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Two-point Paraxial Traveltime in Inhomogeneous Isotropic/Anisotropic Media - Tests of Accuracy

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SUMMARY

We analyze the efficiency and accuracy of the two-point paraxial traveltime formula. It provides the approximate traveltime between two points arbitrarily chosen in a paraxial vicinity of a single reference ray, along which the ray propagator matrix in a ray-centered coordinates has been determined. The reference ray can be traced in an arbitrary, smoothly varying, layered medium of arbitrary anisotropy. We illustrate the potential of the two-point paraxial traveltime formula and its Shanks transform representation, which provides much higher accuracy, on smoothly inhomogeneous, isotropic and weakly anisotropic models. The formulation offers several interesting applications including fast traveltime computations, velocity-independent interpolation and datuming.

Introduction

Estimating traveltime between two points is useful for many applications including traveltime tomography. Alkhalifah and Fomel (2010) suggested using the eikonal equation to calculate traveltime perturbations with respect to the source and receivers locations. Their approach, though very efficient, has the limitations typically associated with finite-difference solutions of the eikonal equation.

Here we study efficiency and accuracy of the two-point paraxial traveltime formula based on ray solutions of the eikonal equation (Červený, Iversen and Pšenčík, 2011). The formula can be used for the approximate determination of traveltimes between two points arbitrarily chosen in a paraxial vicinity of a single reference ray Ω, along which ray propagator matrix in ray-centered coordinates has been determined. The reference ray can be traced in an arbitrary, laterally varying, layered medium of arbitrary anisotropy. The formula has a form of the Taylor expansion of the two-point traveltime to quadratic terms.

We evaluate the performance gains of the two-point paraxial traveltime formula as compared with the eikonal approach used by Alkhalifah and Fomel (2010). We focus on the computation of two-point paraxial traveltimes of P waves propagating in more complex 2D smooth models of inhomogeneous, isotropic or anisotropic media and for different configurations. We begin with tests on an isotropic model and proceed to a weakly anisotropic model using the first-order ray tracing approach (Pšenčík and Farra, 2005). We also illustrate the added accuracy achieved by using Shanks transform (Bender and Orszag, 1978) at virtually no further addition to the computational cost.

Two-point Paraxial Traveltime Formula

The two-point paraxial traveltime formula (Červený et al., 2011) is given as:

$$\begin{aligned}
T(R',S') = {} & T(R,S) + [x_i(R') - x_i(R)]p_i(R) - [x_i(S') - x_i(S)]p_i(S) \\
& + \tfrac{1}{2}[x_i(R') - x_i(R)][f^R_{Mi}(\mathbf{P}_2^{(q)}\mathbf{Q}_2^{(q)-1})_{MN} f^R_{Nj} + (p_i\eta_j + p_j\eta_i - p_i p_j \mathscr{U}_k\eta_k)_R][x_j(R') - x_j(R)] \\
& + \tfrac{1}{2}[x_i(S') - x_i(S)][f^S_{Mi}(\mathbf{Q}_2^{(q)-1}\mathbf{Q}_1^{(q)})_{MN} f^S_{Nj} - (p_i\eta_j + p_j\eta_i - p_i p_j \mathscr{U}_k\eta_k)_S][x_j(S') - x_j(S)] \\
& - [x_i(S') - x_i(S)] f^S_{Mi}(\mathbf{Q}_2^{(q)-1})_{MN} f^R_{Nj}[x_j(R') - x_j(R)].
\end{aligned} \tag{1}$$

