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The traveltime perturbations for seismic body waves in factorized anisotropic inhomogeneous media

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SUMMARY

The traveltime perturbation equations for the quasi-compressional and the two quasi-shear waves propagating in a factorized anisotropic inhomogeneous (FAI) media are derived. The concept of FAI media simplifies considerably these equations. In the FAI medium, the density normalized elastic parameters $a_{ijkl}(x_i)$ can be described by the relation $a_{ijkl}(x_i) = f^2(x_i)A_{ijkl}$, where A_{ijkl} are constants, independent of coordinates x_i, and $f^2(x_i)$ is a continuous smooth function of x_i. The types of anisotropy (A_{ijkl}) and inhomogeneity $[f(x_i)]$ are not restricted. The traveltime perturbations of individual seismic body waves (qP, $qS1$ and $qS2$) propagating in the FAI medium depend, of course, both on the structural pertubations $[\delta f^2(x_i)]$ and on the anisotropy perturbations (δA_{ijkl}), but both these effects are fully separated. The perturbation equations for the time delay between the two qS-waves propagating in the FAI medium are simplified even more. If the unperturbed (background) medium is isotropic, the perturbation of the time delay does not depend on the structural perturbations $\delta f^2(x_i)$ at all. This striking result, valid of course only in the framework of first-order perturbation theory, will simplify considerably the interpretation of the time delay between the two split qS-waves in inhomogeneous anisotropic media. Numerical examples are presented.

Key words: anisotropic medium, delay time between qS-waves, perturbation methods, shear wave splitting.

1 INTRODUCTION

Simple procedures of the solution of direct and inverse kinematic problems in inhomogeneous isotropic structures are based on perturbation theory. They have found broad applications in the solution of many important seismological problems, especially in seismic tomography (Nolet 1987). Similar procedures have been proposed even for *anisotropic* inhomogeneous structures. The general anisotropic inhomogeneous media, however, are specified by 21 density normalized elastic parameters a_{ijkl}, and all these parameters may depend on Cartesian coordinates in a different way. Attempts to determine spatial distributions of 21 density normalized elastic parameters $a_{ijkl}(x_i)$ from the traveltime measurements could hardly lead to reliable results. Some simplifying assumptions should be made.

The parametrization of the anisotropic inhomogeneous structure is considerably simplified if the concept of the factorized anisotropic inhomogeneous (FAI) medium is used. In the FAI medium, the spatial variations of all density normalized elastic parameters are assumed to be the same,

$$a_{ijkl}(x_i) = f^2(x_i)A_{ijkl}. \tag{1}$$

Here A_{ijkl} are constants, independent of coordinates x_i and $f(x_i)$ is an arbitrary smooth function of coordinates. The types of anisotropy (A_{ijkl}) and inhomogeneity $[f^2(x_i)]$ are not restricted, but both effects are factorized. The concept of the FAI medium not only reduces considerably the number of data describing the model, but also simplifies considerably the ray computations. For a detailed treatment of the ray theory computations in the FAI media, see Červený (1989a), where the concept of the FAI medium is introduced. A special form of the FAI medium, in which the function $f(x_i)$ is a linear function of depth, was introduced even earlier by Shearer & Chapman (1988).

In this paper, we shall show that the concept of the FAI medium simplifies also all the traveltime perturbation equations for anisotropic inhomogeneous media. Particularly drastic simplification is obtained for the perturbation of the delay time between the two split quasi-shear waves in

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the FAI medium if the non-perturbed (background) medium is isotropic. In the framework of the first-order perturbation theory, the perturbation of the delay time in the FAI medium with an isotropic background depends only on the perturbation of anisotropy constants δA_{ijkl}, and does not depend on the structural perturbations $\delta f^2(x_i)$. This result will simplify considerably the inversion procedures.

Note that the shear wave splitting plays a very important role in the interpretation of anisotropic structures. For a detailed discussion of this subject see, e.g. Crampin & McGonigle (1981), Crampin (1985) and Liu, Crampin & Booth (1989). The presented papers also give numerous other references. Commonly, two important observations are made from the three-component records showing the shear wave splitting. The first observation is related to the orientation and polarization of the fastest qS-wave, and the second to the time delay between the two split qS-waves. The time delay can be measured directly from the three-component seismograms, or, more accurately, from polarization diagrams. Thus, the polarization diagrams play an extremely important role in the interpretation of shear wave splitting. In general, the polarization of the fastest qS-wave yields information on the *local* anisotropic properties, close to the observation point. Contrary to this, the time delay carries *global* (averaged) information about the anisotropy along the whole ray. Both these types of information may be, of course, different, but both are very important from a seismological point of view. It is obvious that the concept of the FAI medium is particularly suitable for the interpretation of the time delay between the two split quasi-shear waves. For this reason, we intend to discuss the perturbations of the time delay in a greater detail.

We shall discuss here only the kinematic, traveltime perturbations. In isotropic media, the perturbation methods have been recently extended even to the perturbation of rays, ray propagators, etc., see Farra & Madariaga (1987), Nowack & Lutter (1988), Farra, Virieux & Madariaga (1989), etc. Similar investigations have started even for anisotropic inhomogeneous media, see Jech & Pšenčík (1989) and Farra (1989, 1990). The concept of the FAI media simplifies considerably even such extended perturbation equations. This was recently shown by Farra (1989, 1990) for the FAI media with some simpler anisotropy symmetries.

The density normalized elastic parameters $a_{ijkl}(x_i)$ have dimension of squared velocity, i.e. $\mathrm{km}^2\,\mathrm{s}^{-2}$. Thus, the physical meaning and dimensions of $f(x_i)$ and a_{ijkl} can be interpreted in the following two ways.

(1) The dimension of $f(x_i)$ is $\mathrm{km\,s}^{-1}$ and the constants A_{ijkl} are dimensionless. Then, the function $f(x_i)$ has a physical meaning of velocity. The dimensionless constants A_{ijkl} do not actually represent density normalized elastic parameters in this case. To emphasize this fact, we shall also call them the *reduced anisotropy constants*.

(2) The function $f(x_i)$ is dimensionless and the dimension of A_{ijkl} is $\mathrm{km}^2\,\mathrm{s}^{-2}$. Then, A_{ijkl} has a physical meaning of density normalized elastic parameters, but the dimensionless function $f(x_i)$ is not actually the velocity. We can call $f(x_i)$ the *reduced velocity*.

Most of equations derived in this paper are valid for both the above interpretations. In Chapters 4 and 6 only, some equations are specified for the first interpretation, in which $f(x_i)$ represents a velocity and A_{ijkl} the dimensionless reduced anisotropy constants.

