Nonlinear hypocentre determination

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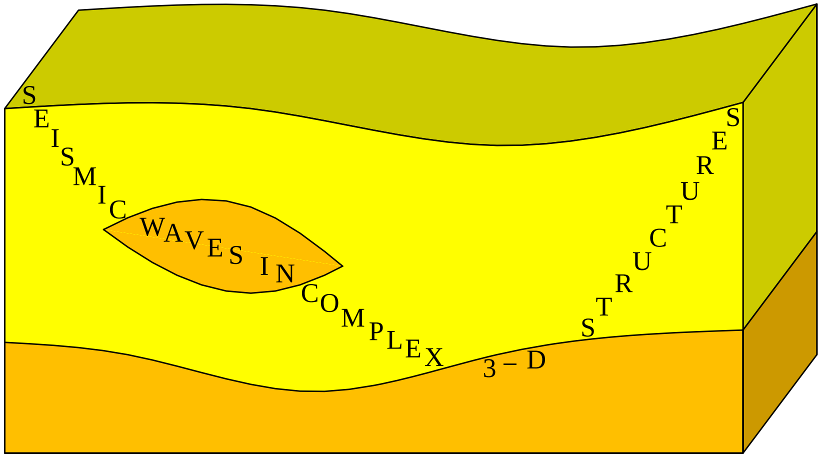
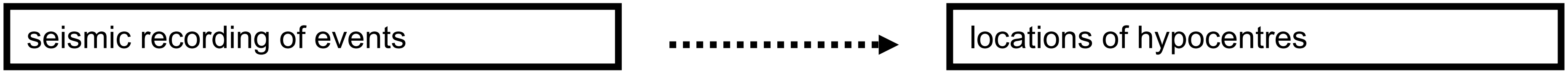


http://sw3d.cz

SW3D meeting, Praha, 15.6.2015

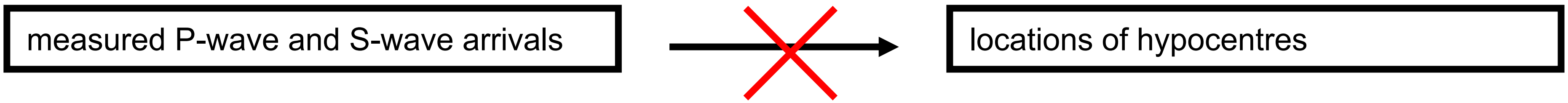
Hypocentre determination



Hypocentre determination

measured P-wave and S-wave arrivals → locations of hypocentres

Hypocentre determination



Hypocentre determination

measured P-wave and S-wave arrivals
velocity model

→

locations of hypocentres

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locations of hypocentres
uncertainty of the hypocentres

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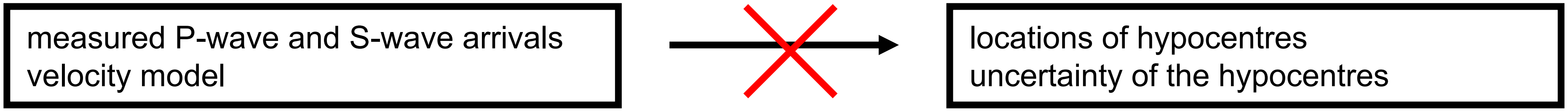

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uncertainty of measured arrival times

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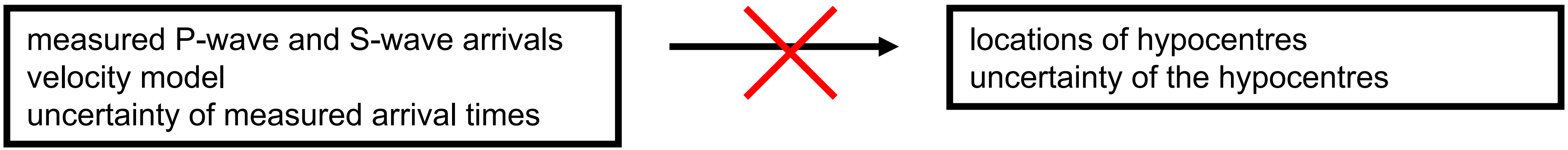

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Tarantola & Valette (1982): nonlinear hypocentre determination consisting in direct evaluation of the nonnormalized 3-D marginal a posteriori density function which describes the relative probability of the seismic hypocentre, discretized at the gridpoints of a sufficiently dense 3-D spatial grid of points

We describe the uncertainty of the velocity model by model covariance function, which is projected onto the uncertainty of the hypocentral position through the geometrical covariances of theoretical travel times calculated in the velocity model (Klimeš 2002, 2008).

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→ geometrical travel-time covariance matrix

nonnormalized 3-D marginal a posteriori density function

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minimum arrival-time residuals
(arrival-time misfit)

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The minimum arrival-time residuals carry information pertinent to the accuracy of the velocity model (Klimeš 1996)

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update model covariance function

