Moveout approximations for P waves in media of monoclinic and higher anisotropy symmetries

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Outline

Introduction

Traveltime approximations

Normal moveout velocity

Tests of the formulae

Conclusions

Introduction

Moveout approximations

standard

- expansion of T^2 in terms of the squared offset

  hyperbolic, non-hyperbolic, ...

alternative

- expansion of T^2 in terms of the deviations

  of anisotropy from isotropy

Introduction

Monoclinic or higher symmetry anisotropic media

Unconverted P wave

reflected from a plane reflector Σ

coinciding with a symmetry plane

$\Rightarrow$ ray of reflected P wave in a plane $\perp$ reflector Σ

2D problem in the (x', z') plane of local x', y', z' coordinate system

Two-step derivation: 1) local, 2) global coordinates

Traveltime approximations

Normal moveout formula

$$T^2(x) = (4H^2 + x^2)/v^2(\mathbf{n})$$

$T(x)$ - traveltime at the offset x

$v(\mathbf{n})$ - ray velocity; the same on down- and up-going leg of ray

$\mathbf{n}$ - unit vector in the direction of the slowness vector

$\mathbf{n}$ may deviate from (x', z') plane

H - depth of the plane reflector

Traveltime approximations

Normalized moveout formula

$$\bar{x} = x/2H \ , \qquad T_0 = 2H/\alpha_0$$

$$T^2(\bar{x}) = \alpha_0^2 T_0^2 \, (1 + \bar{x}^2)/v^2(\mathbf{n})$$

T_0 - two-way zero-offset traveltime

$\bar{x}$ - normalized offset

α_0 - vertical velocity ($\alpha_0^2 = A_{33}$)

$A_{\alpha\beta}$ - density-normalized elastic moduli in the Voigt notation

Traveltime approximations

Problem: Needed: $v^2(\mathbf{n})$

Available: $\mathbf{N}$ and $c^2(\mathbf{N})$

$c(\mathbf{N})$ - phase velocity

$v(\mathbf{n})$ - ray velocity

$\mathbf{n}$ - unit vector in the direction of the slowness vector

$\mathbf{N}$ - unit vector in the direction of the ray-velocity vector $\mathbf{v}$

$\Rightarrow$ need to find the relation between $v^2(\mathbf{n})$ and $c^2(\mathbf{N})$

Traveltime approximations

Approximations of ray velocity

1) Ignore the difference between $\mathbf{n}$ and $\mathbf{N}$

$$v^2(\mathbf{n}) = c^2(\mathbf{N})$$

2) Consider the difference between $\mathbf{n}$ and $\mathbf{N}$

$$v^2(\mathbf{n}) = (c^4(\mathbf{N}) - 4[B_{13}^2(\mathbf{N}) + B_{23}^2(\mathbf{N})])/c^2(\mathbf{N})$$

3) Consider the difference between $\mathbf{n}$ and $\mathbf{N}$

and use 2nd-order approximation of c

$$v^2(\mathbf{n}) = (c^4(\mathbf{N}) + 4a[B_{13}^2(\mathbf{N}) + B_{23}^2(\mathbf{N})])/c^2(\mathbf{N})$$

Traveltime approximations

Approximations of ray velocity

$$B_{13}(\mathbf{N}) = \alpha_0^2 N_1' N_3' [\delta_y' - 2(\delta_y' - \epsilon_x') N_1'^2]$$

$$B_{23}(\mathbf{N}) = \alpha_0^2 N_1' (\chi_z' N_3'^2 + \epsilon_{16}' N_1'^2)$$

$$c^2(\mathbf{N}) = B_{33}(\mathbf{N}) = \alpha_0^2 (1 + 2N_1'^2 [\epsilon_x' + (\delta_y' - \epsilon_x') N_3'^2])$$

$$a = (r^2 - 3/4)/(1 - r^2) \qquad r = \beta_0/\alpha_0 \qquad \beta_0^2 = A_{55}$$

ϵ_x', δ_y', χ_z', ϵ_{16}' - WA parameters in local coordinate system

N_i' - components of $\mathbf{N}$ in local coordinate system ($N_2' = 0$)

Traveltime approximations

Traveltime formulae - local coordinates

1) $T^2(\bar{x}) = T_0^2 \; ((1+\bar{x}^2)^3)/P(\bar{x})$

2) $T^2(\bar{x}) = T_0^2 \; (P(\bar{x})(1+\bar{x}^2)^3)/(P^2(\bar{x}) - Q_1^2(\bar{x}) - (1+\bar{x}^2)Q_2^2(\bar{x}))$

3) $T^2(\bar{x}) = T_0^2 \; (P(\bar{x})(1+\bar{x}^2)^3)/(P^2(\bar{x}) + a[Q_1^2(\bar{x}) + (1+\bar{x}^2)Q_2^2(\bar{x})])$

$$P(\bar{x}) = (1+\bar{x}^2)^2 + 2\delta'_y\bar{x}^2 + 2\epsilon'_x\bar{x}^4$$

$$Q_1(\bar{x}) = 2\bar{x}[2\epsilon'_x\bar{x}^2 + \delta'_y(1-\bar{x}^2)] \qquad Q_2(\bar{x}) = 2\bar{x}(\chi'_z + \epsilon'_{16}\bar{x}^2)$$