In this paper we shall use the following notation. A moving wavefront of a high-frequency elastic body wave is described by the equation $T(x_i) = t$, where t is the time and $T(x_i)$ the traveltime field. The Cartesian components of the slowness vector are denoted by p_i, $p_i = \partial T(x_i)/\partial x_i = N_i/C$ where N_i are Cartesian components of the unit vector perpendicular to the wavefront, and C is the phase velocity. We introduce a 3×3 symmetric positive definite matrix Γ with components $\Gamma_{ik} = a_{ijkl}p_j p_l$, which is often called the Christoffel matrix, and denote its eigenvalues by G_m $(m = 1, 2, 3)$ and its eigenvectors by $\mathbf{g}^{(m)}$ $(m = 1, 2, 3)$. These three eigenvalues and eigenvectors correspond to three elastic high-frequency body waves propagating in an anisotropic inhomogeneous medium. We shall use the notation $m = 1, 2$ for the two quasi-shear waves ($qS1$- and $qS2$-waves), and $m = 3$ for the quasi-compressional wave (qP-wave). For a specified m, the eigenvalue $G_m(x_i, p_i)$ can be determined by solving the cubic equation $\det(\Gamma_{ik} - G_m\delta_{ik}) = 0$, and the relevant eigenvector $\mathbf{g}^{(m)}$ from the three relations $(\Gamma_{ik} - G_m\delta_{ik})g_k^{(m)} = 0$ $(i = 1, 2, 3)$, with an additional normalization condition $g_k^{(m)}g_k^{(m)} = 1$ (no summation over m). If the eigenvalues G_1, G_2 and G_3 are mutually different $(G_1 \neq G_2 \neq G_3)$, the three eigenvectors can be uniquely determined, and we speak about a non-degenerate case. If, however, two eigenvalues are equal, e.g. $G_1 = G_2$, we speak about a degenerate case. In the degenerate case, the two relevant eigenvectors $\mathbf{g}^{(1)}$ and $\mathbf{g}^{(2)}$ cannot be uniquely determined. We only know that they are mutually perpendicular and that they are perpendicular to the remaining eigenvector $\mathbf{g}^{(3)}$. A most important example of the degenerate case which will be needed in this paper is the isotropic medium. In the isotropic medium, $G_1 = G_2 = \beta^2 p_i p_i$, and $G_3 = \alpha^2 p_i p_i$, where α and β are the velocities of compressional and shear waves, respectively. The eigenvector $\mathbf{g}^{(3)}$ corresponding to the compressional wave equals $\mathbf{N}$. The eigenvectors corresponding to the shear wave, $\mathbf{g}^{(1)}$ and $\mathbf{g}^{(2)}$, however, cannot be uniquely determined; we only know that they are mutually perpendicular and situated in a plane perpendicular to $\mathbf{N}$ (i.e., in a plane tangent to the wavefront).

For any of the three seismic body waves propagating in an inhomogeneous anisotropic medium, the traveltime field $T(x_i)$ is a solution of a non-linear partial differential equation of the first-order $G_m(x_i, p_i) = 1$, called also the eikonal equation.

For more details on the computation of high-frequency seismic wavefields in anisotropic inhomogeneous media, particularly on ray tracing, transport equations and computation of amplitudes, see Červený, Molotkov & Pšenčík (1977) and Gajewski & Pšenčík (1987).

2 THE TRAVELTIME PERTURBATIONS IN GENERAL INHOMOGENEOUS ANISOTROPIC MEDIA

In this section, we shall summarize the traveltime perturbations equations for seismic body waves propagating in general anisotropic inhomogeneous media. For homogeneous slightly anisotropic media, a similar problem was first studied by Backus (1965), see also Thomsen (1986). The equations derived by Backus (1965) have found broad applications in seismology. The disadvantage of these equations is that they are limited to homogeneous media. Here we shall discuss the anisotropic inhomogeneous media.

We shall consider a perturbed medium anisotropic which differs only slightly from the unperturbed anisotropic (background) medium. We denote the density normalized elastic parameters in the background medium by a^0_{ijkl} and in the perturbed medium by a_{ijkl}, and the traveltime form a point O to a point S in the background medium by $T^0(O, S)$ and in the perturbed medium by $T(O, S)$. We can write

$$a_{ijkl} = a^0_{ijkl} + \delta a_{ijkl}, \qquad T(O, S) = T^0(O, S) + \delta T(O, S). \quad (2)$$

We assume that the perturbations δa_{ijkl} are small.

The general equation for the traveltime perturbations $\delta T(O, S)$ was derived by Červený (1982):

$$\delta T(O, S) = -\frac{1}{2}\int_{O\,(L_0)}^{S} \delta a_{ijkl} p_i p_l g_j^{(m)} g_k^{(m)}\, dT. \quad (3)$$

Here the integration is performed along the ray L_0 computed in the background medium (frozen ray), and quantities p_i, p_l, $g_j^{(m)}$, $g_k^{(m)}$ and dT correspond to the background medium. Let us emphasize an important property of equation (3): the relation between $\delta T(O, S)$ and δa_{ijkl} is linear. See also Hanyga (1982).

Equation (3) can be used for all three waves, qP, $qS1$ and $qS2$, if the background medium is anisotropic. For qP-waves, it can be used even for an isotropic background medium. In this case, $m = 3$, $\mathbf{g}^{(3)} = \mathbf{N}$, $\mathbf{p} = \mathbf{N}/\alpha$, and equation (3) yields

$$\delta T_{qP}(O, S) = -\frac{1}{2}\int_{O\,(L_0)}^{S} \delta a_{ijkl} N_i N_j N_k N_l \alpha^{-2}\, dT, \quad (4)$$

where α is the velocity of P-waves in the background medium, N_i, N_j, N_k, N_l and $dT = ds/\alpha$ again correspond to the background medium, ds denoting an elementary arclength along the ray L_0 in the background medium.

For the qS-waves ($m = 1, 2$), however, the situation is more complicated. If the background medium is isotropic, the two eigenvectors $\mathbf{g}^{(1)}$ and $\mathbf{g}^{(2)}$ cannot be uniquely determined (degenerate case) and (3) cannot be used. We can, however, easily recognize that

$$g_k^{(1)} g_j^{(1)} + g_k^{(2)} g_j^{(2)} = \delta_{kj} - g_k^{(3)} g_j^{(3)},$$

where δ_{kj} denotes the Kronecker delta, and obtain

$$\delta T_{qS1}(O, S) + \delta T_{qS2}(O, S) = -\frac{1}{2}\int_{O\,(L_0)}^{S} p_i p_l (\delta_{kj} - g_k^{(3)} g_j^{(3)}) \delta a_{ijkl}\, dT. \quad (5)$$

The equation is valid generally, both for isotropic and anisotropic backgrounds. For an isotropic background we can, of course, insert $\mathbf{p} = \mathbf{N}/\beta$, $\mathbf{g}^{(3)} = \mathbf{N}$ and $dT = ds/\beta$. For a more detailed discussion of equations (4) and (5) and for the investigation of the accuracy of these equations see Červený & Jech (1982), Červený & Firbas (1984) and Firbas (1984). First programs for the computation of traveltimes of qP-waves in general 2-D laterally varying slightly anisotropic layered structures based on (4) were written and applied to the solutions of some seismological problems by Firbas (1982).

The kinematic perturbation equations for the two quasi-shear waves are more complex and non-linear if the background medium is isotropic. They were recently derived by Jech & Pšenčík (1989). They read

$$\delta T_{qS1,qS2}(O, S) = -\frac{1}{4}\int_{O\,(L_0)}^{S} \{D_{11} + D_{22} \pm [(D_{11} - D_{22})^2 + 4D_{12}^2]^{1/2}\}\, dT. \quad (6)$$

The integration is performed along the ray L_0 of the shear-wave in the background medium. The equations for $\delta T_{qS1}(O, S)$ and $\delta T_{qS2}(O, S)$ differ only by the sign '+' or '−' in the integral. The functions D_{11}, D_{12} and D_{22} are given by the relations

$$D_{mn} = \delta a_{ijkl} p_i p_l e_j^{(m)} e_k^{(n)}. \quad (7)$$

The quantities p_i, p_l, $e_j^{(m)}$, $e_k^{(n)}$ and dT are again taken in the background medium. The vectors $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are two arbitrary mutually perpendicular unit vectors, perpendicular also to $\mathbf{g}^{(3)} = \mathbf{N}$ in the background medium.

Equation (6) yields, of course, equation (5) for $\delta T_{qS1}(O, S) + \delta T_{qS2}(O, S)$. It, however, also yields an important equation for the *perturbation of the delay time* between the two split shear waves,

$$|\delta T_{qS1}(O, S) - \delta T_{qS2}(O, S)| = \frac{1}{2}\int_{O\,(L_0)}^{S} [(D_{11} - D_{22})^2 + 4D_{12}^2]^{1/2}\, dT. \quad (8)$$

Let us make a note on the selection of unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ along the ray L_0. They can be chosen in different ways. For example, we can take $\mathbf{e}_1 = \mathbf{n}$ and $\mathbf{e}_2 = \mathbf{b}$ where $\mathbf{n}$ and $\mathbf{b}$ are the unit normal and unit binormal vectors related to the ray L_0. Another, more preferable possibility is to take $\mathbf{e}_1$ and $\mathbf{e}_2$ as the polarization vectors of S-waves. It is well known that the polarization vectors can be introduced as the basis vectors of the ray centered coordinate system, see Popov & Pšenčík (1978a, b), Červený & Hron (1980) and Červený (1985). This selection is preferable, as the basis vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are, in fact, commonly computed along the ray L_0, even if we are not interested in perturbations. They are needed to determine the polarization of S-waves, to perform dynamic ray tracing and determine the ray propagator matrix, the ray jacobian, etc.

