Uncertainty analysis for inversion of qP-wave reflection amplitudes in weakly anisotropic elastic media

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Abstract

An approximate equation for $qPqP$ reflection coefficients is used in order to recover contrasts in weak anisotropy (WA) parameters of media surrounding an interface. The approximation fits, within the subcritical region, exact calculations of reflections at a weak-contrast interface separating two homogenous, weakly anisotropic elastic media of arbitrary symmetry. Linear dependence of reflection amplitudes on contrasts in WA parameters is convenient property of this approximation. Hence, the singular value decomposition (SVD) technique is applied to recovering the contrasts, with numerical reflection data from an interface separating an upper isotropic halfspace from a bottom transversely isotropic halfspace with horizontal axis of symmetry (HTI). Reflection amplitudes from a range of incidence angles commonly found in practice and from at least five measurement profiles are needed for recovering the contrasts in WA parameters. Such a conclusion is drawn by performing an uncertainty analysis for the inversion.

Introduction

Amplitude versus offset (AVO) analysis is a powerful tool for lithology characterization and fluid content prediction (Smith and Gidlow, 1987). Due to the fact that seismic anisotropy represents an inherent phenomenon in sedimentary sequences (Winterstein and De, 2001), there is a need for performing AVO analysis along specific directions in which amplitude anomalies may be higher. Bearing this in mind, such a technology is referred to as amplitude versus direction (AVD) analysis (MacBeth and Li, 1999).

The use of approximations for reflection coefficients is the fundamental part in AVO/AVD studies. In isotropic AVO analysis, various modifications of the approximate formula for PP reflections derived in Aki and Richards (1980) is widespreadly used. Ignoring anisotropic effects in sedimentary rocks may lead to erroneous predictions (Wright, 1987). It is the case, for instance, if a vertically fractured reservoir rock is under investigation (Rüger and Tsvankin, 1997). An isotropic AVO analysis will work satisfactorily along the plane of fractures (i.e., the plane of isotropy). On the other hand, anisotropic effects due to the vertical fractures must be considered along the strike direction. This aspect explains the need for incorporating effects of azimuthal anisotropy in AVO studies.

Several approximations for $qPqP$ reflections have been proposed so far. The common assumption among them is the weak anisotropy behavior, which is inherent to most sedimentary formations (Thomsen, 1986). The assumptions of weak contrast across interfaces and weak anisotropy lead to simplification of exact expressions for reflections in anisotropic media.

Basically, the linearized coefficients are written as a sum of the approximation for reflections in the background isotropic media and a correction to weak anisotropy. Approximate equations are mostly restricted to transversely isotropic media with vertical axis of symmetry (Banik, 1987; Thomsen, 1993) or with horizontal axis of symmetry (Rüger, 1997). Approximations in orthorhombic media were studied by Rüger (1998), but only within symmetry planes. A common point in these approximations is the use of both vertical P and S wave velocities for the derivation of the equation for the coefficients. In this contribution, an approximation for $qPqP$ reflections in weakly anisotropic media studied by Pšenčík and Martins (2001) is used for inversion purposes. The approximation holds for arbitrary anisotropic symmetries and for arbitrary background isotropic velocities. For completeness of inversion, uncertainty analysis is implemented according to the same guidelines followed by Cai and McMechan (1999).

The linearized anisotropic reflections

The approximation for the $qPqP$ reflection coefficients used in this numerical study has the following form

$$R_{PP}(\varphi,\theta) = R_{PP}^{iso}(\theta) + \Delta R_{PP}(\varphi,\theta), \qquad (1)$$

where $R_{PP}^{iso}(\theta)$ is the well-known approximate formula for reflections at a weak-contrast interface separating two isotropic media (Aki and Richards, 1980) and