Normal moveout velocity

Definition

$$v_{NMO}^{-2} = dT^2/dx^2|_{x=0}$$

Local coordinates

1) $v_{NMO}^{-2} = \alpha_0^{-2}(1 - 2\delta'_y)$

2) $v_{NMO}^{-2} = \alpha_0^{-2}[1 - 2\delta'_y + 4((\delta'_y)^2 + (\chi'_z)^2)]$

3) $v_{NMO}^{-2} = \alpha_0^{-2}[1 - 2\delta'_y - 4a((\delta'_y)^2 + (\chi'_z)^2)]$

Normal moveout velocity

Global coordinates

$$v_{NMO}^{-2} = W_{11}\cos^2\varphi + 2W_{12}\cos\varphi\sin\varphi + W_{22}\sin^2\varphi$$

1) $W_{11} = \alpha_0^{-2}(1 - 2\delta_y)$ $\quad W_{12} = -2\alpha_0^{-2}\chi_z$ $\quad W_{22} = \alpha_0^{-2}(1 - 2\delta_x)$

3) $W_{11} = \alpha_0^{-2}[1 - 2\delta_y - 4a(\delta_y^2 + \chi_z^2)]$ $\quad W_{12} = -2\alpha_0^{-2}\chi_z[1 + 2a(\delta_x + \delta_y)]$

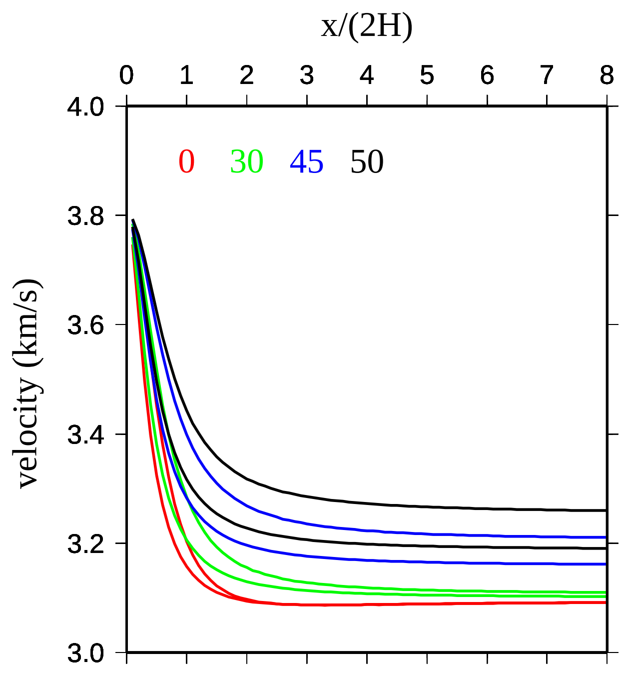
$W_{22} = \alpha_0^{-2}[1 - 2\delta_x - 4a(\delta_x^2 + \chi_z^2)]$

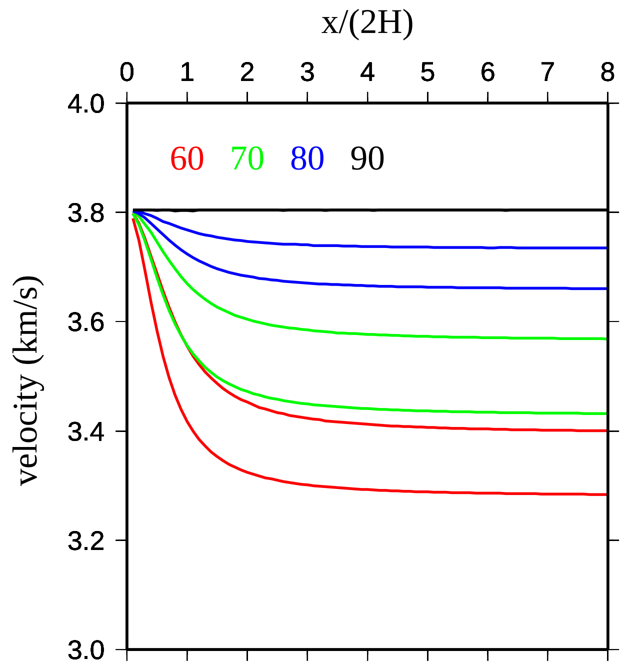
φ - angle of the source-receiver profile with x-axis