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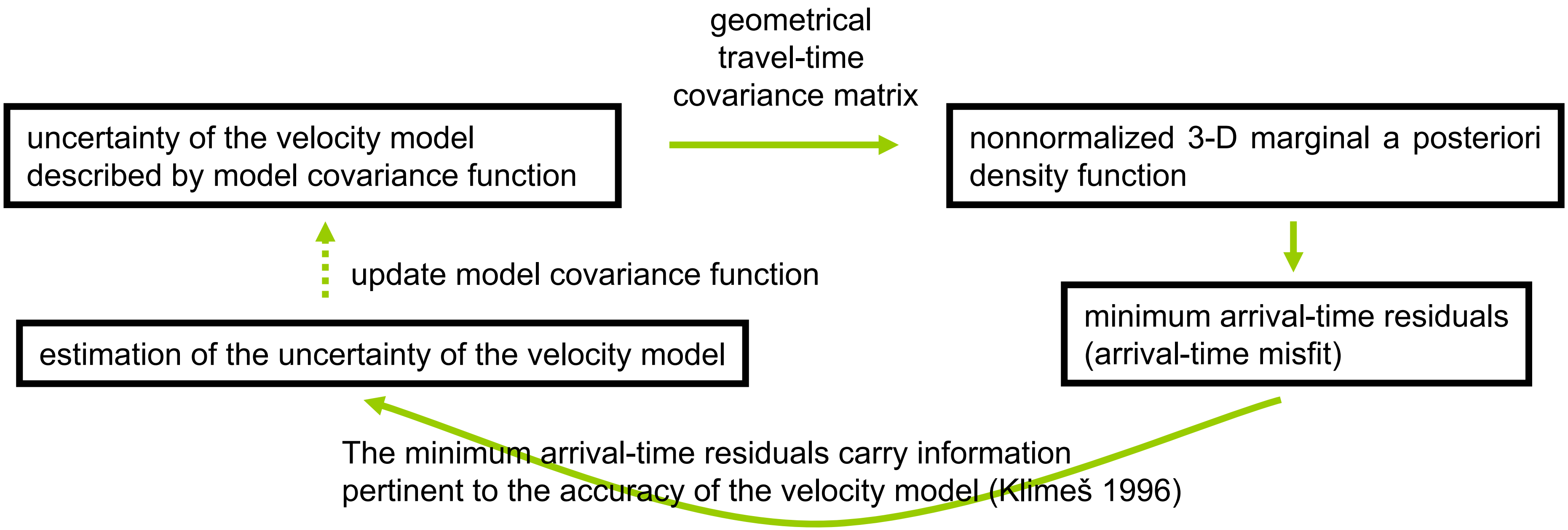
Notes on numerical implementation of the algorithm



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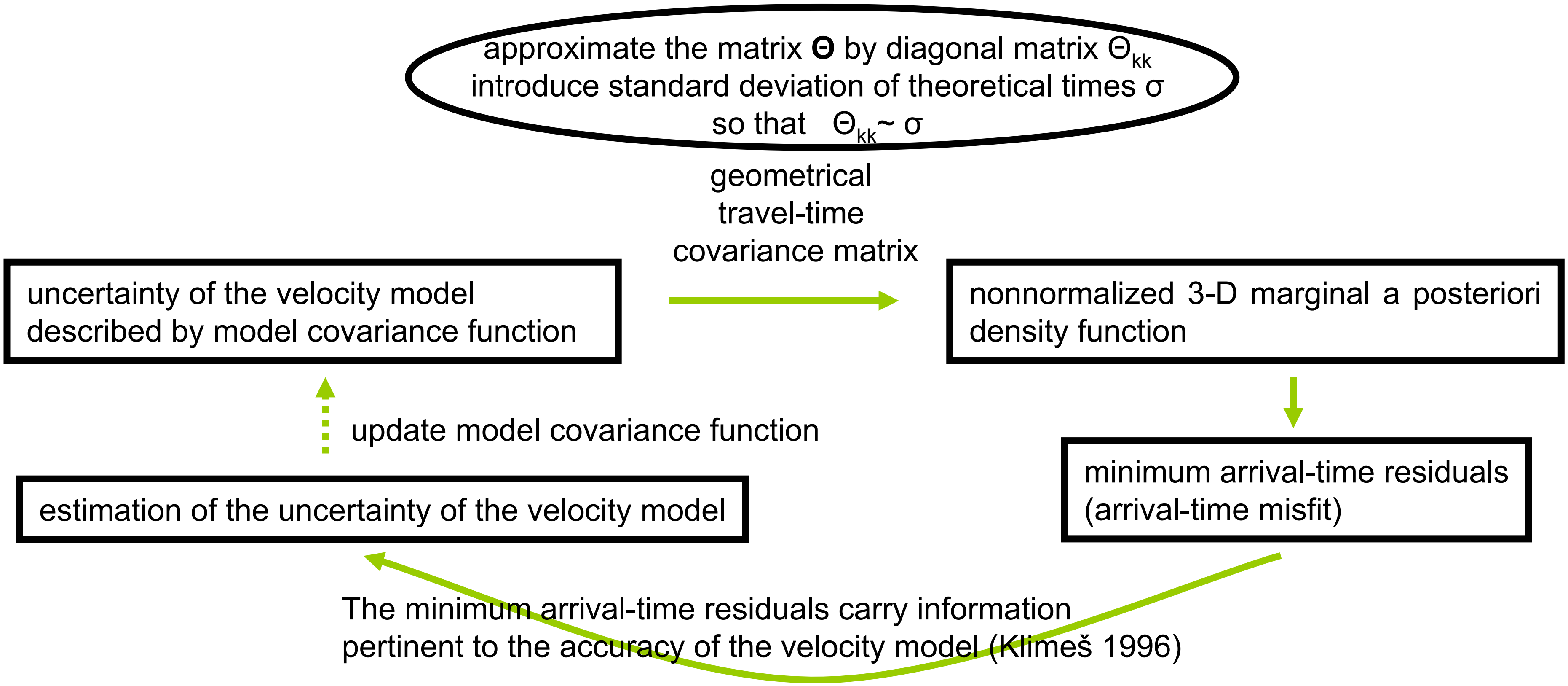