The upper-case indices M,N take values 1 and 2, and the lower-case indices i,j take values 1,2 and 3. The Einstein summation convention is used. Here, $x_i(S)$ and $x_i(R)$ are Cartesian coordinates of two points, S and R, on the reference ray Ω; $x_i(S')$ and $x_i(R')$ are coordinates of points S' and R' situated in close vicinities of S and R, respectively. The symbols $\mathbf{Q}_1^{(q)} = \mathbf{Q}_1^{(q)}(R,S)$, $\mathbf{Q}_2^{(q)} = \mathbf{Q}_2^{(q)}(R,S)$, and $\mathbf{P}_2^{(q)} = \mathbf{P}_2^{(q)}(R,S)$ denote 2×2 submatrices of the 4×4 ray propagator matrix in ray-centred coordinates calculated along Ω from S to R by dynamic ray tracing. The symbols f^S_{Mi} and f^R_{Mi} denote Cartesian components of vectors perpendicular to Ω at S and R, respectively. For their determination, a vectorial, ordinary differential equation must be solved along Ω. The symbols p_i, $\mathscr{U}_i$ and η_i denote Cartesian components of slowness vector, ray-velocity vector and the vector $\mathrm{d}\mathbf{p}(\tau)/\mathrm{d}\tau$, respectively, determined during tracing the reference ray. The superscripts S and R indicate if the corresponding quantities are considered at point S or R. For more details refer to Červený et al. (2011).

Once the reference ray Ω and the above-mentioned quantities calculated along it are available, two-point paraxial traveltimes can be calculated easily. It is worth-mentioning that the above formula will fail to work properly if model parameters variation is too strong or if the matrix $\mathbf{Q}_2^{(q)}$ is singular at point R. The latter problem occurs when there is a caustic at the point R.

In isotropic media, we use exact dynamic ray tracing. However, in weakly anisotropic media, we use the first-order ray tracing approach (Pšenčík and Farra, 2005). Along first-order rays, we perform first-order

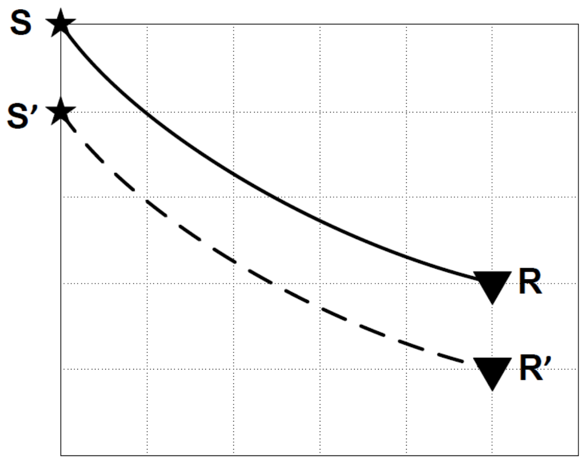
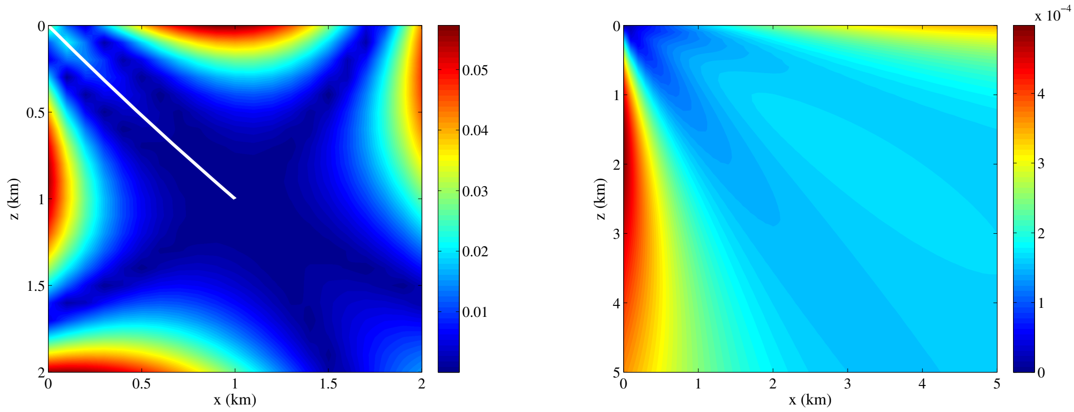


Figure 1 *Schematic plot of reference (between S and R) and paraxial (between S' and R') rays on the 2D grid. Eq.(1) is used to calculate two-point paraxial traveltime between S' and R' using the ray and dynamic ray tracing quantities evaluated between S and R along the reference ray.*

dynamic ray tracing in ray-centered coordinates. For the determination of the two-point paraxial traveltime, we use (1), in which we replace the exact quantities by their first-order counterparts, computed along the reference ray Ω.