Tests of the formulae

HTI model, anisotropy $\sim$ 26%

α_0=3.805 km/s, β_0=1.510 km/s, ϵ_x=–0.169, δ_y=–0.373





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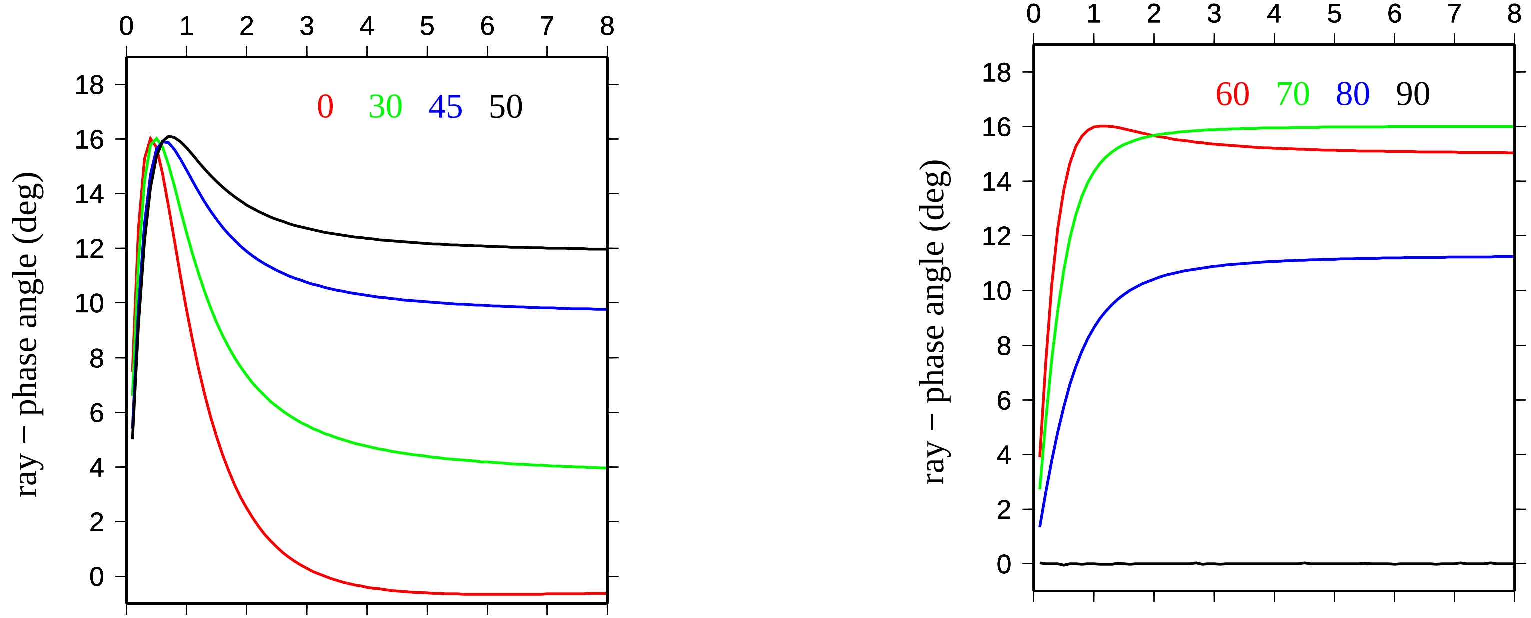

Tests of the formulae

HTI model, anisotropy $\sim$ 26%, formula # 1

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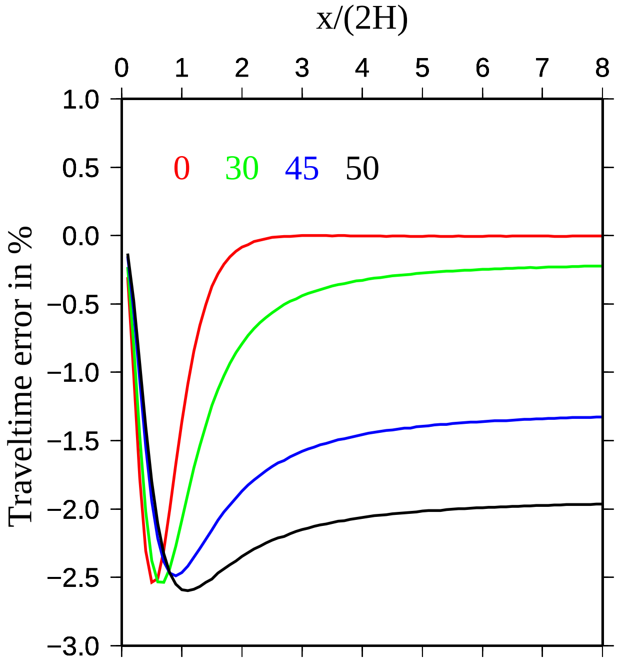