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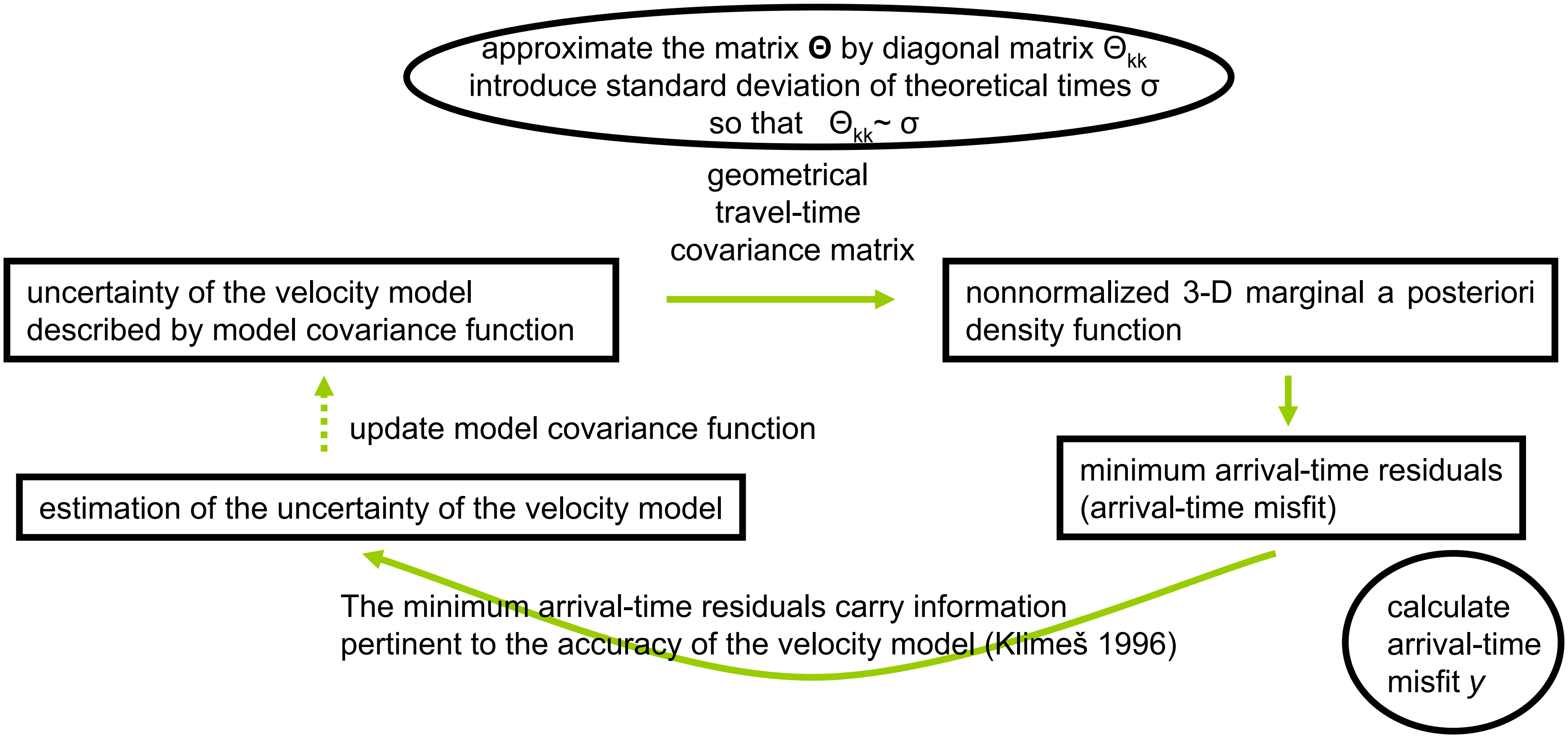

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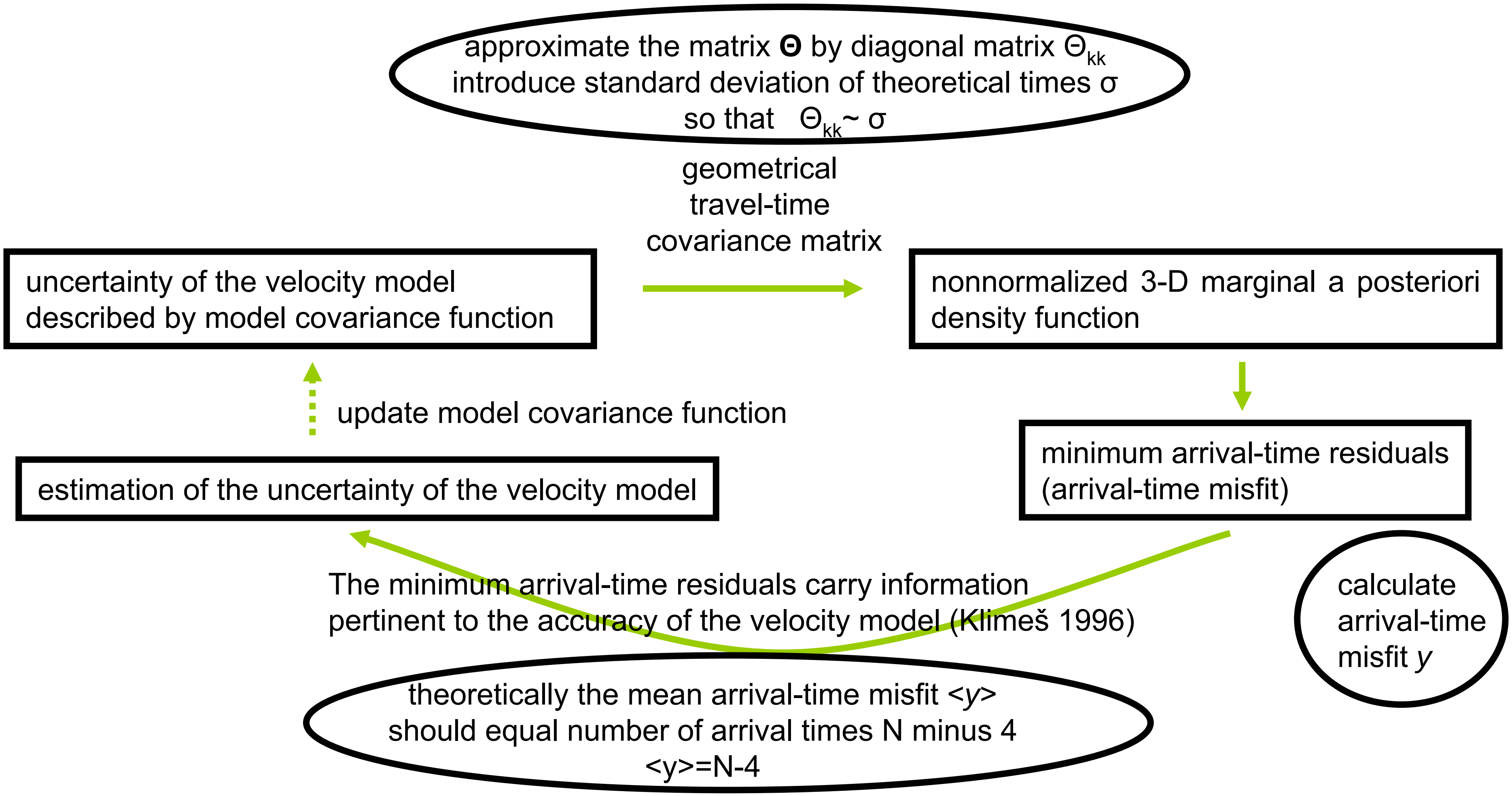