The determination of the unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ is particularly simple if the ray L_0 in the unperturbed medium is a plane ray, see Section 5.

3 THE TRAVELTIME PERTURBATIONS IN THE FAI MEDIUM

The disadvantage of the perturbation equations presented in Section 2 is that they are influenced both by changes of the structure and of the anisotropy. We can obtain a good fit of observed and computed traveltimes if we change the spatial variations of individual elastic parameters (but otherwise leave the main anisotropic properties unchanged), or if we change the anisotropic properties (but leave the spatial variations of elastic parameters unchanged).

A suitable way to separate the perturbations due to spatial variations of elastic parameters and due to perturbations of the elastic parameters themselves is offered by the concept of the FAI medium. Let us assume that both the perturbed and unperturbed media are the FAI media. Then, we can write

$$a^0_{ijkl} = f^{02}(x_i)A^0_{ijkl}, \qquad a_{ijkl} = f^2(x_i)A_{ijkl}. \tag{9}$$

We shall introduce the *structural perturbations* $\delta f^2(x_i)$ and the *anisotropy perturbations* δA_{ijkl} in the following way:

$$f^2(x_i) = f^{02}(x_i) + \delta f^2(x_i), \qquad A_{ijkl} = A^0_{ijkl} + \delta A_{ijkl}. \tag{10}$$

Within the framework of the first-order perturbation theory, we obtain

$$\delta a_{ijkl}(x_i) = A^0_{ijkl}\delta f^2(x_i) + f^{02}(x_i)\delta A_{ijkl}. \tag{11}$$

The dimensions of f^0 and δf are the same as of f, and the dimensions of A^0_{ijkl} and δA_{ijkl} the same as of A_{ijkl}. All the equations which will be derived here are valid for both interpretations of $f(x_i)$ and A_{ijkl} discussed in the Introduction. Inserting (11) into the general expression (3) yields

$$\delta T(O,S) = -\frac{1}{2}A^0_{ijkl}\int_{O\,(L_0)}^{S} \delta f^2(x_i)p_i p_l g_j^{(m)} g_k^{(m)}\,dT - \frac{1}{2}\delta A_{ijkl}\int_{O\,(L_0)}^{S} f^{02}(x_i)p_i p_l g_j^{(m)} g_k^{(m)}\,dT. \tag{12}$$

The first term in equation (12) can be considerably simplified. For a given m, we shall use the equation $(\Gamma_{jk} - G_m\delta_{jk})g_k^{(m)} = 0$ $(j = 1, 2, 3)$, discussed in the Introduction, and multiply it by $g_j^{(m)}$. This yields

$$G_m = \Gamma_{jk}g_j^{(m)}g_k^{(m)} = a^0_{ijkl}p_i p_l g_j^{(m)} g_k^{(m)} = f^{02}(x_i)A^0_{ijkl}p_i p_l g_j^{(m)} g_k^{(m)}.$$

Along the ray, the eikonal equation $G_m = 1$ is satisfied. Thus we have, finally,

$$A^0_{ijkl}p_i p_l g_j^{(m)} g_k^{(m)} = \frac{1}{f^{02}(x_i)}.$$

Inserting this into (12) yields

$$\delta T(O,S) = -\int_{O\,(L_0)}^{S} \frac{\delta f(x_i)}{f^0(x_i)}\,dT - \frac{1}{2}\delta A_{ijkl}\int_{O\,(L_0)}^{S} f^{02}(x_i)p_i p_l g_j^{(m)} g_k^{(m)}\,dT. \tag{13}$$

It is simple to see that the first integral in (13) corresponds to structural perturbations $[\delta f^2(x_i)]$ and the second integral to anisotropy perturbations (δA_{ijkl}). Both effects are fully separated. The most striking effect in (13) is the simplicity of the first integral, corresponding to structural perturbations.

Equation (13) can be used quite universally for quasi-compressional waves, both for isotropic and anisotropic backgrounds. For quasi-shear waves, however, the second integral can be used only if the background is anisotropic. For an isotropic background, the second integral must be replaced by a more complex integral, see next section. The first integral, corresponding to structural perturbations, however, has a quite universal validity. It can be used both for quasi-compressional and quasi-shear waves, for isotropic and anisotropic FAI backgrounds and perturbed media.

For example, let us consider compressional waves with an isotropic background and isotropic perturbed medium. We shall consider $f^0(x_i) = \alpha(x_i)$. Then $\delta A_{ijkl} = 0$, $\delta f = \delta\alpha$, and $dT = ds/\alpha$, and we obtain

$$\delta T_P(O,S) = \int_{O\,(L_0)}^{S} \delta\left(\frac{1}{\alpha}\right) ds. \tag{14}$$

Similarly, for S-waves, with an isotropic background and isotropic perturbed media. We again take $f^0(x_i) = \alpha(x_i)$. Then, we have $\delta A_{ijkl} = 0$, $\delta f = \delta\alpha$, $\delta f/f^0 = \delta\alpha/\alpha = \delta\beta/\beta$ and $dT = ds/\beta$. Then we obtain

$$\delta T_S(O,S) = \int_{O\,(L_0)}^{S} \delta\left(\frac{1}{\beta}\right) ds. \tag{14a}$$

Both these integrals are well known and broadly used in tomographic studies.

Even though (13) cannot be used for the quasi-shear waves if the background medium is isotropic, we can again simply recognize that the expression for $\delta T_{qS1}(O,S) + \delta T_{qS2}(O,S)$ is valid universally. We use the relation $g_j^{(1)}g_k^{(1)} + g_j^{(2)}g_k^{(2)} = \delta_{jk} - g_j^{(3)}g_k^{(3)}$ and obtain

$$\delta T_{qS1}(O,S) + \delta T_{qS2}(O,S) = -2\int_{O\,(L_0)}^{S} \frac{\delta f(x_i)}{f^0(x_i)}\,dT - \frac{1}{2}\delta A_{ijkl} \times \int_{O\,(L_0)}^{S} f^{02}(x_i)p_i p_l(\delta_{jk} - g_j^{(3)}g_k^{(3)})\,dT. \tag{15}$$

This equation is valid for all possible situations: for isotropic and anisotropic backgrounds and perturbed FAI media, similar to (13) for qP-waves.

4 THE TRAVELTIME PERTURBATION EQUATIONS FOR THE FAI MEDIUM WITH AN ISOTROPIC BACKGROUND

We shall now specify all perturbation equations for the FAI medium with an isotropic background. Such perturbation equations offer very important practical applications. We perform all ray tracing and traveltime computations for an isotropic inhomogeneous background, and by perturbation theory we simply recalculate the results for an anisotropic inhomogeneous medium. As time efficient and flexible computer program packages now exist for ray tracing and traveltime computations in very general isotropic 2-D and 3-D laterally varying layered and block structures, this approach can be used to solve a great variety of various forward and inverse problems of anisotropic inhomogeneous media.