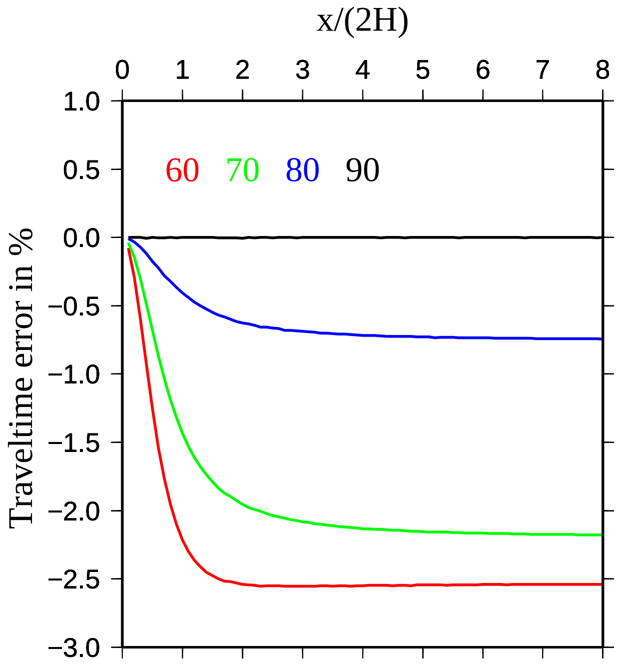

Tests of the formulae

HTI model, anisotropy $\sim$ 26%, formula # 2

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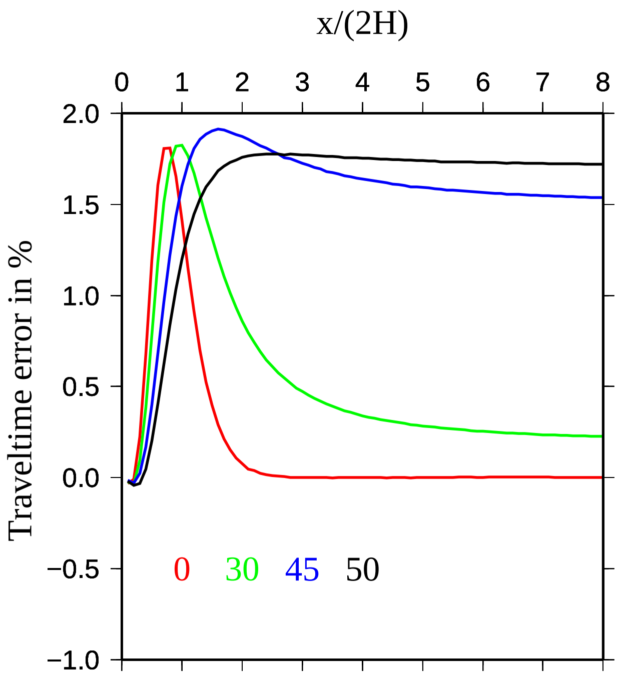

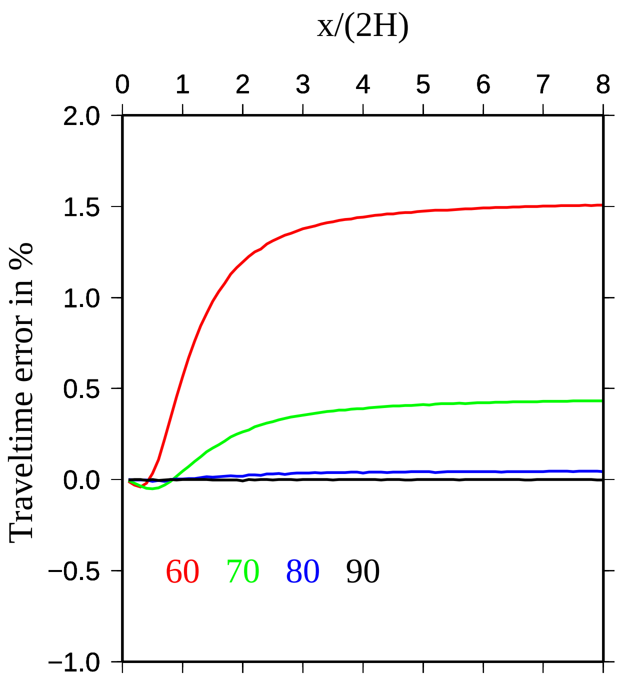

Tests of the formulae

HTI model, anisotropy $\sim$ 26%, formula # 3

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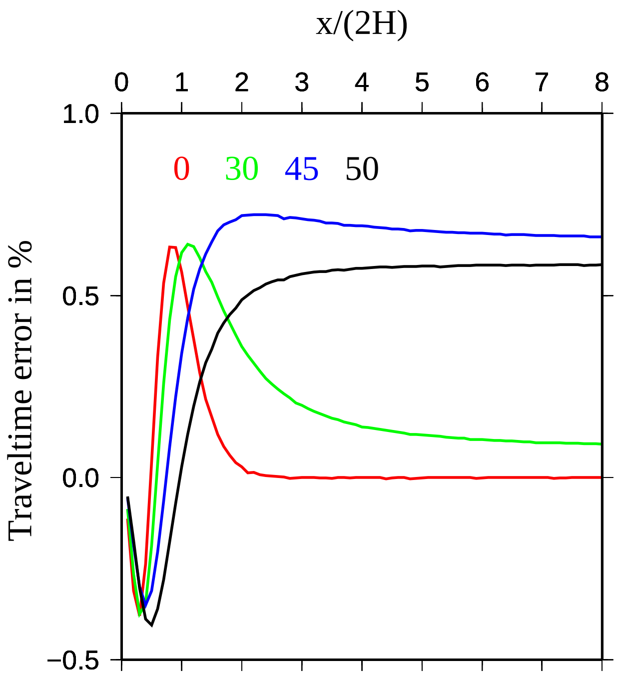