$$\begin{aligned} \Delta R_{PP}(\varphi,\theta) &= A\cos^2\theta + B_1\cos^2\varphi\sin^2\theta \\ &+ B_2\sin^2\varphi\sin^2\theta + B_3\sin\varphi\cos\varphi\sin^2\theta \\ &+ C_1\cos^4\varphi\sin^2\theta\tan^2\theta + C_2\sin^4\varphi\sin^2\theta\tan^2\theta \\ &+ C_3\cos^2\varphi\sin^2\varphi\sin^2\theta\tan^2\theta \\ &+ C_4\cos^3\varphi\sin\varphi\sin^2\theta\tan^2\theta \\ &+ C_5\sin^3\varphi\cos\varphi\sin^2\theta\tan^2\theta, \end{aligned} \qquad (2)$$

represents the first-order perturbation from isotropy to weak anisotropy. To simplify notation, the subscript PP is used henceforth in place of $qPqP$.

The approximation in Eqs. (1) and (2) is linearized version of Eq. (39) of Vavryčuk and Pšenčík (1998). Its dependence on the incidence angle θ and on the direction of the measurement profile (azimuth) φ is clearly observed. Moreover, Eq. (2) explicitly shows *linear* dependence of reflections on constrasts in WA parameters across interface,

$$A = \frac{1}{2}\,\Delta\epsilon_z, \qquad B_1 = \frac{1}{2}\left[\Delta\delta_x - 8\left(\frac{\bar{\beta}}{\bar{\alpha}}\right)^2 \Delta\gamma_x\right],$$

$$B_2 = \frac{1}{2}\left[\Delta\delta_y - 8\left(\frac{\bar{\beta}}{\bar{\alpha}}\right)^2 \Delta\gamma_y\right], \tag{3}$$

$$B_3 = \left[\Delta\chi_z - 4\left(\frac{\bar{\beta}}{\bar{\alpha}}\right)^2 \Delta\epsilon_{45}\right], \quad C_1 = \frac{1}{2}\,\Delta\epsilon_x,$$

$$C_2 = \frac{1}{2}\,\Delta\epsilon_y, \quad C_3 = \frac{1}{2}\,\Delta\delta_z, \quad C_4 = \Delta\epsilon_{16}, \quad C_5 = \Delta\epsilon_{26}.$$

The presence of R_{PP}^{iso} in Eq. (1) and the relationships in (3) also indicate dependence of reflections on upper

$$\alpha_1 = 2.260\ km/s$$
$$\beta_1 = 1.430\ km/s$$
$$\rho_1 = 2.700\ g/cm^3$$

$$\begin{pmatrix} 5.00 & 1.82 & 1.82 & 0.00 & 0.00 & 0.00 \\ & 6.25 & 1.75 & 0.00 & 0.00 & 0.00 \\ & & 6.25 & 0.00 & 0.00 & 0.00 \\ & & & 2.25 & 0.00 & 0.00 \\ & & & & 1.87 & 0.00 \\ & & & & & 1.87 \end{pmatrix}$$

$$\alpha_2 = 2.500\ km/s \qquad \beta_2 = 1.500\ km/s \qquad \rho_2 = 2.700\ g/cm^3$$

$$\Delta\alpha/\bar{\alpha} = 0.101 \qquad \Delta\beta/\bar{\beta} = 0.048 \qquad \Delta\rho/\bar{\rho} = 0.000$$

Figure 1: The IH model corresponding of an isotropic medium over a vertically fractured (HTI) material. Upper and lower elastic parameters indicate weak contrast across the interface. The [weak anisotropy] HTI medium is defined by the matrix of density-normalized elastic parameters in km^2/s^2.