The isotropic background medium, of course, must also satisfy the conditions of the FAI media, so that we have to assume that the ratio of P- and S-wave velocities is constant in the whole medium,

$$\beta = \nu\alpha. \tag{16}$$

Here α and β are P- and S-wave velocities in the background medium, and ν is a constant. We also choose the function $f^0(x_i)$ in the background medium in the following simple way:

$$f^0(x_i) = \alpha(x_i). \tag{17}$$

We shall now use the well-known relation for a^0_{ijkl},

$$a^0_{ijkl} = \frac{\lambda}{\rho}\delta_{ij}\delta_{kl} + \frac{\mu}{\rho}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \tag{18}$$

where λ and μ are Lamé elastic parameters, ρ the density and δ_{ij} the Kronecker delta symbol. Taking into account equations (1), (16) and (17), we can rewrite (18) for the background medium in the following form:

$$A^0_{ijkl} = (1 - 2\nu^2)\delta_{ij}\delta_{kl} + \nu^2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}). \tag{19}$$

As discussed in the Introduction, A^0_{ijkl} represent dimensionless reduced anisotropy constants in this case.

For *quasi-compressional waves,* (13) is universally valid. Thus, we can use it even for an isotropic background. We merely put $f^0 = \alpha$, $\mathbf{p} = \mathbf{N}/\alpha$ and $\mathbf{g}^{(m)} = \mathbf{N}$ to obtain

$$\delta T_{qP}(O, S) = -\int_{O\,(L_0)}^{S} \frac{\delta f(x_i)}{\alpha(x_i)}\,dT - \frac{1}{2}\delta A_{ijkl}\int_{O\,(L_0)}^{S} N_i N_l N_j N_k\,dT. \tag{20}$$

The first integral corresponds to the structural perturbations, and the second to the anisotropy perturbations. If the perturbed medium is also isotropic, we have $\delta A_{ijkl} = 0$ and $\delta f(x_i) = \delta\alpha(x_i)$. Then the first integral yields (14).

Let us now consider *quasi-shear waves.* We shall use equations (6) with (7) and (11). We can also write $\mathbf{p} = \mathbf{N}/\beta$ and obtain

$$D_{mn} = \beta^{-2}E_{mn}\,\delta f^2(x_i) + \beta^{-2}C_{mn}f^{02}(x_i), \tag{21}$$

where

$$E_{mn} = A^0_{ijkl}N_i N_l e^{(m)}_j e^{(n)}_k, \qquad C_{mn} = \delta A_{ijkl}N_i N_l e^{(m)}_j e^{(n)}_k. \tag{22}$$

For E_{mn}, we can write

$$E_{mn} = (1 - 2\nu^2)(N_i e^{(m)}_i)(N_k e^{(n)}_k) + \nu^2(N_i e^{(m)}_i)(N_k e^{(n)}_k) + \nu^2(N_i N_i)(e^{(m)}_k e^{(n)}_k) = \nu^2(e^{(m)}_k e^{(n)}_k).$$

This yields

$$D_{11} = \alpha^{-2}\,\delta f^2 + \nu^{-2}C_{11}, \qquad D_{22} = \alpha^{-2}\,\delta f^2 + \nu^{-2}C_{22},$$
$$D_{12} = \nu^{-2}C_{12}. \tag{23}$$

We further obtain

$$D_{11} + D_{22} = 2\alpha^{-2}\,\delta f^2 + \nu^{-2}(C_{11} + C_{22}),$$
$$(D_{11} - D_{22})^2 + 4D^2_{12} = \nu^{-4}[(C_{11} - C_{22})^2 + 4C^2_{12}]. \tag{24}$$

We now insert (24) into (6) and obtain the final expressions for the traveltime perturbations for both quasi-shear waves in the FAI medium with an isotropic background:

$$\delta T_{qS1,qS2}(O, S) = -\int_{O\,(L_0)}^{S} \frac{\delta f(x_i)}{\alpha(x_i)}\,dT - \frac{1}{4\nu^2}\int_{O\,(L_0)}^{S} (C_{11} + C_{22})\,dT \mp \frac{1}{4\nu^2}\int_{O\,(L_0)}^{S} [(C_{11} - C_{22})^2 + 4C^2_{12}]^{1/2}\,dT, \tag{25}$$

where $dT = ds/\beta$. From (25), we easily obtain two simple formulae,

$$\delta T_{qS1}(O, S) + \delta T_{qS2}(O, S) = -2\int_{O\,(L_0)}^{S} \frac{\delta f(x_i)}{\alpha(x_i)}\,dT - \frac{1}{2\nu^2}\int_{O\,(L_0)}^{S} (C_{11} + C_{22})\,dT, \tag{26}$$

and

$$|\delta T_{qS1}(O, S) - \delta T_{qS2}(O, S)| = \frac{1}{2\nu^2}\int_{O\,(L_0)}^{S} [(C_{11} - C_{22})^2 + 4C^2_{12}]^{1/2}\,dT. \tag{27}$$

It is easy to see that (26) corresponds to (15) for an isotropic background ($f^0 = \alpha$, $\mathbf{p} = \mathbf{N}/\beta$, $\mathbf{g}^{(3)} = \mathbf{N}$).

We can now discuss the physical meaning of individual terms in (25). We recall that dT corresponds to the background medium, so that $dT = ds/\beta$.

The first integral is very simple and corresponds to structural perturbations. It, of course, remains valid even if the perturbed medium is also isotropic, so that $\delta f = \delta\alpha$, $\delta\alpha/\alpha = \delta\beta/\beta$, $dT = ds/\beta$ and the first term yields (14a).

The second term corresponds to one half of the sum of the time perturbations of both quasi-shear waves, for the structural perturbations vanishing ($\delta f = 0$), see equation (26). As we know, this term has quite regular behaviour and can be used universally.

Finally, the third term is related to the perturbation of the time delay between the two split quasi-shear waves. In absolute value, it corresponds to one half of this perturbation.

The most striking result is given by equation (27). It shows that the perturbation of the traveltime delay in the FAI medium with an isotropic background does not depend at all on the structural perturbations $\delta f^2(x_i)$, but depends only on the anisotropy perturbations δA_{ijkl}. Thus, if we consider a FAI medium with an isotropic background and perturb both the structure $[\delta f^2(x_i)]$ and anisotropic properties (δA_{ijkl}), the time delay is not influenced at all by the perturbation of the structure, but only by the perturbation of anisotropic properties.

Note that the fact that the perturbation of the delay time in the FAI medium with an isotropic background does not depend on structural perturbations $[\delta f^2(x_i)]$ was already predicted, without a derivation, by Červený (1989c).

5 PERTURBATIONS ALONG A PLANE RAY L_0

All expressions are simplified if the unperturbed ray L_0 is a plane ray. This situation has a great practical importance, as many computer program packages exist for isotropic 2-D media. These programs packages can be simply used to perform the computations for traveltime perturbations in the FAI medium with an isotropic background. For this reason, we shall specify all such equations in greater detail. As the specification of the perturbation equation (20) for qP-waves is straightforward, we shall pay attention only to the perturbation equations for qS-waves.

As usually, we shall consider a ray L_0 situated in the plane (x_1, x_3), i.e. in the plane $x_2 = 0$. Along the whole plane, the component N_2 of the vector normal to the unperturbed wavefront vanishes,

$$N_2 = 0. \tag{28}$$

If we choose the unit vector $\mathbf{e}_2$ perpendicular to the plane $x_2 = 0$ at the initial point of the unperturbed ray, it is perpendicular to this plane along the whole ray. Thus we have

$$e_1^{(2)} = e_3^{(2)} = 0, \qquad e_2^{(2)} = 1. \tag{29}$$

The unit vector $\mathbf{e}^{(1)}$ is situated in the plane $x_2 = 0$ and is perpendicular both to $\mathbf{N}$ and $\mathbf{e}_2$. We choose it as follows:

$$e_1^{(1)} = -N_3, \qquad e_2^{(1)} = 0, \qquad e_3^{(1)} = N_1. \tag{30}$$

In the same way, we can take $e_1^{(1)} = N_3$ and $e_3^{(1)} = -N_1$; the results do not depend on the change of signs.