Test Examples

As an illustration of the accuracy of the two-point paraxial traveltime formula, we present several results of tests performed, for simplicity, on 2D models of inhomogeneous, isotropic and weakly anisotropic (HTI) media. The models are covered by rectangular grids with 0.1 km separation in the x and z directions. P-wave two-point paraxial traveltime between points S' and R' is calculated from (1) using quantities determined along the reference ray between S and R, see Figure 1.

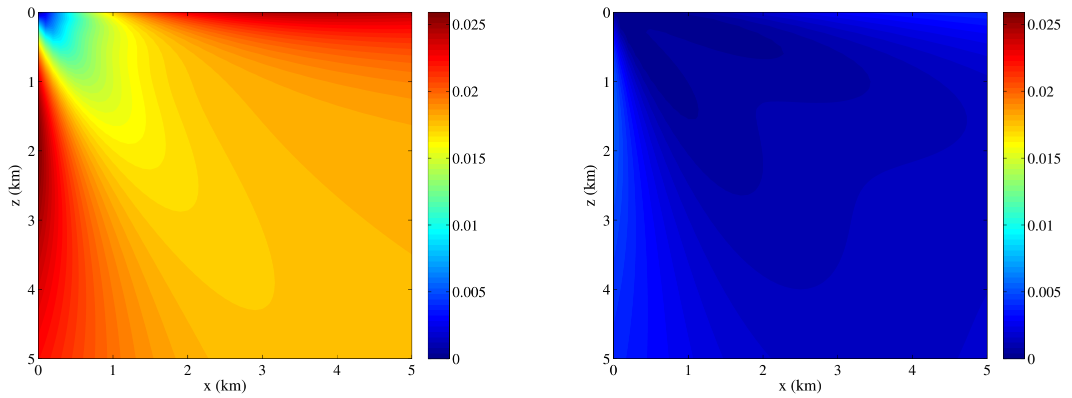


(a) (b)

Figure 2 *Absolute differences (in seconds) of P-wave paraxial and exact two-point traveltimes for an isotropic model with velocity varying horizontally and vertically (a) Two-point paraxial traveltimes to all grid points are computed from the point R (1,1) on the ray (white curve) connecting S (0,0) and R. (b) The source and receivers perturbed vertically by 0.2 km (see Figure 1). The perturbed source is at (0,0).*

Figure 2a illustrates the efficiency of the above two-point traveltime formula for the case $S' \equiv S$ and points R' situated at all grid points. The points S (0,0) and R (1,1) are situated on the reference ray Ω (white curve). Here, an isotropic model with P-wave velocity of 2 km/s at the origin and linear vertical gradient of 0.7 s^{-1} and a linear horizontal gradient of 0.5 s^{-1} is used. Figure 2a shows differences of paraxial and exact traveltimes. We can see that highly accurate results are obtained around the ray Ω between S and R, along its extrapolation and around the wavefront passing through the point R.

Figure 2b shows differences of paraxial and exact traveltimes in the same model as in Figure 2a, but for the source and receivers shifted by 0.2 km in the positive z direction (as in Figure 1), so that the perturbed source is at (0,0). Notice that for most part of the model studied, traveltime errors are less than 0.00002 s ($\sim$0.04% relative error). The largest errors are around 0.00005 s ($\sim$0.18% relative error), owing to negligible variation in the whole studied region.



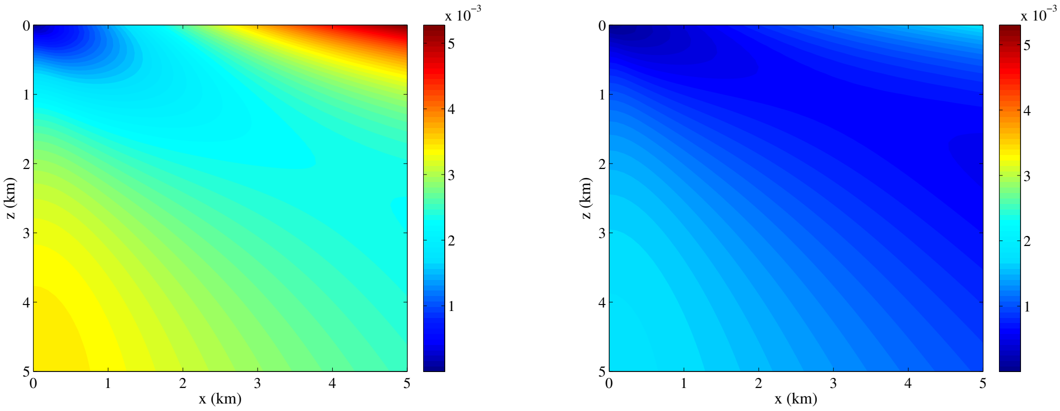
(a) (b)