and lower background isotropic media, which are represented by α and β (P and S wave phase velocities, respectively). Contrasts across the interface are denoted by Δ (for example, density contrast: $\Delta\rho = \rho_2 - \rho_1$), and averages by a bar over the corresponding quantity. Subscripts 1 and 2 index upper and lower medium, respectively. We can see from Eq. (3) that the reflection coefficient (1) also depends on nine P-wave WA parameters (Pšenčík and Gajewski, 1998)

$$\delta_x = \frac{A_{13} + 2A_{55} - \alpha^2}{\alpha^2}, \quad \delta_y = \frac{A_{23} + 2A_{44} - \alpha^2}{\alpha^2},$$

$$\delta_z = \frac{A_{12} + 2A_{66} - \alpha^2}{\alpha^2}, \quad \epsilon_{16} = \frac{A_{16}}{\alpha^2},$$

$$\epsilon_{26} = \frac{A_{26}}{\alpha^2}, \quad \chi_z = \frac{A_{36} + 2\,A_{45}}{\alpha^2},$$

$$\epsilon_x = \frac{A_{11} - \alpha^2}{2\alpha^2}, \quad \epsilon_y = \frac{A_{22} - \alpha^2}{2\alpha^2}, \quad \epsilon_z = \frac{A_{33} - \alpha^2}{2\alpha^2},$$

and on three S-wave WA parameters

$$\gamma_x = \frac{A_{55} - \beta^2}{2\beta^2}, \quad \gamma_y = \frac{A_{44} - \beta^2}{2\beta^2}, \quad \Delta\epsilon_{45} = \frac{A_{45}}{\beta^2}.$$

WA parameters are generalizations of anisotropy parameters introduced by Thomsen (1986). As "observed" data for the inversion for contrasts in WA parameters (see below), reflection coefficients were calculated for the model proposed by Rüger (1997), see Figure 1. Exact and approximate reflections determined along three azimuths are shown in Figure 2. Although anisotropy is weak for this model, reflections vary significantly along azimuths.

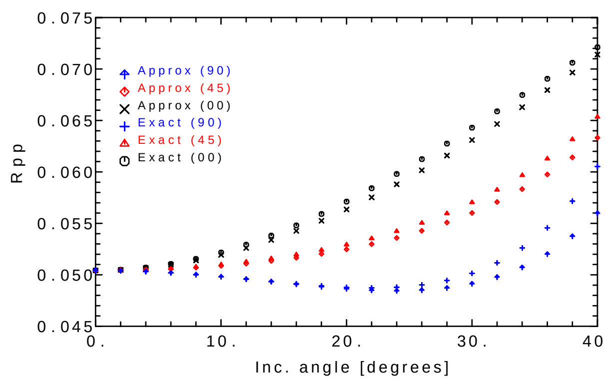

Figure 2: Exact and approximate reflection coefficients for the IH model in Figure 1. Approximate coefficients were calculated by using Eq. (1).

The inverse problem formulation

For a given data set of m observations, Eq. (2) represents a linear system for n sought parameters. The system can be written in matrix form as

$$\delta\mathbf{r} = \mathbf{L}\ \delta\mathbf{w}, \tag{4}$$

where $\delta\mathbf{r}$ is a vector of differences of observed reflection coefficients $R_{PP}(\varphi, \theta)$ and coefficients $R_{PP}^{iso}(\theta)$. Each observation contributes to one equation in (4).

The elements of matrix $\mathbf{L}$ in Eq. (4) can be easily obtained from Eq. (2), i.e.

$$l_{i,1} = \cos^2\theta, \quad l_{i,2} = \cos^2\varphi\,\sin^2\theta, \quad l_{i,3} = \sin^2\varphi\,\sin^2\theta,$$

$$
\begin{aligned}
l_{i,4} &= \sin\varphi \cos\varphi \sin^2\theta, \quad l_{i,5} = \cos^4\varphi \sin^2\theta \tan^2\theta, \\
l_{i,6} &= \sin^4\varphi \sin^2\theta \tan^2\theta, \\
l_{i,7} &= \cos^2\varphi \sin^2\varphi \sin^2\theta \tan^2\theta, \qquad (5) \\
l_{i,8} &= \cos^3\varphi \sin\varphi \sin^2\theta \tan^2\theta, \\
l_{i,9} &= \sin^3\varphi \cos\varphi \sin^2\theta \tan^2\theta.
\end{aligned}
$$

