# COMPUTATION OF RAY INTEGRALS AND RAY AMPLITUDES IN RADIALLY SYMMETRIC MEDIA

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Summary: *A new approximation of the velocity-depth distribution in radially symmetric media is suggested. This approximation guarantees the continuity of velocity and its first and second derivatives, and does not generate false low-velocity layers. It removes false anomalies from the amplitude-distance curve and considerably increases its stability. The evaluation of ray integrals and ray amplitudes using this velocity-depth approximation does not require the computation of any transcendental function and is, therefore, very fast. Numerical examples are presented.*

The ray amplitudes of seismic body waves are very sensitive to the method used to approximate the velocity-depth distribution. The commonly used methods of approximation in vertically inhomogeneous media and also in radially symmetric media (such as the piece-wise linear approximation or the Bullen law) do not guarantee the continuity of the first and second derivatives and can produce false oscillations. These features cause false anomalies on the amplitude-distance curve (such as zeros or shadow zones) so that the form of the amplitude-distance curve is then often very complicated and does not represent the medium under consideration properly.

Therefore, in recent years new methods of approximation have been sought that would be more suitable for the computation of the ray amplitudes. Some of the suggested approximations have

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removed the mentioned disadvantages, but their practical application is connected with other difficulties. Let us mention the necessity of a numerical evaluation of the ray integrals in the case of the approximation that uses the cubic splines.

Recently a new method of approximation for vertically inhomogeneous media was suggested by Červený [1]. It reads

$$z = a_i + b_i v^{-2} + c_i v^{-4} + d_i v^{-6}, \qquad (1)$$

where $z$ is the depth, $v = v(z)$ denotes the velocity and $i$ is the index corresponding to the $i$-th layer. The coefficients $a_i$, $b_i$, $c_i$, $d_i$ for all layers are calculated applying the smoothed spline approximation [2]. This approximation guarantees the continuity of the velocity-depth distribution with its first and second derivatives and the stability of the resultant amplitude-distance curve; it does not generate false low-velocity layers and the ray integrals can be evaluated analytically. It is sufficient to evaluate just the square roots; no transcendental functions are required. Thus the computation is even faster than for the piece-wise linear approximation and not much slower than for a system of homogeneous parallel layers.

In view of the numerous advantages of the approximation given by formula (1), a similar approximation is suggested here for radially symmetric media. It reads

$$\ln(r) = a_i + b_i(r/v)^2 + c_i(r/v)^4 + d_i(r/v)^6, \qquad (2)$$

where $r$ is the distance from the Earth's centre and $v = v(r)$ denotes the velocity. The coefficients $a_i$, $b_i$, $c_i$, $d_i$ for all spherical layers are again calculated by the smoothed spline approximation. Using the approximation given by formula (2) one can profit in the radially symmetric media from all the advantages mentioned previously. Due to these properties, this form of approximation may find many applications in radially symmetric media in seismology. It might even be useful to use approximation (2) with a smaller number of terms. For example, this approximation with two terms was used recently by Cormier [3]. According to [3], parametrizing the logarithm of

Table 1. The part of the B1 model used in calculations.

| $h$(km) | $v_p$(km/s) | $v_s$(km/s) | $\varrho$(Mg/m$^3$) |
|---|---|---|---|
| 0 | 6·300 | 3·550 | 2·84 |
| 33 | 6·300 | 3·550 | 2·84 |
| 33 | 7·750 | 4·353 | 3·32 |
| 182 | 8·198 | | |
| 200 | 8·256 | | |
| 400 | 8·923 | | |
| 413 | 8·970 | | |
| 600 | 10·243 | | |
| 800 | 11·009 | | |
| 984 | 11·420 | | |
| 1200 | 11·706 | | |
| 1400 | 11·992 | | |
| 1600 | 12·262 | | |
| 1800 | 12·530 | | |
| 2000 | 12·794 | | |
| 2200 | 13·034 | | |
| 2400 | 13·270 | | |
| 2600 | 13·495 | | |
| 2695 | 13·600 | | |
| 2878 | 13·640 | | |

the radius in the Bullen parameter $r/v$ was suggested in his Ph. D. thesis by J. H. Woodhouse in 1974.

In this paper we present some numerical examples based on approximation (2) and comparisons with some other approximations. The detailed formulae are not presented here, they will be published elsewhere. As the demonstration model the upper part of the classical Earth model **Bl** [4] between the surface and the depth of 2878 km (the core-mantle boundary) is chosen (Tab. 1). With the exception of the Fig. 5, the grid points at the depths of 182, 400, 1400, 1800 and 2200 km are omitted.