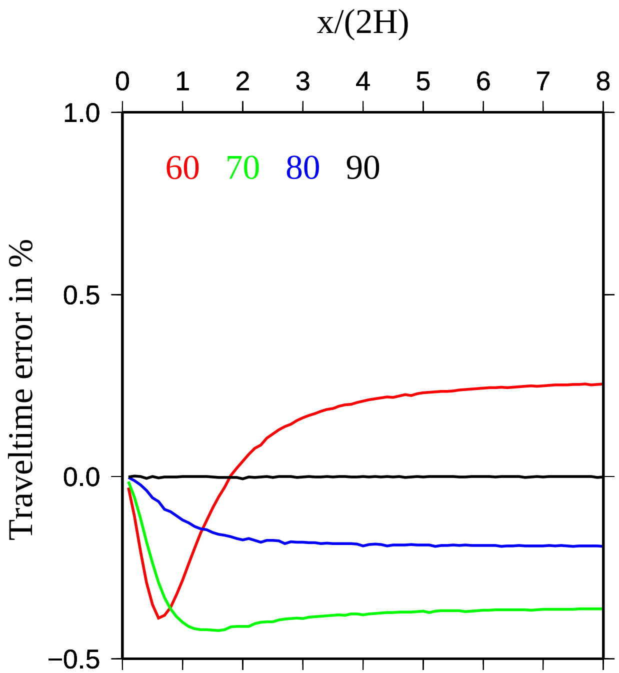

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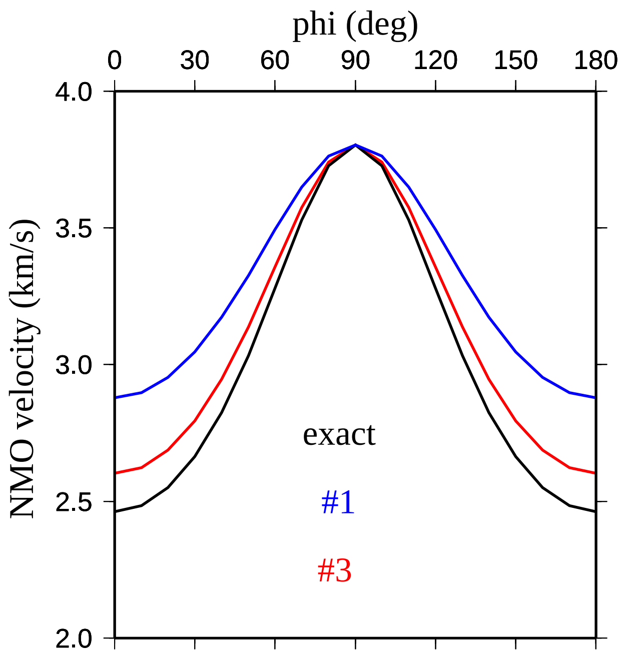
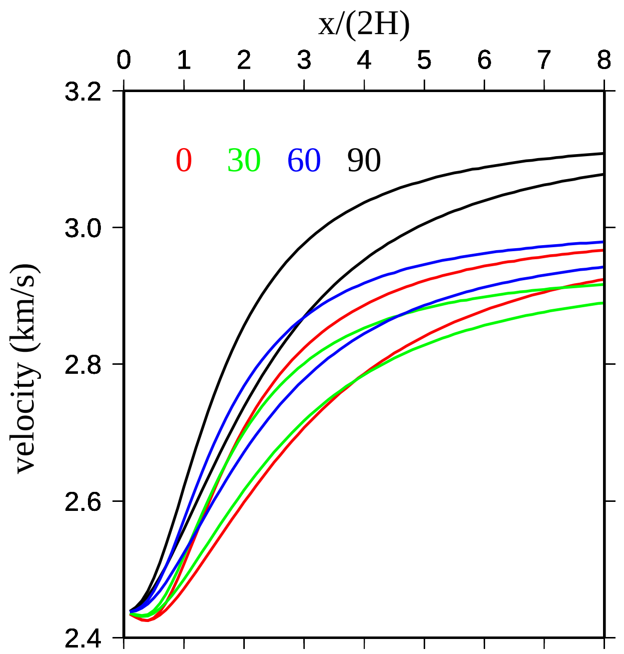