Check of the model covariance function

if we have sufficiently numerous set of events with sufficiently large numbers of arrivals:

step1: - estimate uncertainty of the velocity model by choosing a value of σ
- calculate the a posteriori density function
- calculate arrival-time misfits *y* for all the events and calculate average $\underline{y}$
- compare with average value of $\underline{N-4}$, update σ
- continue until we find σ for which $\underline{y} \sim \underline{N-4}$

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step 2: - perform step 1 using P-wave arrivals to find σ_P
- perform step 1 using S-wave arrivals to find σ_S
- check whether the location with both P and S arrivals provides reasonable $\underline{y}_{S+P}$
so that $\underline{y_{S+P}} \sim \underline{N_S+N_P-4}$

Numerical example

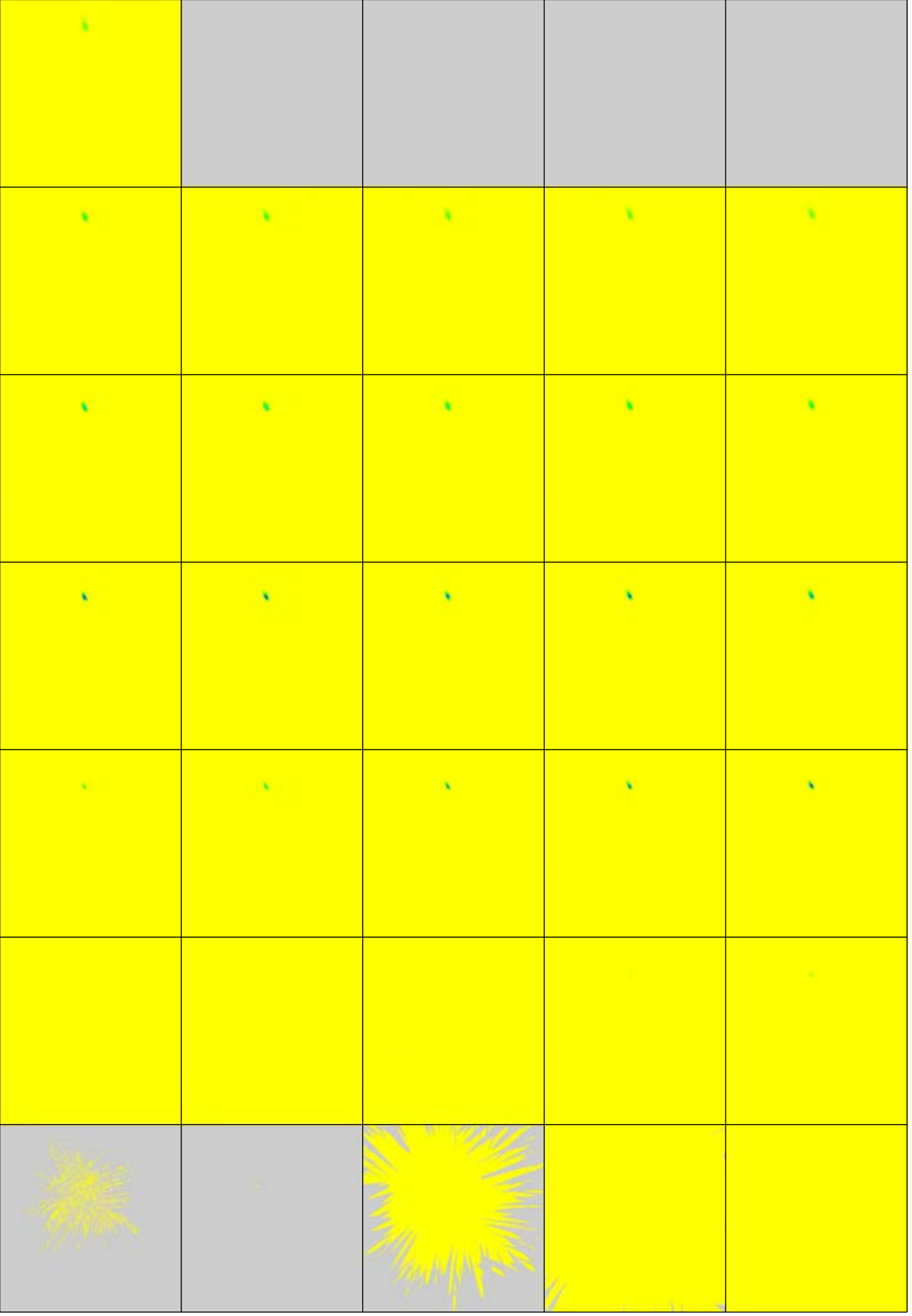
- microseismic monitoring of natural events, 33 events registered
- 15 stations, but each event was registered only on some of the stations (3 to 9 stations)
- simple 1-D layered velocity model consisting of 4 layers
- no information about the velocity model uncertainty

