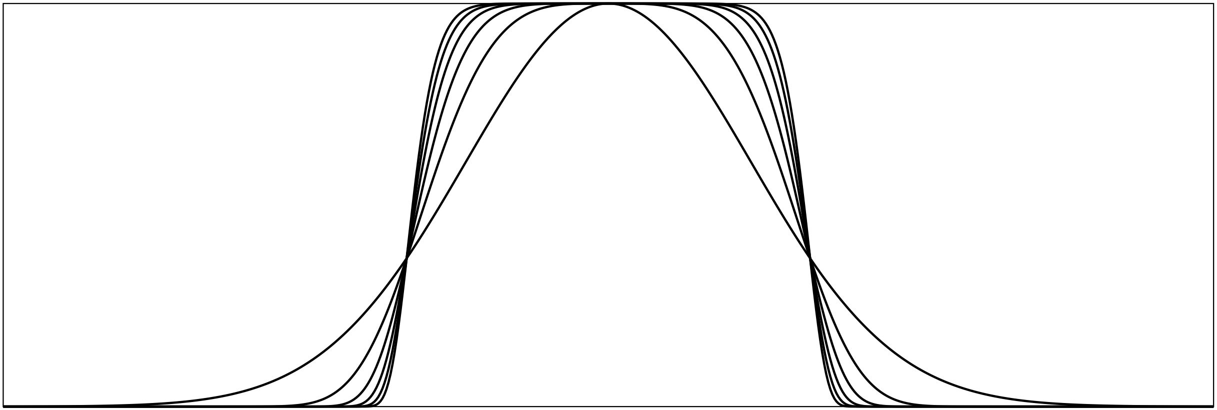
Amplitude cross–section:

$$\exp(-ax^N)\ , \quad N = 4, 6, 8, \ldots \ .$$



Plot of Gaussian function $\exp(-x^2)$ and supergauss functions $\exp(-x^4)$, $\exp(-x^6)$, $\exp(-x^8)$, $\exp(-x^{10})$ and $\exp(-x^{12})$.

Matrix

$$Q_{ia} = \frac{\partial x_i}{\partial \gamma_a}$$

of geometrical spreading is real-valued.

Travel-time derivatives (Klimeš, 2002):

$$\tau_{,ij\ldots n} = T_{ab\ldots f}\, Q^{-1}_{ai}\, Q^{-1}_{bj} \cdots Q^{-1}_{fn} \quad ,$$

$$T_{ab\ldots f}(\gamma) = T_{ab\ldots f}(\gamma^0) + \int_{\gamma^0}^{\gamma} \mathrm{d}\gamma\; K_{ij\ldots n}\, Q_{ia} Q_{jb} \cdots Q_{nf} \quad .$$

The integration kernels $K_{ij\ldots n}$ corresponding to the N^{th}-order derivatives of travel time are real-valued:

$$\mathrm{Im}[T_{ab\ldots f}(\gamma)] = \mathrm{Im}[T_{ab\ldots f}(\gamma^0)] \quad .$$

The lowest–order (N^{th}-order) paraxial approximation of the imaginary part of the travel time is constant along all paraxial rays.

The Super–Gaussian beams are thus equivalent to the zero–order ray–theory wavefield with the real–valued travel time and with the initial Super–Gaussian amplitude profile, without the diffracted wavefield which could result from the representation theorem.

The diffraction of a beam is thus satisfactorily included in the case of paraxial Gaussian beams, but not in the case of paraxial Super–Gaussian beams.

References (online at "http://sw3d.cz")

Klimeš, L. (2002): Second–order and higher–order perturbations of travel time in isotropic and anisotropic media. *Stud. geophys. geod.*, **46**, 213–248.

Klimeš, L. (2013): Paraxial Super-Gaussian beams. In: *Seismic Waves in Complex 3-D Structures, Report 22*, pp. 145–148, Dep. Geophys., Charles Univ., Prague.

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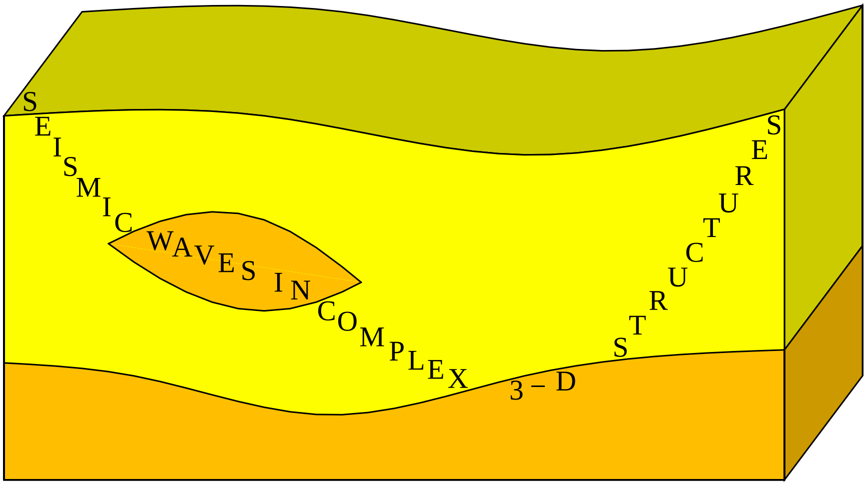

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