Figure 1 shows the velocity-depth distribution and the amplitude-distance curve for the approximation that uses the Bullen law. The grid points are given by dots. This velocity-depth distribution with the discontinuities of the first derivative at the grid points causes the singularities on the amplitude-distance curve. We can remove the discontinuities of the first derivative of the velocity by using the formula with more terms. But in this case we usually get false oscillations on the velo-

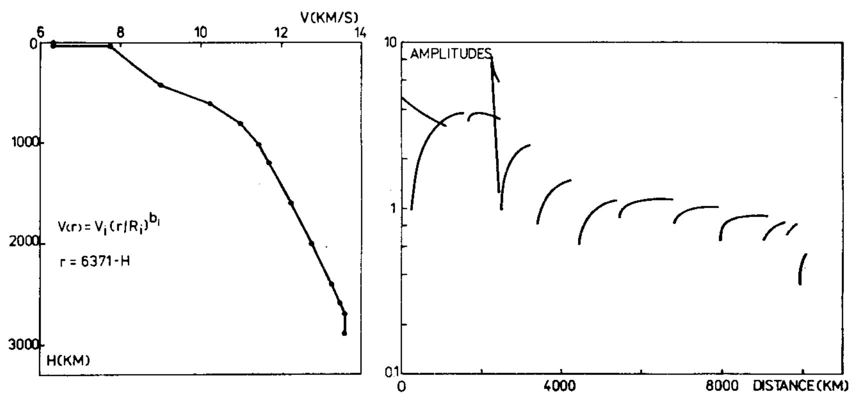


Fig. 1. Velocity-depth distribution and corresponding amplitude-distance curve. The velocity within individual layers is specified by the Bullen law. The discontinuities of the first derivative of velocity cause the singularities in the amplitude-distance curve.

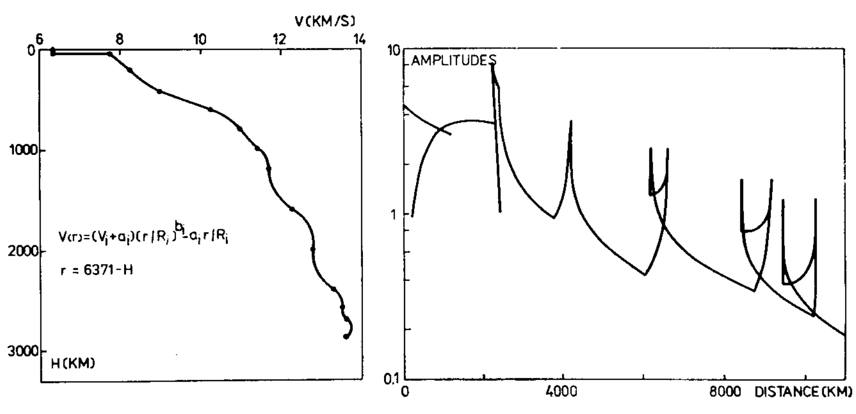


Fig. 2. The same as in Fig. 1. The velocity within the individual layer is specified by the formula shown on the left-hand side of the figure. This formula removes the discontinuities of the first derivative of velocity, but it generally introduces false oscillations, so that the resulting amplitude--distance curve is quite chaotic.

city-depth distribution and, therefore, false caustics on the amplitude-distance curve which becomes very chaotic, as shown in Fig. 2. This approximation formula can generally give a useful amplitude-distance curve only if we are able to determine a suitable number and the positions of the grid points by a time consuming trial. Figure 3 shows an example where the grid points at the depths of 600, 984, 1200, 1600, 2000, 2400, 2600 and 2698 km have been omitted and a new one has been introduced at the depth of 450 km. The corresponding amplitude-distance curve is then smoother; the singularities have practically vanished. The caustic at the distance of about 2000 km is a real one, caused by the sudden change of the velocity gradient at the depth of 413 km. In both these approximations a small change in the position or the number of the grid points can cause a great change in the amplitude-distance curve.

Figures 4 and 5 show the results of the calculation for the approximation given by formula (2). In Fig. 4 we can see the influence of two different degrees of smoothing by splines in the calculation of the coefficients $a_i$, $b_i$, $c_i$, $d_i$ in (2). The full line corresponds to the higher accuracy, the dashed to the lower accuracy of the spline approximation. The difference in the velocity-depth distribution is negligible except for the vicinity of the depth of 200 km and for the bottom of the demonstration

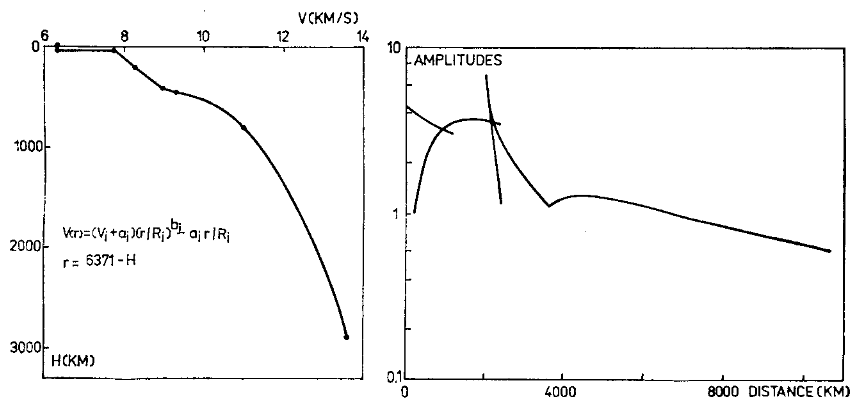


Fig. 3. The same as in Fig. 2. To remove the false oscillations in the velocity-depth distribution, the number and positions of grid points were adapted by trial-and-error.