- we assumed power-law model covariance functions, used the value of Hurst exponent from Western Bohemia
- estimated separately P-wave and S-wave model inaccuracy, and then calculated the a posteriori density function using all the arrivals (as described on previous slide)

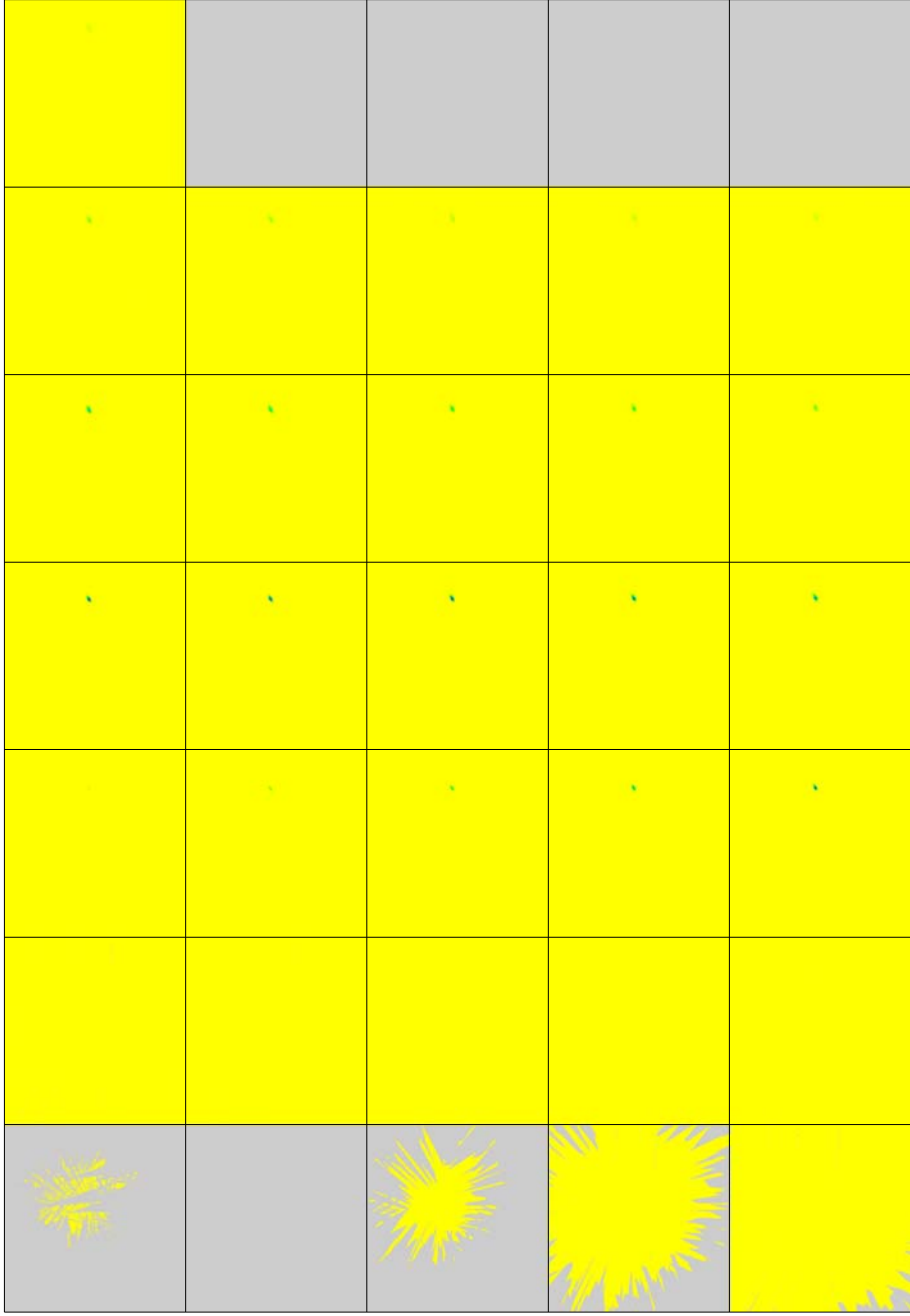
Event	N_P	N_S	y_P	y_S	y_{P+S}
01	8	7	2.693	1.471	6.656
02	9	7	4.631	2.776	11.189
03	8	7	3.172	1.823	7.673
04	8	7	3.527	2.082	8.486
05	8	6	2.452	1.590	5.636
06	8	7	2.955	1.662	7.038
07	8	7	3.767	2.167	9.060
08	8	7	3.186	1.809	7.601
09	9	7	4.360	1.580	9.181
10	5	5			
11	8	7	3.000	2.969	8.833
12	4	4			
13	8	7	7.124	3.752	21.344
14	9	8	6.625	4.533	25.573
15	9	9	5.674	8.439	16.794
16	9	9	5.062	6.335	13.235
17	9	7	10.407	1.342	17.272
18	9	9	4.211	5.522	12.666
19	9	8	3.830	3.445	8.691
20	9	8	2.756	2.846	6.656
21	9	7	3.893	1.741	6.970
22	9	8	3.154	2.140	6.200
23	9	9	3.869	3.084	8.964
24	9	8	6.564	6.355	15.421
25	9	9	10.732	12.631	25.050
26	8	8	5.837	8.220	18.836
27	9	9	5.518	8.105	14.547
28	9	8	3.916	1.932	7.890
29	5	3			
30	8	8	3.675	4.818	12.946
31	8	7	3.591	3.988	11.129
32	8	6	5.314	1.011	7.535
33	7	7	1.869	3.084	7.751
Average	8.5	7.6	4.579	3.775	11.561