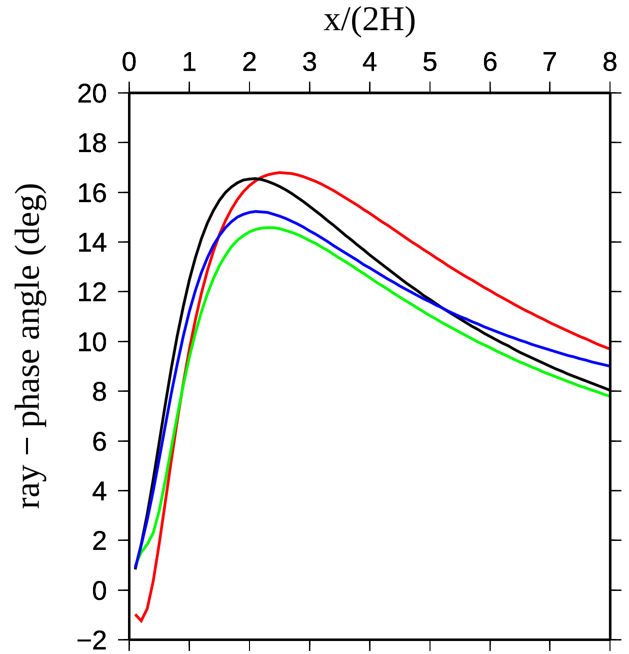
Tests of the formulae

ORT model, anisotropy $\sim$ 25%

α_0=2.437 km/s, β_0=1.414 km/s, ϵ_x=0.258, ϵ_y=0.328

δ_x=0.077, δ_y=–0.083, δ_z=0.340





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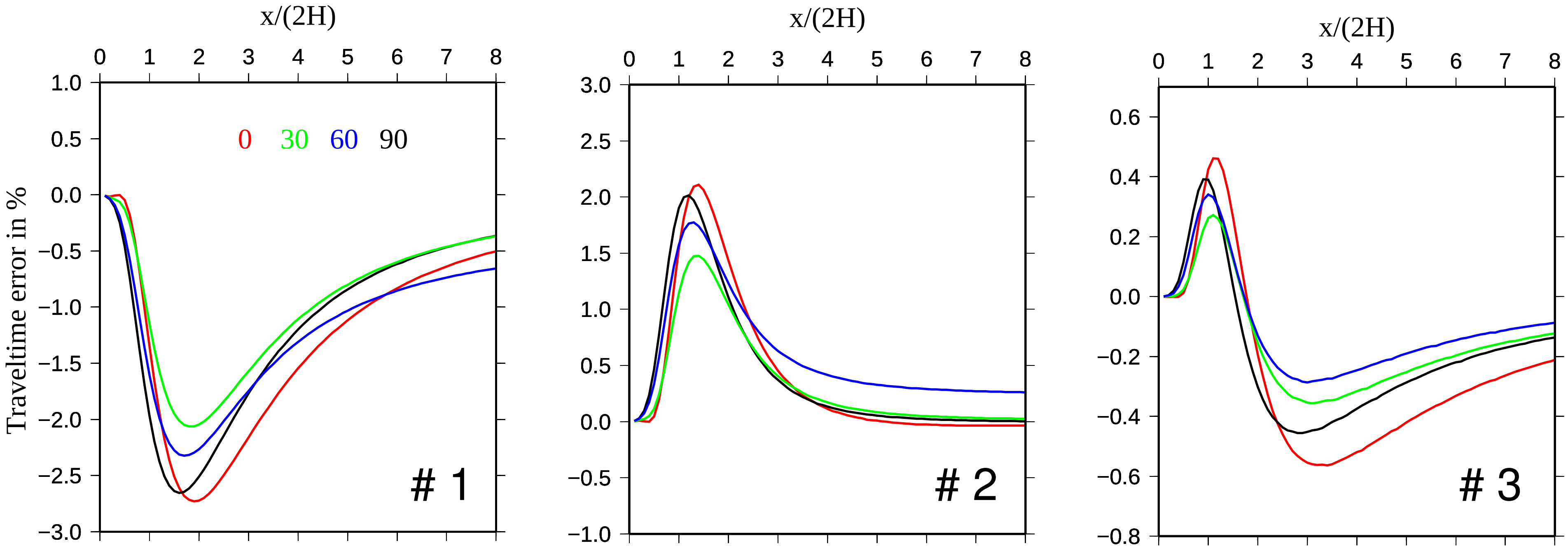

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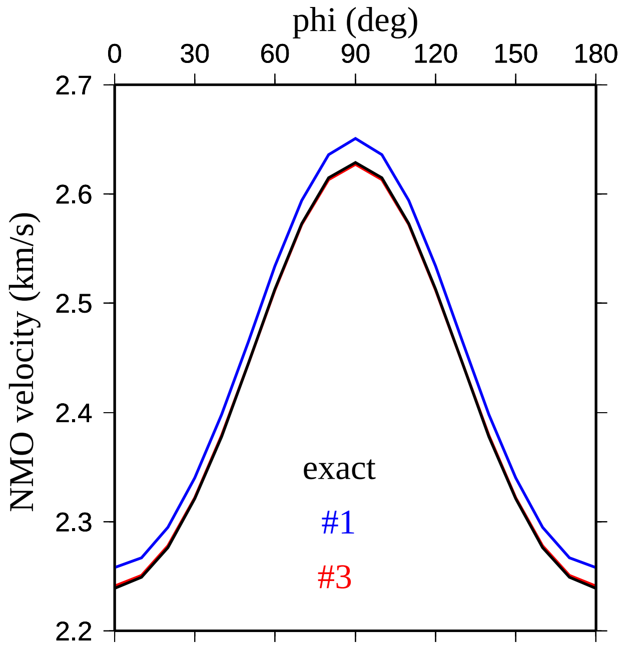
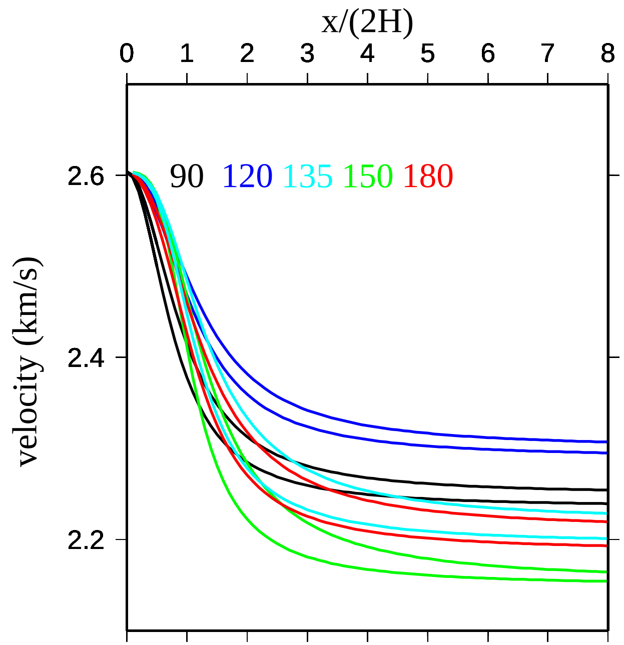