Hence, $n = 9$ is the number of unknowns [Eqs. (3)] to be inverted, which are the elements of vector

$$\delta\mathbf{w} = (\, A \;\; B_1 \;\; B_2 \;\; B_3 \;\; C_1 \;\; C_2 \;\; C_3 \;\; C_4 \;\; C_5 \,)^T. \qquad (6)$$

As a consequence of the configuration-dependence feature of the matrix $\mathbf{L}$, there should be a *minimum number of profiles* to be considered in the inversion so as to achieve a unique solution. Note that unknowns B_1, B_2 and B_3 are, in fact, combinations of contrast of two WA parameters.

Uncertainty analysis for the IH model

The model in Figure 1 was used for calculating exact reflection coefficients at each 2^o, in order to perform an iterative SVD inversion for contrasts in WA parameters according to the formulation described above. Velocities and the density of the isotropic medium in the upper halfspace were assumed to be known. Velocities

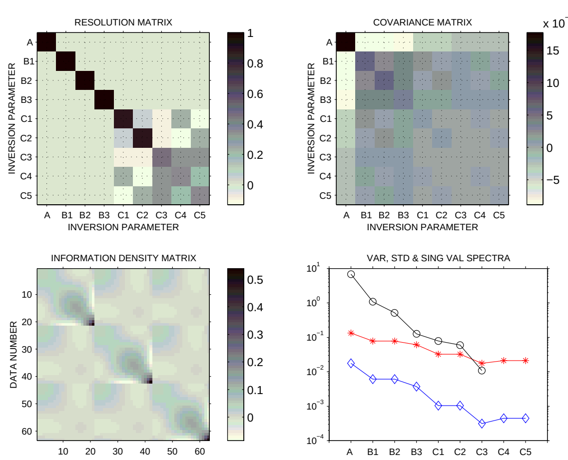

Figure 3: Uncertainty analysis for inversion of noise-free reflection coefficients with incidence information from 0^o to 40^o along three azimuths ($30^o/45^o/60^o$). Symbols $\diamond$, $*$ and $\circ$ denote variance, standard deviation and singular value, respectively.

of the background isotropic medium in the lower halfspace were iteratively updated during the process of inversion. The iterative process was stopped when the differences $\delta\mathbf{r}$ on the LHS of Eq. (4) were less than a predefined constant. The density of the lower halfspace was assumed to be known.

Two noise-free data sets were used for inversion. The first data set was obtained along three azimuths; the other data set gathered reflections from five azimuths. Both data sets contained incidence angles ranging from 0^o to 40^o. An inversion procedure is considered incomplete without performing an uncertainty analysis (Wiggins, 1972). Hence, we followed the same guidelines as in Cai and McMechan (1999) to analyse the resolution of the inversions.

The results of the uncertainty analysis for both inversions are shown in Figures 3 and 4. The resolution matrix is a measure of uniqueness of the solution; if the diagonal elements are units, the solution is said to be unique and the elements of the vector of sought parameters are said to be well resolved. The covariance matrix measures the uncertainty of the solution estimate. Its diagonal elements are variances, i.e., measures of uncertainty; if individual sought parameters have the same variance, they are said to be interdependent. The information density matrix is a measure of interdependence of the data used in the inversion. This matrix has zeros in all off-diagonal positions if each data observation is independent of all others. The more the information density matrix approaches a diagonal matrix, the more independent the observations.

