Prevailing-frequency approximation of the coupling ray theory for S waves along the SH and SV reference rays

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Abstract

The coupling ray theory S-wave tensor Green function is frequency dependent. Its prevailing–frequency approximation removes this frequency dependence and allows us to introduce the coupling–ray–theory travel times and the coupling–ray–theory amplitudes, and to process the coupling–ray–theory wave field in the same way as the anisotropic–ray–theory wave field. This simplification may be decisive when storing the tensor Green function at the nodes of dense grids.

The coupling ray theory is usually applied to anisotropic common reference rays, but it is more accurate if it is applied to reference rays which are closer to the actual wave paths. In a generally anisotropic medium, the actual wave paths may be approximated by the anisotropic–ray–theory rays if these rays behave reasonably. In an approximately transversely isotropic medium, we can define and trace the SH and SV reference rays, and use them as reference rays for the prevailing–frequency approximation of the coupling ray theory.

We test the accuracy of the proposed prevailing–frequency approximation of the coupling ray theory numerically. The additional inaccuracy introduced by the prevailing–frequency approximation is smaller than the inaccuracy of the standard frequency–domain coupling ray theory.

Introduction

There are two different high–frequency asymptotic ray theories for S waves with frequency–independent amplitudes: the isotropic ray theory based on the assumption of equal velocities of both S waves, and the anisotropic ray theory assuming both S waves strictly decoupled. Here the term “different” means that the isotropic ray theory is not a special case of the anisotropic ray theory for decreasing anisotropy, and that both theories yield different S waves in equal velocity models.

In the isotropic ray theory, the S–wave polarization vectors do not rotate about the ray, whereas in the anisotropic ray theory they coincide with the eigenvectors of the Christoffel matrix which may rotate rapidly about the ray.

In “weakly anisotropic” media, at moderate frequencies, the actual S–wave polarization tends to remain unrotated round the ray, but is partly attracted by the rotation of the eigenvectors of the Christoffel matrix. The intensity of the attraction increases with frequency. This behaviour of the actual S–wave polarization is described by the coupling ray theory proposed by Coates & Chapman (1990). The frequency–dependent coupling ray theory is the generalization of both the zero–order isotropic and anisotropic ray theories and provides continuous transition between them. The coupling ray theory is applicable to S waves at all degrees of anisotropy, from isotropic to considerably anisotropic velocity models. The numerical algorithm for calculating the frequency–dependent coupling–ray–theory S–wave Green tensor has been designed by Bulant & Klimeš (2002).

The coupling–ray–theory S–wave Green tensor is frequency dependent, and is usually calculated for many frequencies. This frequency dependence represents no problem in calculating the Green tensor, but may represent a great problem in storing the Green tensor at the nodes of dense grids (Klimeš & Bulant, 2013), typical for applications such as seismic migrations, Born approximation, or hypocenter determination. This contribution is devoted to the approximation of the coupling–ray–theory Green tensor, which eliminates this frequency dependence within a reasonably broad frequency band.

The accuracy of the coupling ray theory depends on the reference rays (Bulant & Klimeš, 2008). The coupling ray theory is usually applied to anisotropic common reference rays (Bakker, 2002; Klimeš, 2006). On the other hand, the coupling ray theory is more accurate if it is applied to reference rays which are closer to the actual wave paths (Klimeš & Bulant, 2014a; 2015). In a generally anisotropic medium, the actual wave paths may be approximated by the anisotropic–ray–theory rays if these rays behave reasonably, which is not always the case, mainly in the presence of singularities, as demonstrated by Klimeš & Bulant, 2014b. In an approximately transversely isotropic medium, we can define and trace the SH and SV reference rays, and use them as reference rays for the prevailing–frequency approximation of the coupling ray theory, as described in the second part of this contribution.

Prevailing-frequency approximation

In the vicinity of a given prevailing frequency, we approximate the frequency–dependent frequency–domain coupling–ray–theory tensor Green function (G^{CRT}) by two dyadic Green functions corresponding to two waves described by their travel times and amplitudes calculated for the prevailing frequency. We refer to these travel times and amplitudes as the coupling–ray–theory travel times and the coupling–ray–theory amplitudes. This prevailing–frequency approximation of the coupling ray theory tensor Green function (G^{PFA}) is uniquely defined by two conditions:

a) at the given prevailing frequency, we require $G^{PFA}=G^{CRT}$ and

b) at the given prevailing frequency, we require the derivative of G^{PFA} with respect to the frequency to equal the derivative of G^{CRT}.