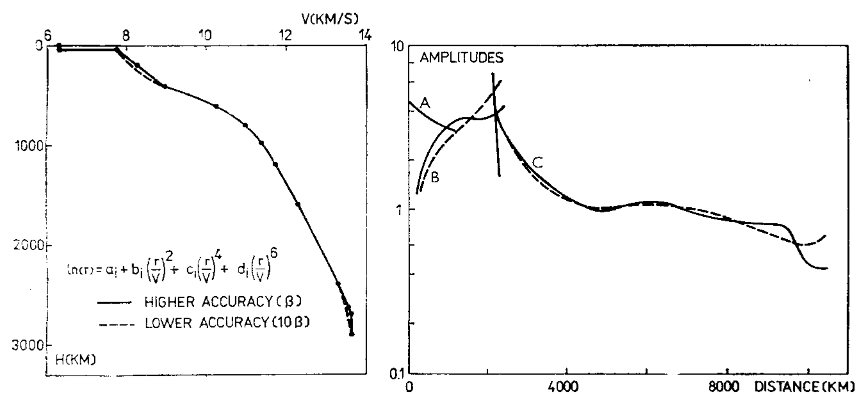


Fig. 4. Velocity-depth distributions and corresponding amplitude-distance curves. The velocity distribution within the individual layers is specified by formula (2); the coefficients $a_i$, $b_i$, $c_i$, $d_i$ are determined by the smoothed spline approximation. Two different degrees of smoothing are compared. For details, see text.

model. These two regions of differences cause the differences in the amplitude-distance curves roughly from 300 to 2300 km and from 8500 to 10 600 km. But it is evident that the differences are not too great allthough the degree of smoothing differs 10 times. Part A of the amplitude-distance curves corresponds to the rays with a minimum in the crust, part B to the rays with a minimum above 413 km and part C to the rays with a minimum below 413 km in the mantle. It must be pointed out again that the caustic at the distance of about 2000 km is real and cannot be removed within the scope of the ray method. In the computation the same degree of smoothing was applied to all grid points. Of course, one can use different smoothings for different grid points.

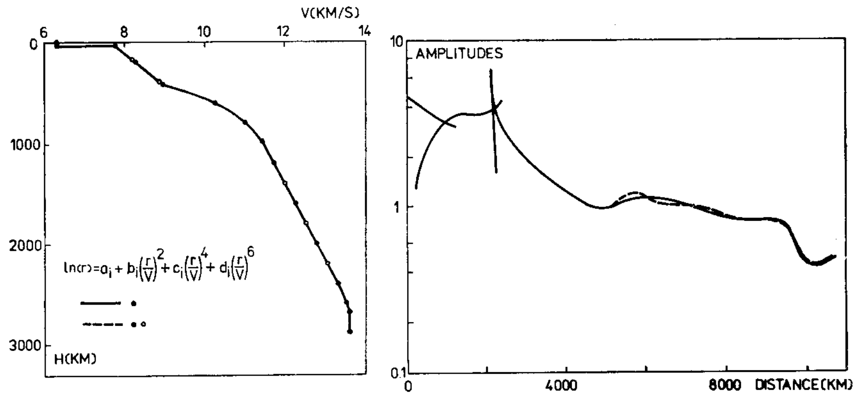


Fig. 5. The same as in Fig. 4. The amplitude-distance curves for two different numbers of grid points in the velocity-depth distribution are compared.

Figure 5 shows the influence of the number of grid points used. The full line is the same as that in Fig. 4, i.e. without the grid points at the depths of 182, 400, 1400, 1800 and 2200 km. (These grid points are denoted by circles.) The dashed line corresponds to the case when we use all the grid points from the upper part of the $B_1$ model, i.e. both the dots and the circles. The differences between these two velocity-depth distributions are negligible and cannot be seen in the velocity-depth graph. A higher number of grid points causes a higher number of oscillations in the amplitude-distance curve. But both amplitude-distance curves are very similar and the differences are not too distinct. This demonstrates that the suggested method of approximation is suitably stable. It is not too sensitive to the number of grid points and to their positions. With respect to the properties of radially symmetric media it is recommended to increase the density of grid points with increasing depth.

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