To simplify the final equations, we shall use a two-suffix notation for the density normalized elastic parameters, instead of the four-suffix notation. For C_{11}, C_{22} and C_{12}, we obtain from (22)

$$\begin{aligned} C_{11} &= \delta A_{55}N_1^4 + \delta A_{55}N_3^4 + (\delta A_{11} + \delta A_{33} - 2\delta A_{13} - 2\delta A_{55}) \\ &\quad \times N_1^2N_3^2 + 2N_1N_3(N_3^2 - N_1^2)(\delta A_{15} - \delta A_{35}), \\ C_{22} &= \delta A_{66}N_1^2 + \delta A_{44}N_3^2 + 2\delta A_{46}N_1N_3, \\ C_{12} &= \delta A_{56}N_1^3 - \delta A_{45}N_3^3 + (\delta A_{45} + \delta A_{36} - \delta A_{16})N_1^2N_3 \\ &\quad + (\delta A_{34} - \delta A_{14} - \delta A_{56})N_1N_3^2. \end{aligned} \tag{31}$$

Inserting the expressions (31) for C_{11}, C_{22}, and C_{12} into (25), (26) and (27), we obtain the final formulae for the traveltime perturbations of both qS-waves in the FAI medium with an isotropic background, if the ray L_0 is situated in the plane $x_2 = 0$.

Let us make an interesting note to (31). If the perturbations δA_{14}, δA_{16}, δA_{34}, δA_{36}, δA_{45} and δA_{56} vanish, we obtain $C_{12} = 0$. This yields

$$[(C_{11} - C_{22})^2 + 4C_{12}^2]^{1/2} = |C_{11} - C_{22}|,$$

and all the expressions for δT_{qS1}, δT_{qS2} and $|\delta T_{qS1} - \delta T_{qS2}|$ are linear with respect to the remaining perturbations δA_{ij}. This was recognized even earlier by Červený & Jech (1982), where such linear equations for δT_{qS1}, δT_{qS2} and $|\delta T_{qS1} - \delta T_{qS2}|$ were derived and discussed.

The computer program package BEAM87 designed for the numerical modelling of seismic wavefields in 2-D isotropic complex structures by the ray method or by the Gaussian beam method, described briefly in Červený (1989b), has been modified to incorporate the kinematic perturbation computations in the FAI media, both for qP- and qS-waves. Equations (20) and (25), with (31), were used. We shall call this modified version BEAM87-A. The program BEAM87 considers rather general 2-D isotropic laterally varying layered models. Models with vanishing layers, block structures, fractures and isolated bodies can be handled. Within any layer, the velocity distributions may vary both laterally and vertically. Any multiply reflected, possibly converted waves can be treated in the package.

Even the perturbed FAI medium considered in BEAM87-A is rather general. The anisotropic parameters A_{ijkl} of the FAI medium may be different in different layers. They are, of course, independent of coordinates in any layer, but may vary from layer to layer.

Some simple numerical examples computed by the program BEAM87-A will be presented and discussed in the next section.

6 NUMERICAL EXAMPLES

In this section we shall present several simple numerical examples of the computation of traveltimes of two qS-waves in anisotropic inhomogeneous media by the perturbation method, starting from an isotropic inhomogenous background with $v = \beta/\alpha = \text{constant}$. We shall concentrate our attention particularly on the time delay between the two split quasi-shear waves. We wish to demonstrate that the perturbations of the traveltime delay depend strongly on the anisotropy perturbations (δA_{ijkl}), but that they are not so sensitive to the structural perturbations $\delta f^2(x_i)$. Of course, the traveltimes of individual quasi-shear waves depend considerably both on the structural perturbations $\delta f^2(x_i)$ and on the anisotropy perturbations δA_{ijkl}.

To demonstrate these results in a very simple way, we shall consider a simple type of an isotropic background structure, particularly an isotropic vertically inhomogeneous medium with a constant gradient of velocity. The function $f^0(x_i)$ in the definition of the FAI medium will be specified as proposed by equation (17),

$$f^0(z) = \alpha(z) = \alpha_0 + cz. \tag{32}$$

Here z is the depth (oriented downwards), α_0 is the compressional velocity on the surface of the model (for $z = 0$), and c is the gradient of compressional velocity. Note that the density normalized elastic parameters $a_{ijkl}^0(z)$ in the background medium do not depend linearly on depth, but quadratically,

$$a_{ijkl}^0(z) = (\alpha_0 + cz)^2 A_{ijkl}^0, \tag{33}$$

see (1). As the ratio of the shear and compressional velocities $v = \beta/\alpha$ in the model is constant, the surface shear wave velocity is $\beta_0 = v\alpha_0$ and the gradient of the shear wave velocity is vc.

The dimensionless reduced anisotropic constants A_{ijkl}^0 in the isotropic background are chosen according to the rule (19). In a two-suffix notation, the 6×6 symmetric matrix of reduced anisotropic constants is as follows:

$$A^0_{ij}=\begin{pmatrix} 1 & 1-2v^2 & 1-2v^2 & 0 & 0 & 0 \\ & 1 & 1-2v^2 & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & v^2 & 0 & 0 \\ & & & & v^2 & 0 \\ & & & & & v^2 \end{pmatrix}. \tag{34}$$

The *perturbed anisotropic model* can be chosen arbitrarily, by the specifications of $\delta f^2(x_i)$ and δA_{ijkl}. We shall perform the following numerical experiment: we shall compute exactly the rays and traveltimes of seismic S-waves for serveral isotropic background models (32) with different gradients c. All these individual models can be, in fact, considered as the perturbed isotropic models of a homogeneous isotropic background model ($c=0$). This will give us the possibility to appreciate exactly the traveltime differences of S-waves (δT_S) due to the change of the structure (δf^2). Then, for each of the isotropic models (corresponding to a specified gradient c), we shall consider the same anisotropy perturbations δA_{ijkl} and compute the traveltimes of the two split quasi-shear waves, including their traveltime delay. Thus, for each P-velocity gradient c, the perturbed model will be specified by the relation

$$a_{ijkl}(z)=f^{02}(z)A_{ijkl}=(\alpha_0+cz)^2A_{ijkl}, \tag{35}$$

with $A_{ijkl}=A^0_{ijkl}+\delta A_{ijkl}$. We shall show that the traveltimes of individual split quasi-shear waves $qS1$ and $qS2$ will change considerably with a change of c, but the time delay will not be so sensitive to the change of c.

In principle, it would be possible to consider arbitrary δA_{ijkl} to test this fact. However, just to be closer to reality, we shall consider a model close to that proposed by Shearer (1988) as a possible interpretation VSP data from the Geysers, California. A detailed description of the VSP experiment is contained in Majer *et al.* (1988), and a brief summary can be found in Shearer (1988). The values of the density normalized elastic tensor at the surface, obtained by the interpretation of the VSP data, are as follows: $a_{1111}=1.10$, $a_{2222}=1.94$, $a_{1212}=0.51$, $a_{2323}=0.68$ and $a_{1122}=0.37\,\mathrm{km^2\,s^{-2}}$. The model is hexagonal with a horizontal, (100) symmetry axis, situated in the saggital plane. In a two-suffix notation, the corresponding 6×6 matrix of density normalized elastic parameters $a_{ij}(z)$ at the surface $z=0$ is as follows:

$$a_{ij}(0)=\begin{pmatrix} 1.10 & 0.37 & 0.37 & 0 & 0 & 0 \\ & 1.94 & 0.58 & 0 & 0 & 0 \\ & & 1.94 & 0 & 0 & 0 \\ & & & 0.68 & 0 & 0 \\ & & & & 0.51 & 0 \\ & & & & & 0.51 \end{pmatrix}. \tag{36}$$

The background model is specified by the following surface values of individual parameters: $\alpha_0=1.23\,\mathrm{km\,s^{-1}}$, $\beta_0=0.73\,\mathrm{km\,s^{-1}}$, $\rho_0=2.5\,\mathrm{Mg\,m^{-3}}$. Note that $v^2=\beta^2/\alpha^2=\beta_0^2/\alpha_0^2=0.35$ throughout the model. Then, for the dimensionless reduced anisotropic constants A^0_{ij}, we obtain from (34)

$$A^0_{ij}=\begin{pmatrix} 1 & 0.30 & 0.30 & 0 & 0 & 0 \\ & 1 & 0.30 & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & 0.35 & 0 & 0 \\ & & & & 0.35 & 0 \\ & & & & & 0.35 \end{pmatrix}. \tag{37}$$