These two conditions uniquely determine coupling-ray-theory travel times and coupling-ray-theory polarization vectors. We numerically calculate G^{CRT} at the given prevailing frequency using the algorithm by Bulant & Klimeš (2002), and we calculate the derivatives of G^{CRT} with respect to the frequency using the derivative of this algorithm (Klimeš & Bulant 2016). The prevailing–frequency approximation of the coupling ray theory allows us to process the coupling–ray–theory wave field in the same way as the anisotropic–ray–theory wave field. This simplification may be decisive when storing the tensor Green function at the nodes of dense grids.

SH and SV reference rays

The coupling ray theory is usually applied to anisotropic common reference rays, but it is more accurate if it is applied to reference rays which are closer to the actual wave paths. In a generally anisotropic medium, the actual wave paths may be approximated by the anisotropic–ray–theory rays if these rays behave reasonably. In an approximately transversely isotropic medium, we can define and trace the SH and SV reference rays, and use them as reference rays for the prevailing–frequency approximation of the coupling ray theory (Klimeš & Bulant 2015). The coupling ray theory tensor Green function along the SH reference rays or along the SV reference rays can be calculated using the algorithm by Bulant & Klimeš (2002). The decomposition of the coupling ray theory tensor Green function into two arrivals can be determined using the algorithm by Klimeš & Bulant (2016).

We thus obtain two arrivals along each SH reference ray, and two arrivals along each SV reference ray, and have to select the correct ones. We may try to select the proper arrival according to its polarization or travel time. If one of the arrivals is close to the reference SH wave in its polarization and its travel time, we may assume that it has probably been calculated with good accuracy and we may select it. Analogously for each SV reference ray. Unfortunately, the criteria based on polarization and travel time may be contradictory. If this selection fails, the coupling ray theory along the SH and SV reference rays cannot be used, and we should calculate the coupling–ray–theory approximation along the anisotropic common reference rays.

Examples

We tested the accuracy of the proposed prevailing–frequency approximation of the coupling ray theory numerically using elastic S waves in several anisotropic velocity models. The synthetic seismograms generated by a vertical force are calculated at the receivers located in a vertical well at a distance of 1 km from the source. The source–receiver configuration is displayed in Figure 1.

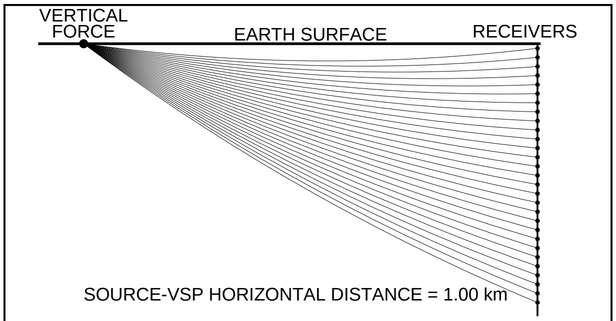

Figure 1: The source–receiver configuration for the calculation of synthetic seismograms, with a sketch of the reference rays. The source is located at the Earth's surface, the receivers are placed in the vertical well. The numbers and depths of the receivers are presented in the figures with seismograms.

The source time function is the Gabor signal with reference frequency 50Hz, bandpass filtered by a cosine filter specified by frequencies 0Hz, 5Hz, 60Hz and 100Hz. The receivers record the following 3 components of displacement: radial component (along the line connecting the source and the top of the well, positive away from the source), transverse component (perpendicular to the source–receiver plane), and vertical component (positive downwards). The recording system is right–handed. For the prevailing–frequency approximation of the coupling ray theory, we naturally use the prevailing frequency 50Hz. The synthetic seismograms are compared with the Fourier pseudospectral method which is considered here as a nearly exact reference.

A vertically heterogeneous 1–D anisotropic velocity model QI is approximately transversely isotropic in a vertical plane which forms a 45° angle with the source–receiver vertical plane and is situated between the positive radial and positive transverse seismogram components. Velocity model SC1_I is very close to transversely isotropic, but is slightly tetragonal. Velocity model SC1_II is analogous to SC1_I, but the reference symmetry axis of its approximately transversely isotropic component is tilted. The model thus contains split intersection singularity positioned in the source–receiver plane close to the horizontal slowness vectors. In the weakly orthorhombic velocity model ORT, the slowness surface contains four conical singularities. The rays leading from the source to the middle part of the receiver profile pass close to one of these singularities.

