Superposition of Gaussian packets in heterogeneous anisotropic media

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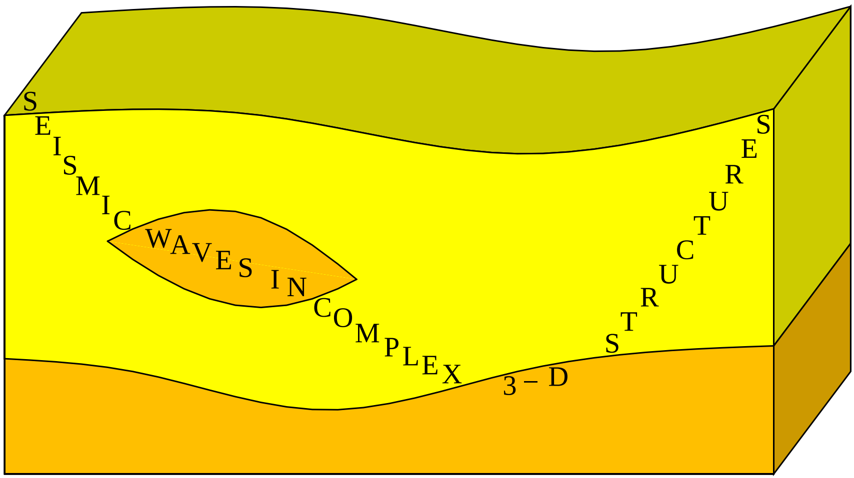

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Ray coordinates

We consider Cartesian or curvilinear coordinates x^i in 3-D space. We assume an orthonomic system $x^i = \tilde{x}^i(\gamma^a)$ of rays parametrized by ray coordinates γ^a, where γ^1 and γ^2 are the ray parameters, and γ^3 is the parameter along rays determined by the form of the Hamiltonian function.

The integral superposition of Gaussian packets will be performed with respect to ray coordinates γ^a.

We also define the matrix

$$X_a^k = \frac{\partial x^k}{\partial \gamma^a}$$

of geometrical spreading corresponding to the orthonomic system of rays.

Gaussian packets

The Gaussian packet centred at point $\tilde{x}^i(\gamma^a)$ reads

$$g_{i[j]}(x^m,\gamma^a,\omega) = A^{\mathrm{GP}}_{i[j]}(\gamma^b,\omega)\, g(x^m,\gamma^a,\omega) \quad ,$$

where $A^{\mathrm{GP}}_{i[j]}(\gamma^b,\omega)$ is the complex-valued vectorial or tensorial amplitude of the Gaussian packet.

The envelope of the Gaussian packet is

$$g(x^m,\gamma^a,\omega) = \exp\left\{\mathrm{i}\omega\left[\tau + (x^k - \tilde{x}^k)\, p_k + \tfrac{1}{2}(x^k - \tilde{x}^k)\, f_{kl}\, (x^l - \tilde{x}^l)\right]\right\} \quad ,$$

where ω is the circular frequency, $\tilde{x}^k = \tilde{x}^k(\gamma^a)$ is the central point of the Gaussian packet, $\tau = \tau(\gamma^a)$ is the travel time corresponding to the orthonomic system of rays, $p_k = p_k(\gamma^a)$ is the slowness vector corresponding to the orthonomic system of rays, and $f_{kl} = f_{kl}(\gamma^a)$ is the complex-valued matrix with positive imaginary part describing the shape of the Gaussian packet.

Integral superposition of Gaussian packets

The time-harmonic superposition of Gaussian packets at frequency ω reads

$$u_{i[j]}(x^m,\omega) = \iiint \mathrm{d}\gamma^1 \mathrm{d}\gamma^2 \mathrm{d}\gamma^3 \, A^{\mathrm{GP}}_{i[j]}(\gamma^a,\omega)\, g(x^m,\gamma^a,\omega) \quad ,$$

where

$$A^{\mathrm{GP}}_{i[j]}(\gamma^a,\omega) \;=\; \left(\tfrac{\omega}{2\pi}\right)^{\frac{3}{2}} A_{i[j]}(\gamma^a,\omega)\, |\det(X^n_c)|\, \sqrt{\det}\!\left[\mathrm{i}\left(N_{ab}-f_{ab}\right)\right] \quad .$$

Here ω is the circular frequency, $A_{i[j]}(\gamma^a,\omega)$ is the complex-valued vectorial or tensorial ray-theory amplitude, $X^n_c = X^n_c(\gamma^a)$ is the matrix of geometrical spreading, $N_{kl} = N_{kl}(\gamma^a)$ is the matrix

$$N_{kl} = \frac{\partial^2 \tau}{\partial x^k \partial x^l}$$

of the second-order partial derivatives of travel time, and $f_{kl} = f_{kl}(\gamma^a)$ is the complex-valued matrix with positive imaginary part describing the shape of the Gaussian packet.

Function $\sqrt{\det}\left(M_{ab}\right)$ is the product of the square roots of the eigenvalues of matrix M_{ab}. The individual square roots are taken with positive real parts.

Conclusions

The presented integral superposition of Gaussian packets may correspond to the anisotropic ray theory, to the frequency-dependent coupling ray theory for S waves, to the prevailing-frequency approximation of the coupling ray theory for S waves, or to the isotropic ray theory.

Reference

Klimeš, L. (2014): Superposition of Gaussian packets in heterogeneous anisotropic media. *Seismic Waves in Complex 3-D Structures*, **24**, 127–130, online at "http://sw3d.cz".

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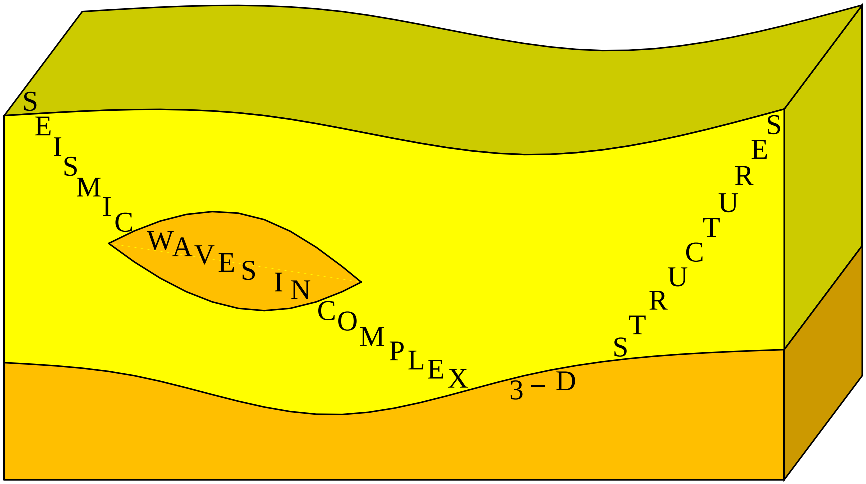

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