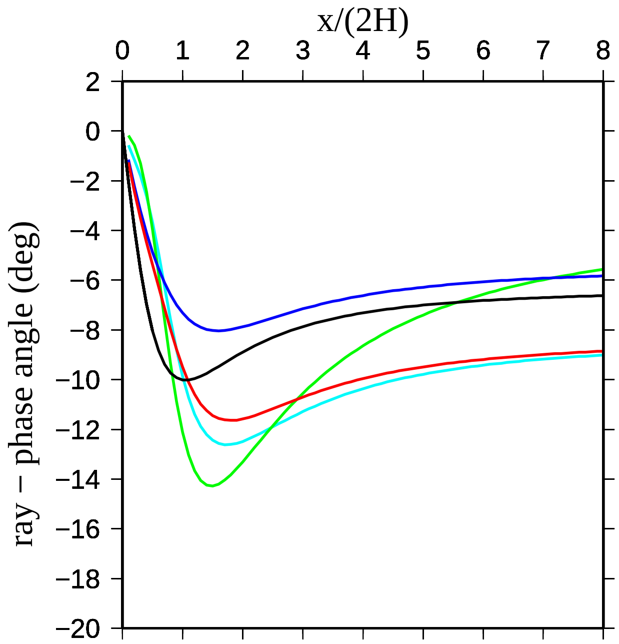
Tests of the formulae

MONO model, anisotropy $\sim$ 15%

α_0=2.604 km/s, β_0=1.566 km/s, ϵ_x=–0.135, ϵ_y=–0.124, δ_x=–0.128,

δ_y=–0.057, δ_z=–0.241, ϵ_{16}=0.057, ϵ_{26}=–0.043, χ_z=–0.071





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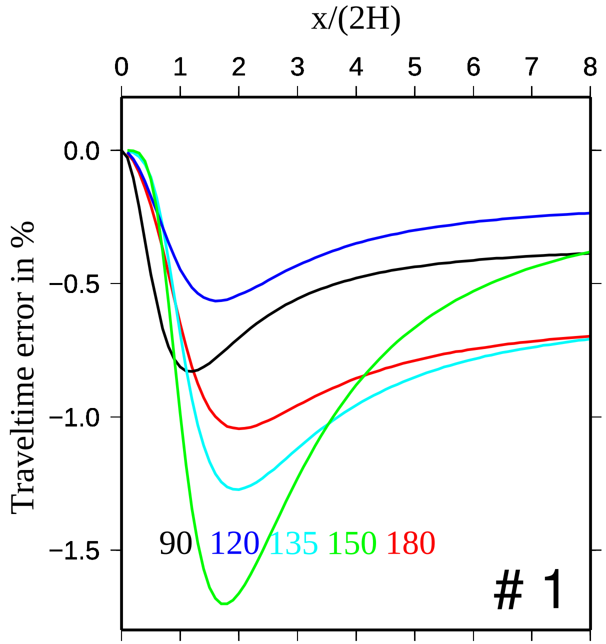