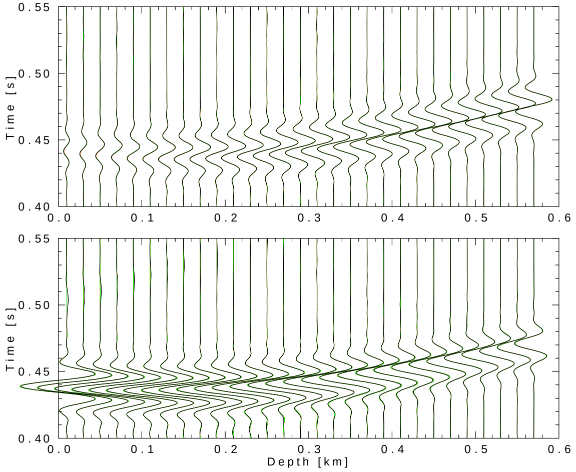

Figure 2: Transverse (top) and vertical (bottom) component of the synthetic seismograms in model QI. The prevailing–frequency seismograms are plotted in **red**, then coupling-ray-theory seismograms are plotted in **green**, and they are overlaid by the **black** Fourier pseudospectral method seismograms considered here as a nearly exact solution. All seismograms are in good agreement, with the prevailing–frequency approximation seismograms obscured by the standard–coupling–ray–theory seismograms.

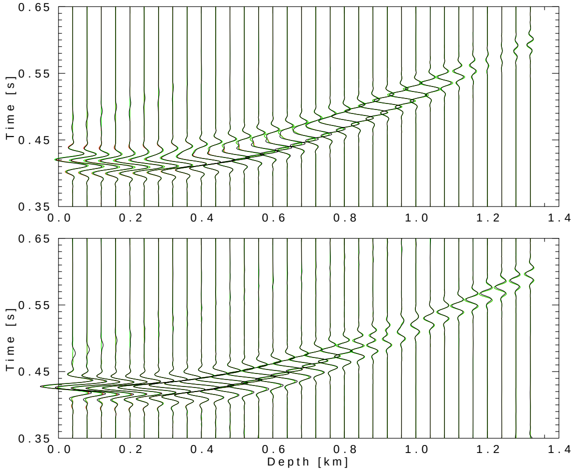

Figure 3: Transverse (top) and vertical (bottom) component of the synthetic seismograms in model SC_I. The prevailing–frequency seismograms are plotted in **red**, then coupling-ray-theory seismograms are plotted in **green**, and they are overlaid by the **black** Fourier pseudospectral method seismograms. The prevailing–frequency approximation seismograms are mostly obscured by the standard–coupling–ray–theory seismograms. Both the coupling–ray–theory seismograms are close to the Fourier pseudospectral seismograms.

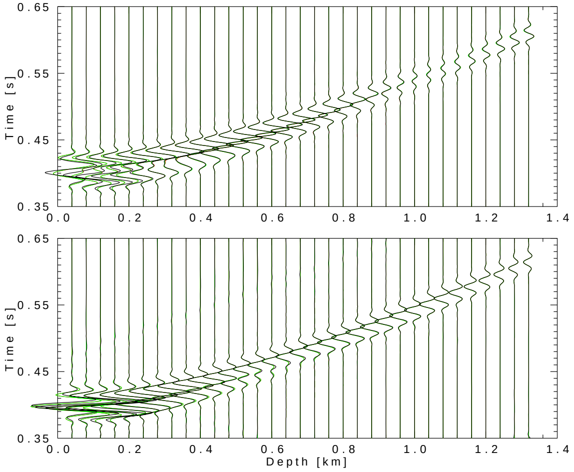

Figure 4: Transverse (top) and vertical (bottom) component of the synthetic seismograms in model SC_II. The prevailing–frequency seismograms are plotted in **red**, then coupling-ray-theory seismograms are plotted in **green**, and they are overlaid by the **black** Fourier pseudospectral method seismograms. The seismograms from the shallow receivers indicate problems with the inaccurate reference polarization vectors and the inaccurate reference geometrical spreading. However, the prevailing–frequency approximation represents very good approximation to the standard coupling ray theory even in this situation.