Numerical example - results

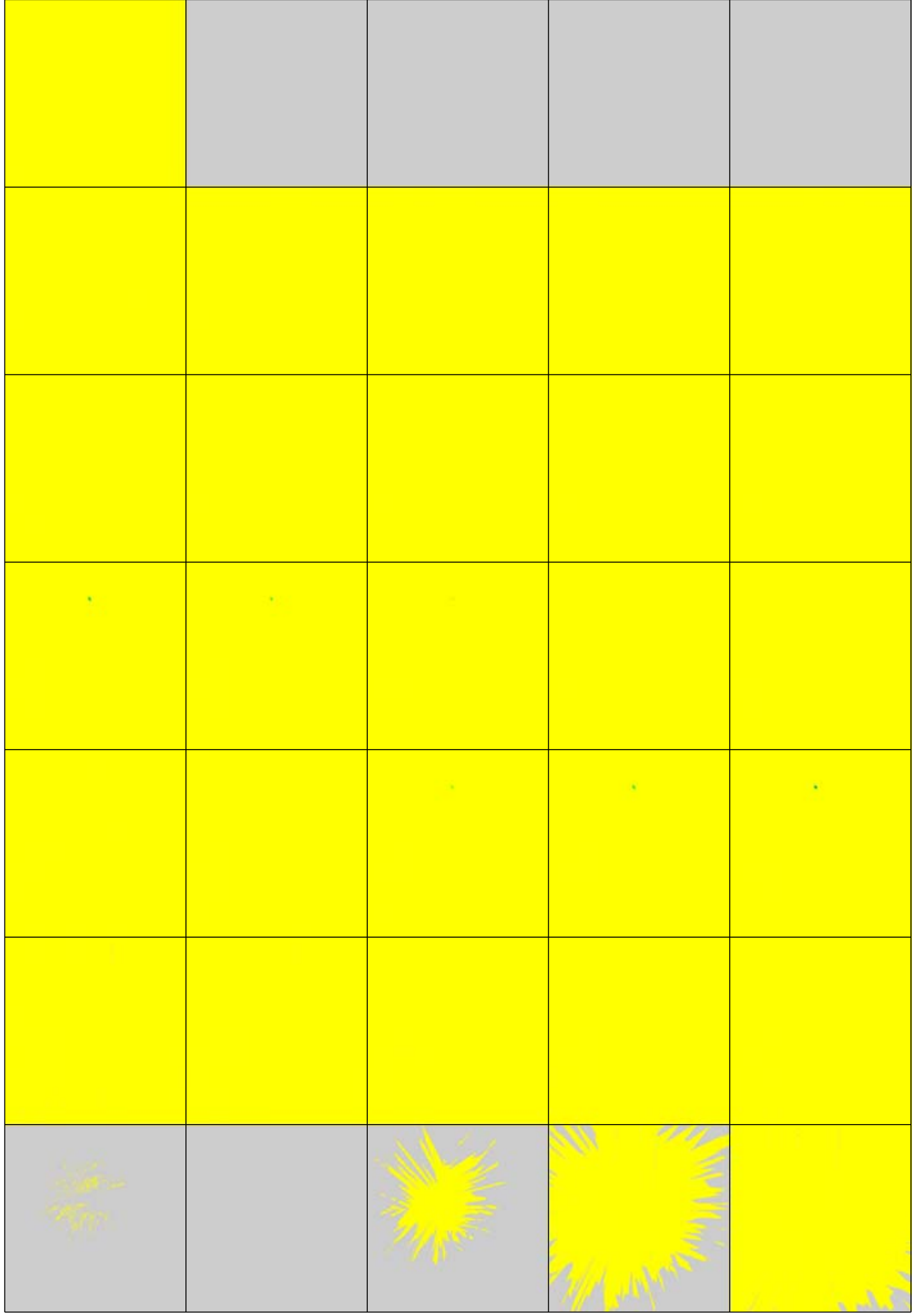
- although we used the incorrect P-wave and S-wave geometrical travel time covariance matrices restricted just to diagonal elements, the behavior of the nonlinear hypocentre determination is reasonable – the average arrival-time misfit determined from both the P-wave and S-wave arrivals behaves in the same way it should behave for the correct geometrical travel-time covariance matrices