Using $\alpha_0^2=1.51\,\mathrm{km^2\,s^{-2}}$, and the relation $A_{ij}=\alpha_0^{-2}a_{ij}(0)$, see (35), we obtain from (36)

$$A_{ij}=\begin{pmatrix} 0.73 & 0.24 & 0.24 & 0 & 0 & 0 \\ & 1.28 & 0.38 & 0 & 0 & 0 \\ & & 1.28 & 0 & 0 & 0 \\ & & & 0.45 & 0 & 0 \\ & & & & 0.34 & 0 \\ & & & & & 0.34 \end{pmatrix}. \tag{38}$$

The anisotropy perturbations $\delta A_{ij}=A_{ij}-A^0_{ij}$ are as follows:

$$\delta A_{ij}=\begin{pmatrix} -0.27 & -0.06 & -0.06 & 0 & 0 & 0 \\ & 0.28 & 0.08 & 0 & 0 & 0 \\ & & 0.28 & 0 & 0 & 0 \\ & & & 0.10 & 0 & 0 \\ & & & & -0.01 & 0 \\ & & & & & -0.01 \end{pmatrix}. \tag{39}$$

The maximum anisotropy perturbations are rather high, but they do not exceed 30 per cent. In this paper, however, we are interested only in some general properties, not actual inversions. For this reason, we shall use this model, even though the perturbations are large. Many other examples for different anisotropy models will be presented in Simões-Filho & Červený (1991).

We shall perform here computations for five different models of media, with different gradients. The first model corresponds to a homogeneous medium ($c=0\,\mathrm{s^{-1}}$), and the other four models to gradients of velocity $c=0.5$, 1.0, 1.5 and $2.0\,\mathrm{s^{-1}}$. The relevant gradients of S-velocity are $vc=0.0$, 0.30, 0.59, 0.89 and $1.18\,\mathrm{s^{-1}}$.

The results of computations for the five different gradients are shown in Figs 1–5. We use a different source–receiver configuration than in the VSP experiments (source at depth, receivers along the surface), but this has no effect on our conclusions. Each figure shows the ray diagram of the shear wave propagating in an isotropic medium with a relevant gradient. The plotted rays correspond to the 'frozen rays L_0'. Along these rays, the traveltime perturbations due to anisotropy perturbations δA_{ijkl} [given by (39)] were computed using (25), with $\delta f(x_i)=0$ and with C_{ij} given by (31). In addition to the ray diagram, the reduced traveltimes of S-waves in the isotropic medium (symbol '+') and of the two qS-waves in a relevant perturbed anisotropic inhomogeneous medium (symbol '×') are shown.

The presented pictures allow us to make some conclusions related to the behaviour of traveltimes of individual waves.

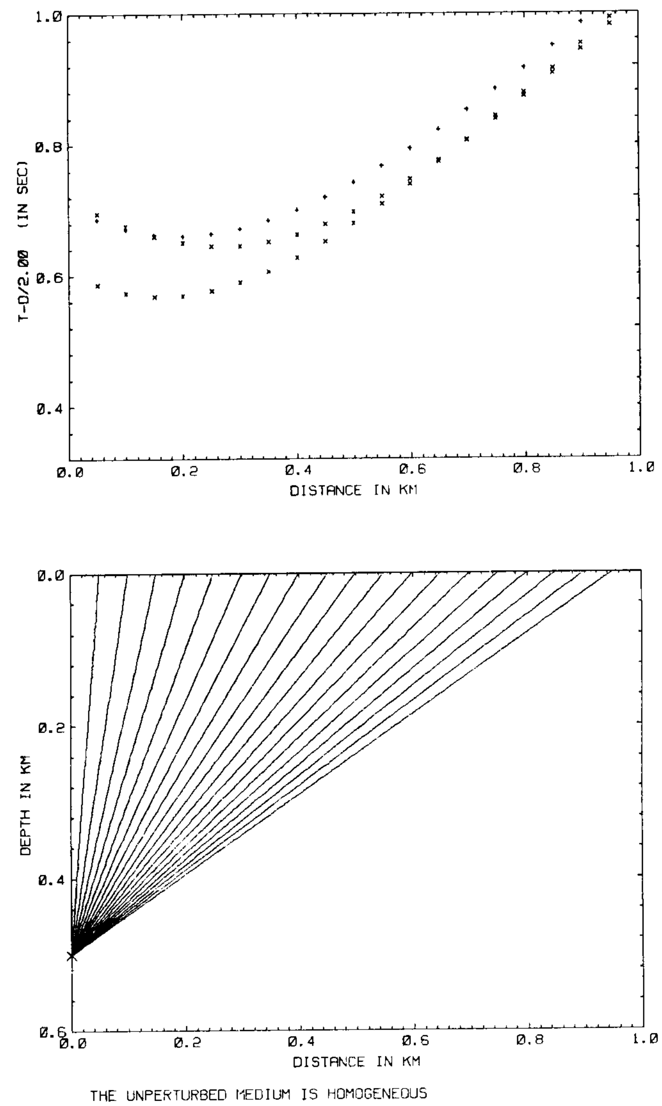

Figure 1. Bottom: the ray diagram of shear waves propagating in a homogeneous isotropic background medium with the shear velocity $\beta_0 = 0.73\ \mathrm{km\,s^{-1}}$ $(\nu = \beta/\alpha = 0.59)$. Top: corresponding reduced traveltimes of S-waves in the background medium (denoted by +) and the reduced traveltimes of two qS-waves in the relevant perturbed anisotropic medium (denoted by ×). The reduced velocity is $2.0\ \mathrm{km\,s^{-1}}$. The traveltimes of qS-waves are calculated by perturbation methods. The elastic parameters of the perturbed anisotropic medium are specified by equation (36).

As we can see, the traveltimes of S-waves and of both qS-waves depend considerably on the gradient of velocity. In other words, we can say that they depend considerably on structural perturbations. The time delay between the two split qS-waves, however, is not so sensitive to the changes of the structure.

As we wish to see these properties in a more objective way, we shall present here Tables 1 and 2 corresponding to minimum epicentral distance of 50 m and the maximum epicentral distance of 950 m. The tables show the traveltimes of S-waves in the isotropic background medium and both $qS1$- and $qS2$-waves in the anisotropic perturbed medium, for the five gradients of velocity under consideration. In addition, the time delay between the two qS-waves is also shown. All traveltimes and time delays are given in miliseconds. In addition to the absolute traveltimes, the relative traveltimes are also presented. The absolute traveltimes have a clear physical meaning. The relative traveltimes of S-, $qS1$-, and $qS2$-waves are related to the background homogeneous isotropic medium (gradient $0\ \mathrm{s^{-1}}$). In other words, the quantities 691 and 1475 ms are subtracted from the absolute traveltimes in Tables 1 and 2, respectively. Thus, the relative traveltimes may be considered as traveltime perturbations δT caused by the structural perturbations $\delta f^2(x_i)$, for a homogeneous