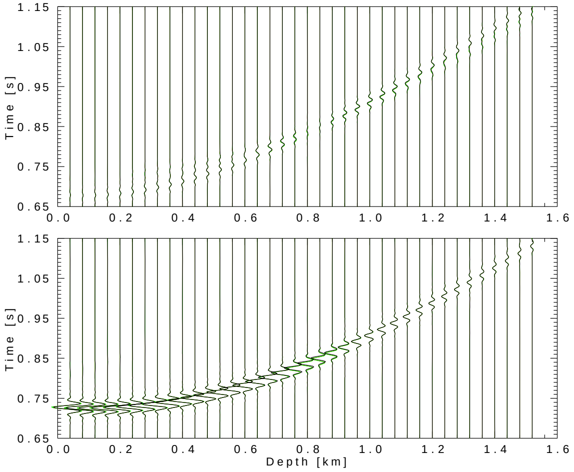

Figure 5: Transverse (top) and vertical (bottom) component of the synthetic seismograms in model ORT. The prevailing–frequency seismograms are plotted in **red**, then coupling-ray-theory seismograms are plotted in **green**, and they are overlaid by the **black** Fourier pseudospectral method seismograms. The prevailing–frequency approximation seismograms are obscured by the standard–coupling–ray–theory seismograms. Both the coupling–ray–theory seismograms are close to the Fourier pseudospectral seismograms despite the existence of conical singularities in this velocity model.

The additional inaccuracy introduced by the prevailing–frequency approximation is smaller than the inaccuracy of the standard frequency–domain coupling ray theory, see the examples of synthetic seismograms in the four models on Figures 2 to 5.

We used SH and SV reference rays in several models which are approximately transversely isotropic, and obtained more accurate synthetic seismograms compared to the case of using the common anisotropic reference rays. In velocity model SC1 II containing a split intersection singularity in the source–receiver plane, we observe a great inaccuracy of the coupling–ray–theory seismograms calculated along the anisotropic common reference rays (Figure 5). The coupling–ray–theory seismograms calculated along the SH and SV reference rays (Figure 6) represent a considerable accuracy improvement.

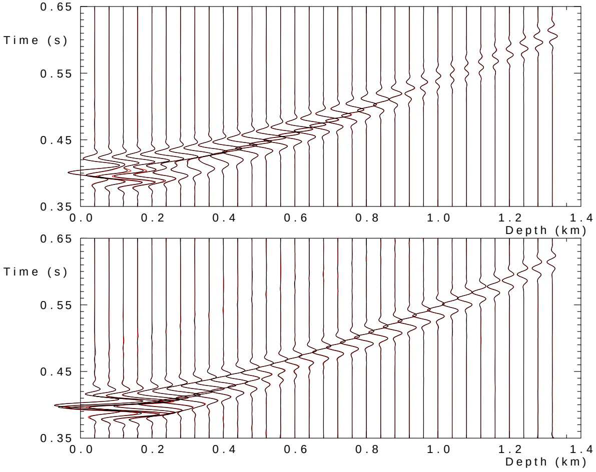

Figure 6: Transverse (top) and vertical (bottom) component of the seismograms calculated in velocity model SC1 II. The **red** seismograms are calculated using the prevailing–frequency approximation of the coupling ray theory along the SH and SV reference rays. They are overlaid by the **black** seismograms calculated using the Fourier pseudospectral method. In comparison with Figure 5, the differences of the coupling ray theory from the Fourier pseudospectral method have considerably been reduced.

Conclusions

Within a limited frequency band, we may efficiently approximate the frequency–dependent frequency–domain coupling–ray–theory tensor Green function by two dyadic Green functions corresponding to two waves, described by their coupling–ray–theory travel times and the corresponding vectorial amplitudes calculated for the prevailing frequency. The additional inaccuracy introduced by this prevailing–frequency approximation is usually smaller than the inaccuracy of the standard frequency– domain coupling ray theory. This simplification may be decisive when storing the tensor Green function at the nodes of dense grids, which is typical for applications such as the Born approximation. The prevailing–frequency approximation with its coupling–ray–theory travel times also offers a new way of understanding the results of the coupling ray theory.

The SH and SV reference rays may represent very accurate reference rays for the coupling ray theory in approximately transversely-isotropic media. Improvement of the coupling–ray–theory synthetic seismograms calculated along the SH and SV reference rays have been demonstrated in velocity model SC1_II.

Acknowledgments

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