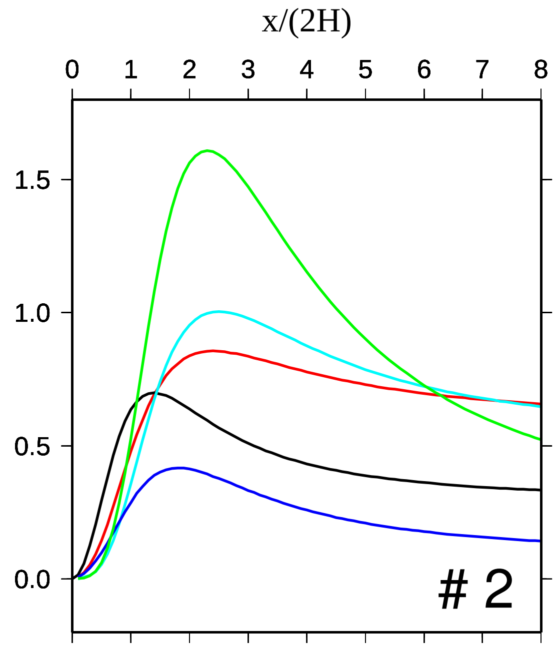

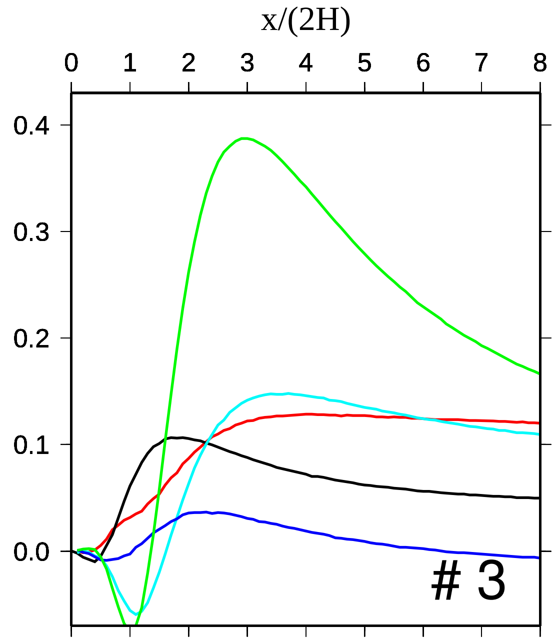

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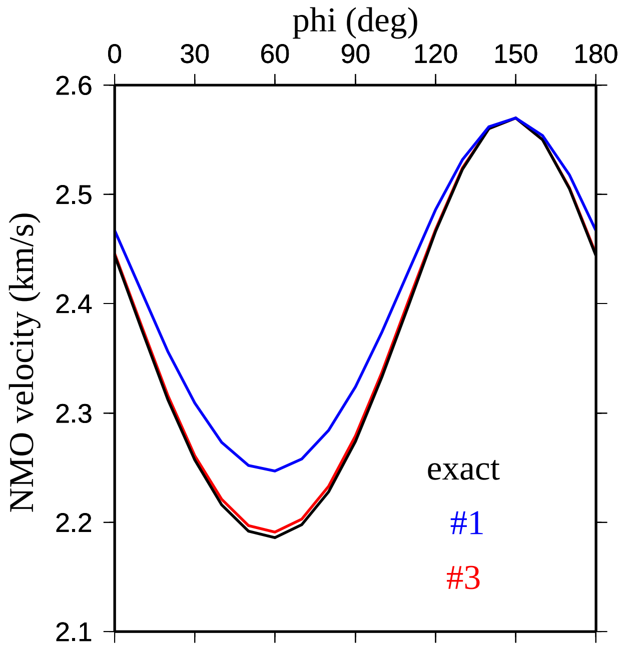


Conclusions

- based on WA approximation
- relatively simple formulae; no non-physical assumptions
- applicable to VTI, HTI, orthorhombic or even monoclinic media

  for ther reflector $\|$ with a plane of symmetry
- inaccuracies for large deviations of $\mathbf{n}$ and $\mathbf{N}$
- for small and large offsets accurate
- second-order formulae very accurate; $<$ 1% for $\sim$ 25% anisotropy
- reduced number of WA parameters (WA/exact):

  monoclinic - 8/12, orthorhombic - 5/9, TI - 2/4
- byproduct: simple expressions for NMO velocities

Generalizations

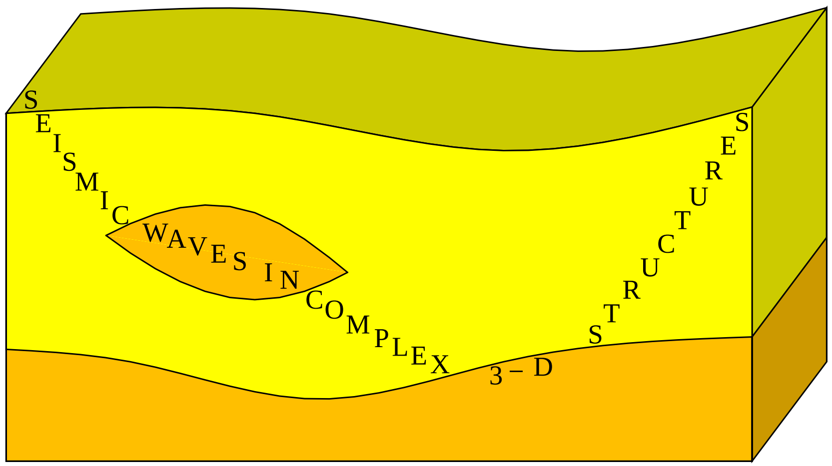
Straightforward

- dip-constrained media with an inclined reflector
  parallel to a symmetry plane
- unconverted coupled S wave along a common ray

Possible

- converted waves

Acknowledgements



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