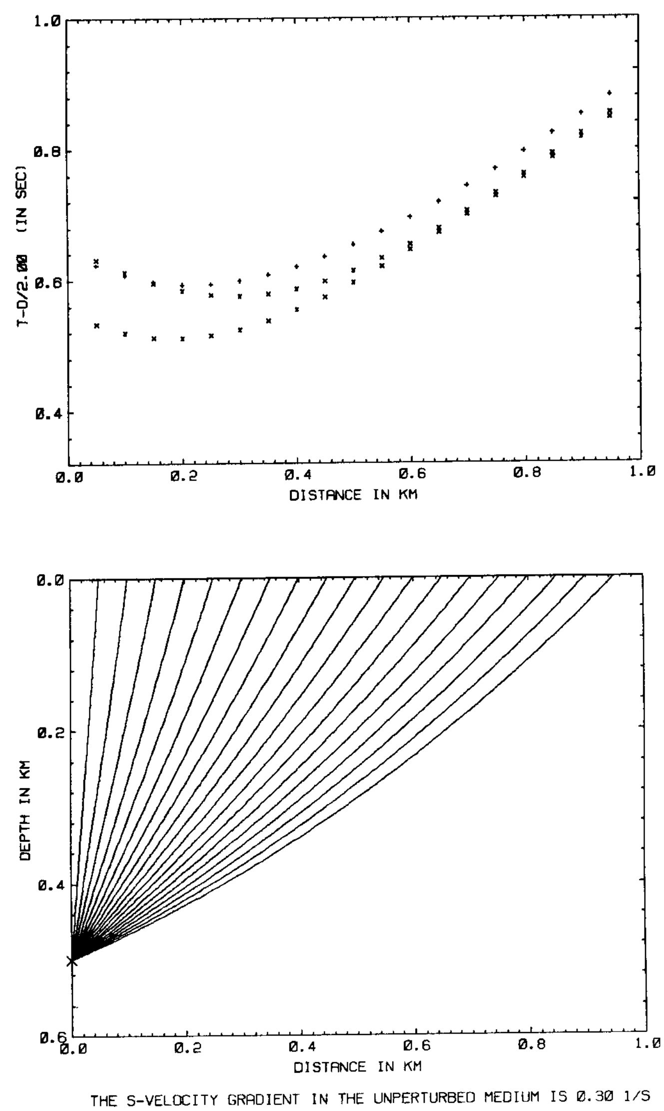

Figure 2. Bottom: the ray diagram of shear waves propagating in an isotropic vertically inhomogeneous background medium with the surface S velocity $\beta_0 = 0.73\ \mathrm{km\,s^{-1}}$ and a S velocity gradient of $0.30\ \mathrm{s^{-1}}$. The ratio $\nu = \beta/\alpha$ is constant throughout the model, $\nu = 0.59$. Top: corresponding reduced traveltimes of S-waves in the background medium (denoted by +) and the reduced traveltimes of two qS-waves in the relevant perturbed anisotropic medium (denoted by ×). The reduced velocity is $2.0\ \mathrm{km\,s^{-1}}$. The traveltimes of qS-waves are calculated by perturbation methods. The surface elastic parameters of the perturbed anisotropic medium are specified by equation (36).

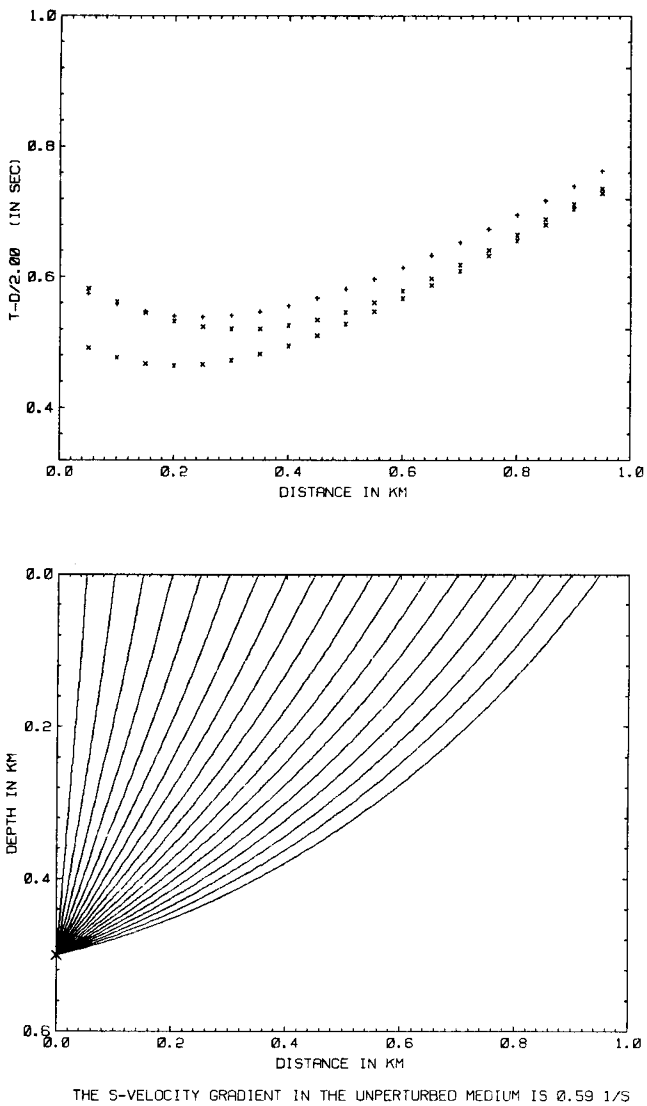

Figure 3. As Fig. 2, for an S velocity gradient $vc = 0.59\,\mathrm{s}^{-1}$.

isotropic background. Similar quantities are also presented for the time delay; the only difference is that the relative value of time delay is taken with respect to the homogeneous *anisotropic* medium. In other words, the quantities 108 and 10 ms are subtracted from the absolute traveltime delays in Tables 1 and 2, respectively. Thus, the relative time delays show us the changes of the time delay with a change of the structure (changes of velocity gradient). The reason why we select two very different epicentral distances in Tables 1 and 2 is as follows: as we can see in Figs 1–5, the rays at small epicentral distances are not greatly influenced by the gradient of velocity, but they are considerably more influenced by it at large epicentral distances. Moreover, the time delay between the two quasi-shear waves is relatively high at small epicentral distances, but considerably smaller at large epicentral distances. In Tables 1 and 2 we wish to consider two epicentral distances with such great differences in ray and traveltime behaviours.

Tables 1 and 2 offer the following two interesting observations.

(1) The traveltimes of individual qS-waves are considerably more influenced by the structural perturbations than by the anisotropy perturbations. Just the opposite is true for the delay time between the two qS-waves. It is considerably more influenced by the anisotropy perturbations than by the structural perturbations.

(2) The influence of the structural perturbations on the time delay between the two quasi-shear waves is at least one order of magnitude smaller than its influence on the traveltimes of individual qS-waves.

These are the numerical observations which support the result derived theoretically in Section 4. A more detailed numerical investigation will be presented in Simões-Filho & Červený (1991).

7 CONCLUSIONS

The traveltimes of qP-, $qS1$-, and $qS2$-waves propagating in inhomogenous anisotropic media are strongly influenced not only by local anisotropic properties at some points of the medium (source, receiver, etc.), but also by their spatial