By analysing Figures 3 and 4, the resolution matrices show that at least five measurement profiles

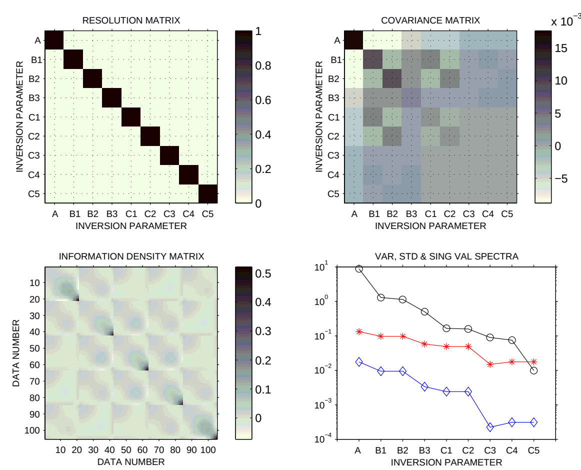

Figure 4: Uncertainty analysis for inversion of noise-free reflection coefficients with incidence information from 0^o to 40^o along five azimuths ($0^o/30^o/45^o/60^o/90^o$). Symbols $\diamond$, $*$ and $\circ$ denote variance, standard deviation and singular value, respectively.

are needed for a satisfactory estimation of contrasts in WA parameters. The data are highly interdependent, since the information density matrices exhibit off-diagonal trends. Interdependence of some unknowns (i.e., estimated contrasts) is observed in the covariance matrix plot. The curve of variances shows interdependence of contrasts B_1 and B_2, C_1 and C_2, and C_4 and C_5. The contrasts controlled by the *critical* term $\sin^2\theta \tan^2\theta$ in Eq. (1) are poorly resolved. This follows from the analysis of the singular values, see Figures 3 and 4. As can be seen in Figure 3, small singular values were neglected.

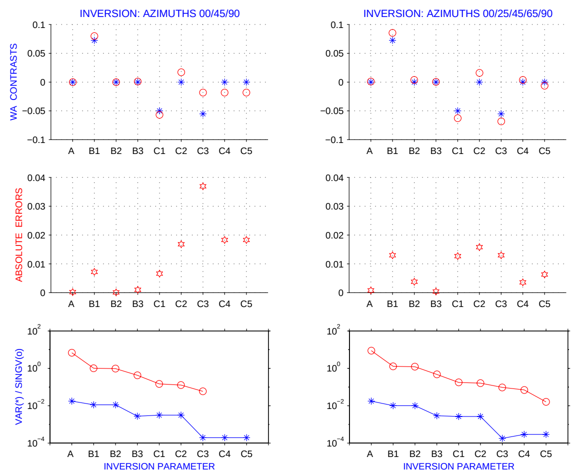

Figure 5: Outcomes from inversion of noise-corrupted reflection amplitudes using three $(0^o/45^o/90^o)$ and five $(0^o/25^o/45^o/65^o/90^o)$ profiles. Data with incidence information from 0^o to 40^o have been used for inversion. Upper plots show actual ($*$) and estimated ($\circ$) contrasts in WA parameters.

Gaussian noise ($\mu = 2.5 \times 10^{-3}$ and $\sigma = 7.5 \times 10^{-4}$) was added to exact reflection coefficients from the two previous experiments. In Figure 5 the confidence on the SVD inversion is analysed in terms of absolute errors, singular values and variances. Conclusions of previous inversions are confirmed.

Conclusions

Use of the approximation for reflections in weakly anisotropic media expressed in Eq. (1) allows a satisfactory estimation of contrasts in WA parameters even with noisy data. The inversion requires: (1) weak contrast and weak anisotropy for the media under study; (2) a priori information of background isotropic velocities, or at least an accurate estimation for those quantities; and (3) reflection data from offsets usually found in practice and from at least five measurement profiles. Under these assumptions, even small offset information can lead to estimation of important contrasts in WA parameters (i.e., B_1, B_2 and B_3).

Acknowledgments

The authors express their gratitude for the consortium project "*Seismic Waves in Complex 3-D Structures* (SW3D)", Czech Republic. Jorge L. Martins acknowledges support from ANP (proc. 48610.002779/99-31) and from CNPq/RHAE (proc. 610154/99-0). This abstract was prepared at GTEP - Group of Tech. and Petroleum Engineering, PUC-Rio.

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