9 P-wave arrivals



9 S-wave arrivals



9 P-wave arrivals + 9 S-wave arrivals

Numerical example - results

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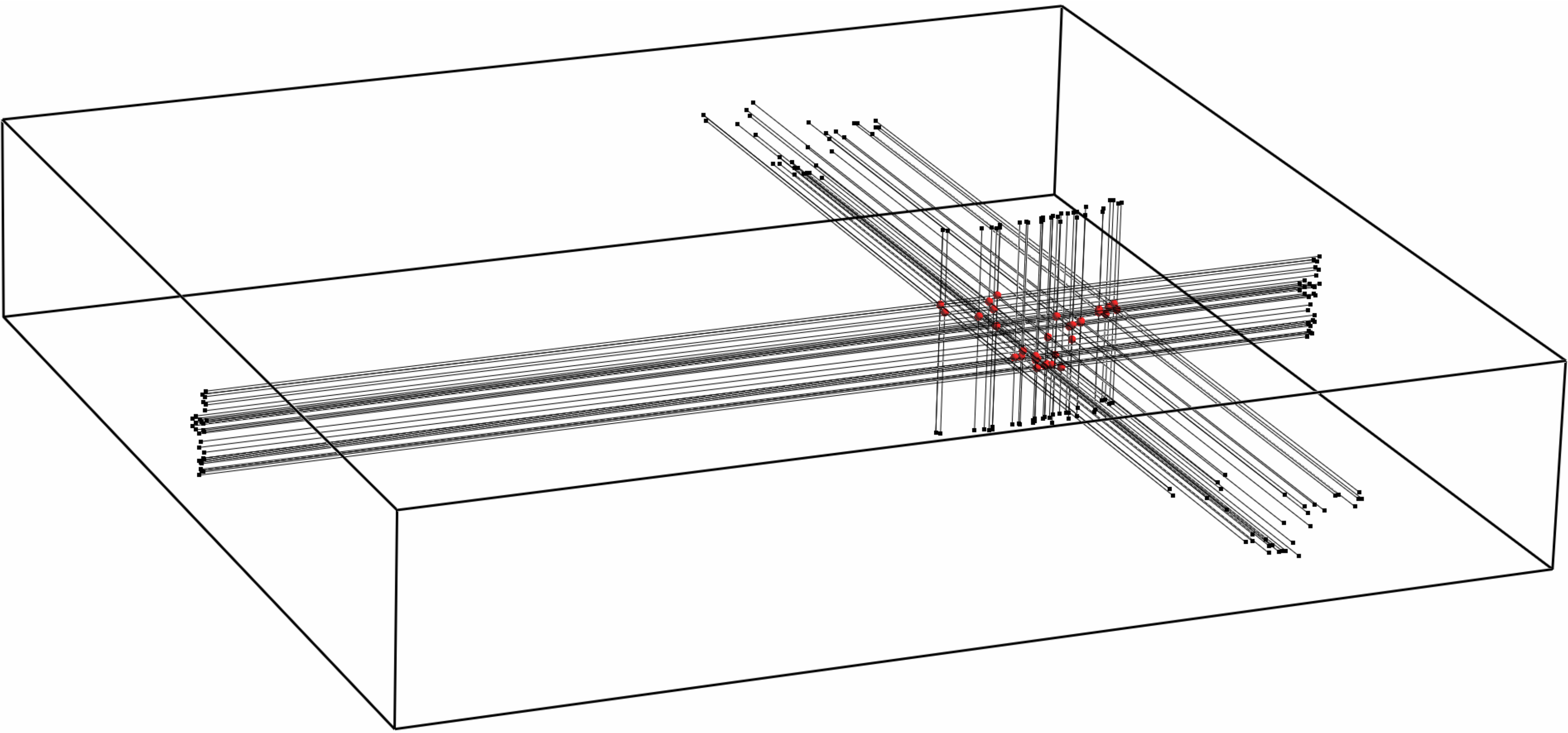
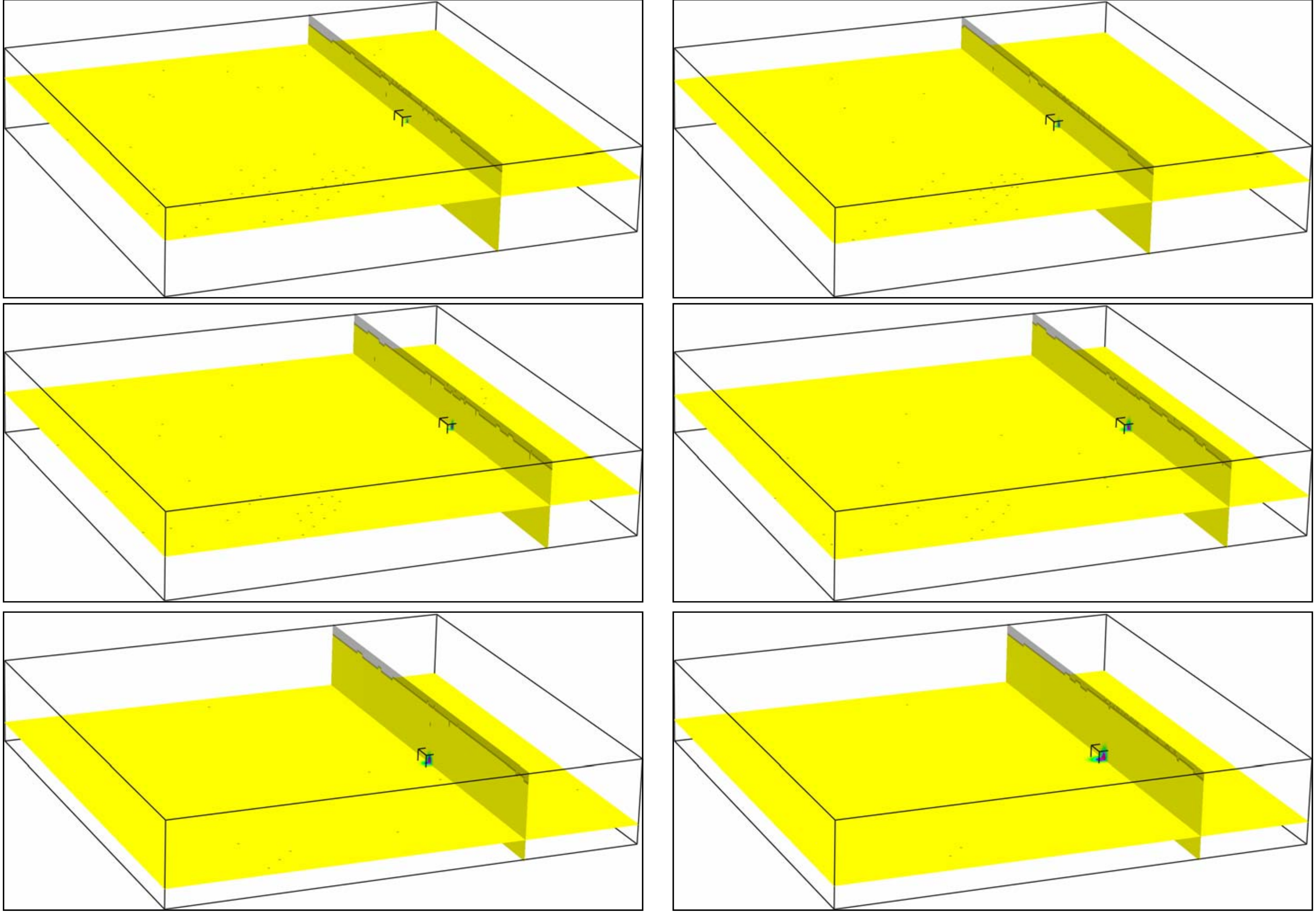
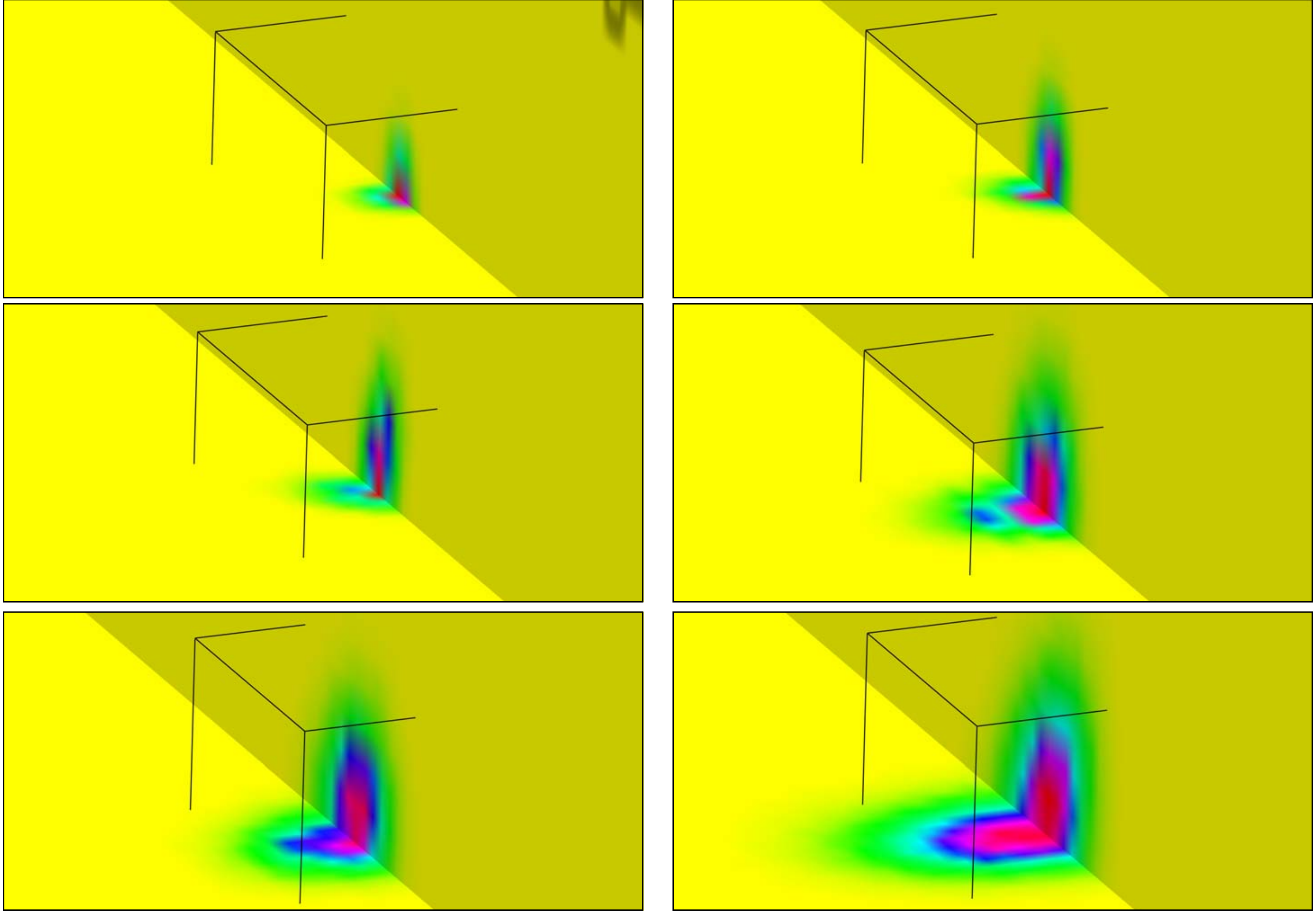
- when we use just P-wave or just S-wave arrival times, the depth of locations is uncertain for approximately 75% of events (including events with 9 arrivals)





No. of arrivals:	9+9	8+6
	9+9	8+7
	9+7	4+4



No. of arrivals: 9+9 8+6
9+9 8+7
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Numerical example - results

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- when we use just P-wave or just S-wave arrival times, the depth of locations is uncertain for approximately 75% of events (including events with 9 arrivals)

- when we use both P-wave and S-wave arrivals, we observe that the uncertainty of the hypocentral location increases with increasing depth and decreasing number of arrivals

Conclusions

We considered the robust nonlinear approach to hypocentre determination proposed by Tarantola & Valette (1982), consisting in direct evaluation of the nonnormalized 3–D marginal a posteriori density function which describes the relative probability of the seismic hypocentre, and proposed the corresponding numerical algorithm.

The nonnormalized 3–D marginal a posteriori density function allows for testing the model covariance function describing the uncertainty of the velocity model.

If we were able to use the whole geometrical travel-time covariance matrix, we could estimate the uncertainty of the model.

Acknowledgments

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