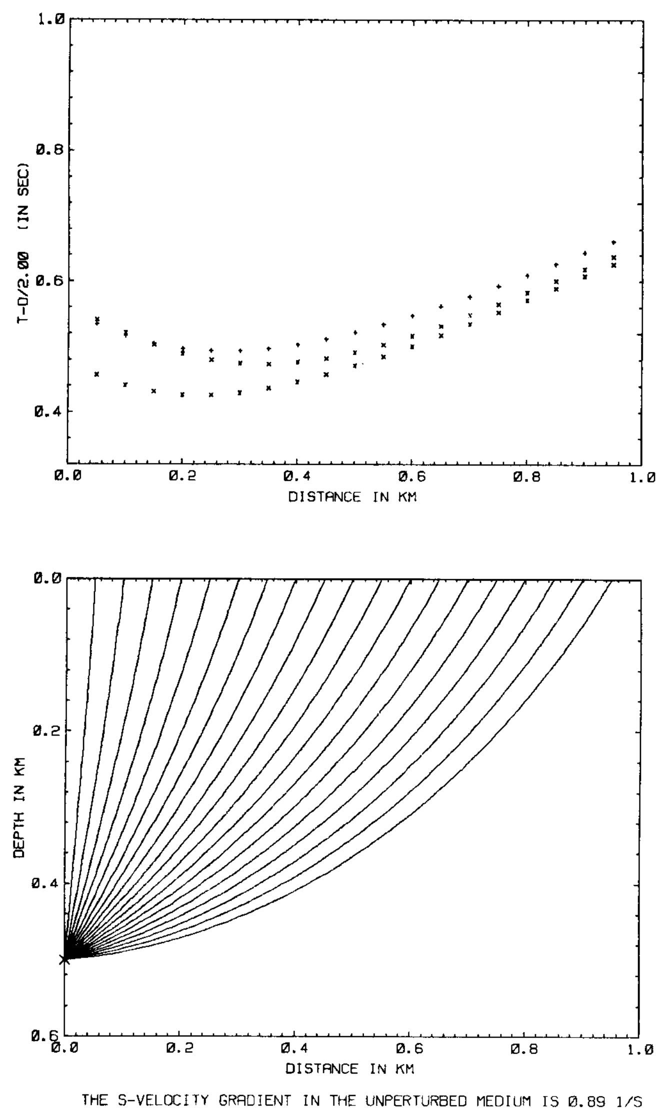

Figure 4. As Fig. 2, for an S velocity gradient $vc = 0.89\,\mathrm{s}^{-1}$.

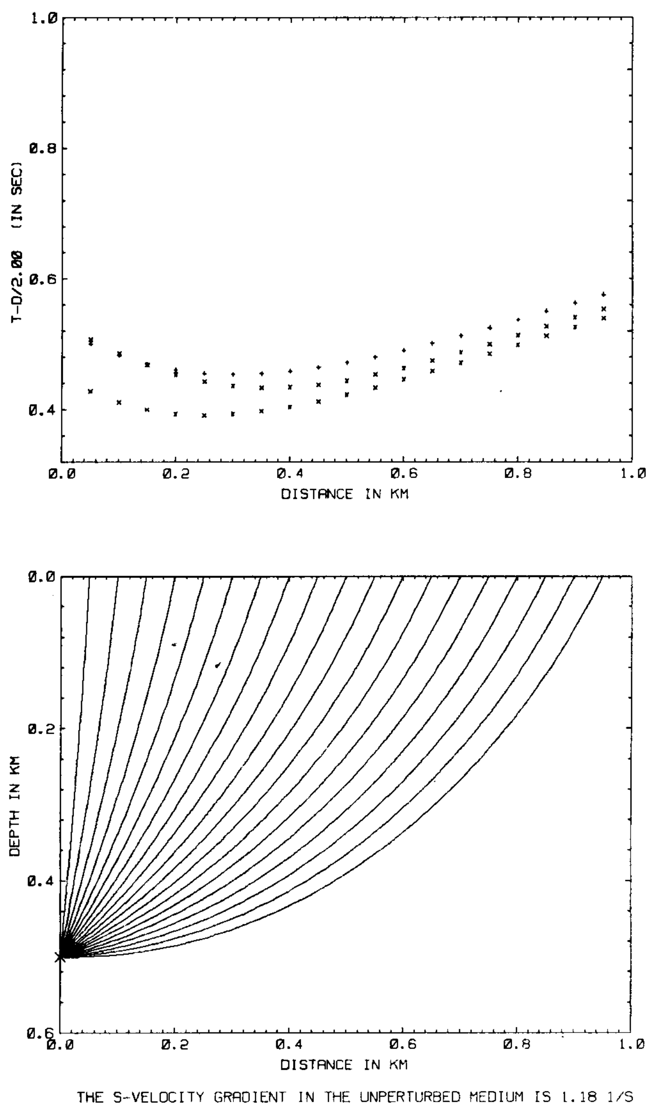

Figure 5. As Fig. 2, for an S velocity gradient $vc = 1.18\,s^{-1}$.

vatiations. This is the main reason why the kinematic inversion of observed traveltimes in terms of anisotropic properties does not usually give reliable results. Roughly speaking, a good fit of observed and computed traveltimes can be always obtained either by structural changes of the model or by changing the anisotropic properties of the model. In other words, the traveltime perturbations of these waves depend both on structural perturbations and anisotropy perturbations. The words 'structural perturbations' and 'anisotropy perturbations', however, do not have a specific meaning in general anisotropic inhomogeneous media. They are strictly defined only if we consider the factorized anisotropic inhomogeneous (FAI) medium; see Chapter 3 of this paper.

In the framework of the first-order perturbation theory, there is one exception from the above general rule. This exception applies to the time delay between the two quasi-shear waves. Under certain conditions, the perturbations of the time delay *do not depend on the structural perturbations*. There are two such conditions:

(a) the background (unperturbed) medium is isotropic, possibly inhomogeneous, with a constant ratio of S- and P-velocities throughout the whole model ($v = \beta/\alpha =$ constant); and

(b) the perturbed anisotropic inhomogeneous medium represents a factorized anisotropic inhomogeneous medium [in which all the density normalized elastic parameters depend on coordinates in the same way, see equation (1)].

If the two above conditions are satisfied, the perturbation of the time delay depends only on the perturbation of several constants (which do not depend on coordinates), specifying an effective anisotropy in the whole medium under study. Thus, the information is not local, but global; the local anisotropy properties may deviate from these averaged characteristics.

It is quite obvious that the most important information about the anisotropic properties of the medium can be obtained from the three-component records of shear wave splitting, particularly from the polarization analysis of these records. As discussed in the Introduction, the polarization diagrams are commonly used to study the polarization of the first quasi-shear wave ($qS1$-wave), and to determine the time delay. The polarization of the $qS1$-wave can be used to obtain information on local anisotropic properties close to the receiver. On contrary, the time delay offers global information on anisotropy along the whole ray.

The authors propose to interpret the time delay between the two qS-waves in the framework of the factorized

Table 1. Traveltimes of S waves in the isotropic background, and of $qS1$- and $qS2$-waves in the anisotropic perturbed medium, and the delay time between the two qS-waves, in miliseconds, at an epicentral distance of 50 m. Five models with a constant gradient of S velocity are considered. The relative traveltimes of S-, $qS1$- and $qS2$-waves are related to the homogeneous *isotropic* background medium (zero gradient of velocity). The relative time delay is related to the perturbed homogeneous *anisotropic* medium.

	S		qS1		qS2		qS2 - qS1	
S-velocity gradient (s^{-1})	absolute	relative	absolute	relative	absolute	relative	absolute	relative
0.0	691	–	592	−99	700	−21	108	–
0.30	629	−62	539	−152	637	−54	98	−10
0.59	579	−112	496	−195	587	−104	91	−17
0.89	539	−152	462	−229	546	−145	84	−24
1.18	505	−186	433	−258	512	−179	79	−29

Table 2. As Table 1, for the epicentral distance of 950 m.

S-velocity gradient (s^{-1})	S		qS1		qS2		qS2 - qS1	
	absolute	relative	absolute	relative	absolute	relative	absolute	relative
0.0	1475	–	1437	−38	1447	−28	10	–
0.30	1336	−139	1302	−173	1309	−166	7	−3
0.59	1217	−258	1184	−291	1191	−284	7	−3
0.89	1117	−358	1082	−393	1093	−382	11	1
1.18	1031	−444	995	−480	1009	−466	14	4

anisotropic inhomogeneous medium, expressed in terms of the two conditions (a) and (b) specified above. Such an inversion procedure will be very robust and will be considerably simpler than standard inversion procedures, as it will completely remove the structural perturbations from the inversion. The authors believe that the information obtained will be valuable even if the actual medium is not strictly factorizable, and will be considerably more reliable than any information obtained by a method which mixes structural and anisotropy perturbations.

In any case, after the FAI inversion, the obtained FAI medium can be used as a background and standard perturbation techniques for inhomogeneous anisotropic medium can be applied to obtain more detailed information on anisotropy and structure. Such an inversion, however, will require a considerably larger amount of traveltime data. Perhaps the traveltime data will not be sufficient at all, and waveform inversion will be necessary.

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