Figure 3 *Absolute differences (in seconds) of P-wave paraxial and exact two-point traveltimes as in Fig. 2b, but (a) with the source and receivers perturbed vertically by 0.8 km and (b) using Shanks transform representation of (1) for configuration in (a). The perturbed source is at (0,0). Same colorbars are used for illustration purpose.*

Figure 3a shows a map of the traveltime errors for source and receivers shifted by 0.8 km in the positive z direction in the same model as in Figure 2. With hardly any additional computational cost, we can use the Shanks transform representation of (1) and in this case the errors are further reduced (Fig. 3b). In most of studied region, the traveltime errors are around 0.02 s ($\sim$ 3% relative error), the highest error being 0.026 s ($\sim$ 10% relative error). Again, variation of the error is relatively small in most parts of the model studied. Figure 3b underscores the efficiency of the two point paraxial traveltime formula used in conjunction with the Shanks transform. The largest error has been significantly reduced to 0.0055 s ($\sim$ 2.4% relative error) whereas for most part of the model studied, errors are below 0.002 s ($\sim$ 0.5% relative error). These errors are negligible in view of a relatively large perturbation of 0.8 km for the source and receivers.

In Figure 4, we present results similar to those in Figure 3, but for the model of a transversely isotropic medium. The weak-anisotropy parameters for the VTI model at z = 0 km are $\varepsilon = 0.0866$ and $\delta = 0.07692$, and $V_{P0} = 3.66$ km/s. At z = 5 km, these parameters take on values $\varepsilon = 0.0866$ and $\delta = 0.0773$, and $V_{P0} = 5.49$ km/s. The VTI models are then rotated by 90^o so that axes of symmetry coincide with x-axis. Moreover, the axis of symmetry at the surface (z = 0 km) is further rotated by 45^o from x-axis. As depth increases between the two levels, axis of symmetry rotates smoothly in the horizontal plane. The resulting model is thus the "twisted crystal" model.

Figure 4a shows traveltime errors for the source and receivers shifted by 0.8 km in the positive z direction. Figure 4b uses Shanks transform representation of (1) for the same model and shift. We see that the accuracy is even higher in this case than in the isotropic model, cf. Figure 3. For most part of model



(a) (b)

Figure 4 *Absolute differences (in seconds) between P-wave paraxial and exact two-point traveltimes for the HTI "twisted crystal" model with a linear vertical gradient. For details see the text. (a) Source and receivers perturbed vertically by 0.8 km (b) using Shanks transform representation of (1) for configuration of (a). The perturbed source is at (0,0). The same colorbars are used for illustration purpose.*

in Figure 4a, traveltime errors are below 0.0035 s ($\sim$ 0.5% relative error) and the largest error is 0.0053 s ($\sim$1.26% relative error). In Figure 4b, we observe that Shanks transform yields significant improvement gain with the largest error being reduced to 0.0018 s ($\sim$ 0.6% relative error).

Concluding Remarks

The above test examples demonstrate the great potential of the two-point paraxial traveltime formula. Its advantage, when compared, for example, to approaches based on most eikonal solvers, is that it provides traveltimes related to energetic arrivals. The approach is not expected to work properly in strongly varying media, in which multipathing occurs. Generalization to more realistic models is straightforward. We are going to consider 2D or 3D models of laterally inhomogeneous isotropic or anisotropic media. Let us note that layered media can also be considered. It is only necessary to take into account the effects of interfaces on dynamic ray tracing. Results of such tests will be presented at the 74^{th} EAGE conference.

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