Smoothing 2-D model HESS for Kirchhoff migrations

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Summary

This is an example how to fit the gridded velocities and the gridded interfaces by a model with interfaces, suitable for ray tracing.

We take the gridded velocities and interfaces of the data set 'hess.dat', provided by Amerada Hess, and build the 2-D model suitable for ray tracing.

1 Data and model parametrization

File 'hess.dat' has the form of a file with input data for the MODEL package. The dimensions of the 2-D model are 40200 times 21000 feet. Two interfaces in the model, limiting a salt body, are discretized on the grid of 64 points with spacing 200 feet. Background velocities are discretized on the grid of 202×106 points with spacing 200 feet $\times$ 200 feet, the velocity inside the salt body is constant. See Figure 1.

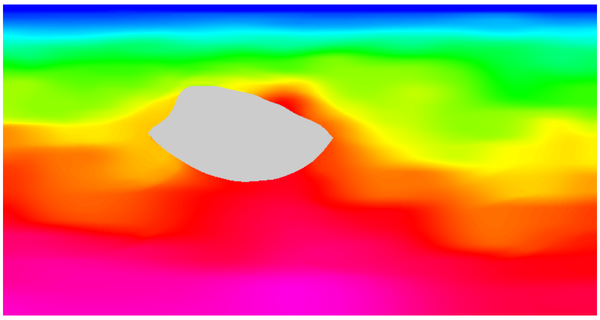

Figure 1: P-wave velocity in the original model (before smoothing). The background velocity is varying from 5000 feet s^{-1} at the top of the model to 9800 feet s^{-1} at the bottom. The velocity of the salt body is constant and equals to 14800 feet s^{-1}.

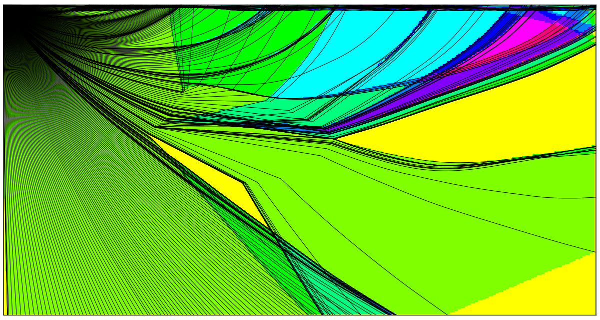

Figure 2: Numbers of travel times and rays computed for the refracted wave in the original model. The point source is located in the top left corner of the model. Yellow colour corresponds to no travel time, green to 2 arrivals, cyan to 4, blue to 6, magenta to 8, red to 10 arrivals.

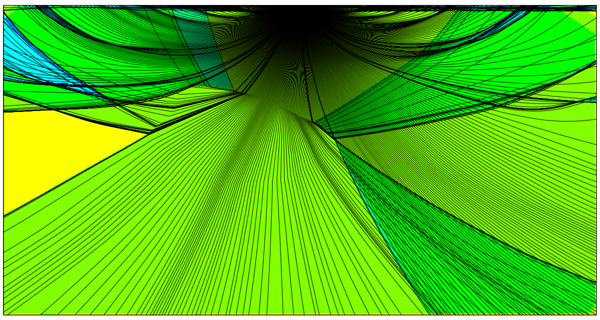

Figure 3: Numbers of travel times and rays computed for the source located in the middle of the top of the model. The colour scale is the same as in Fig. 2.

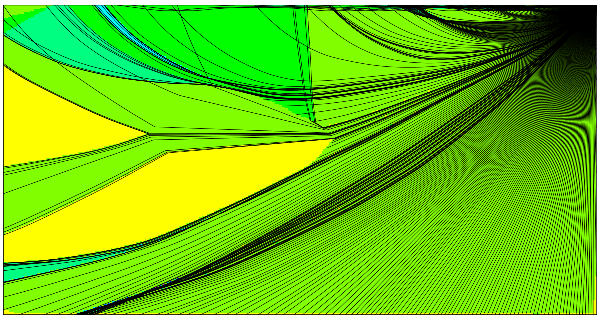

Figure 4: Numbers of travel times and rays computed for the source located in the top right corner of the model. The colour scale is the same as in Fig. 2.

In: Seismic Waves in Complex 3-D Structures, Report 10 (Department of Geophysics, Charles University, Prague, 2000), pp. 75–82

We perform several tests of ray tracing in the original model. The behaviour of the ray field seems to be too complicated with too many caustics, especially in a low velocity channel near the surface of the model, see Figures 2 to 4. We thus decide to smooth the model.

In Figure 1, we can see that the horizontal variation of the velocity field is less pronounced than the vertical variation, and that the vertical variation is greater in the shallow parts of the model than in its deeper parts. The interfaces are quite smooth, but the edges are sharp. According to these observations, we manually choose a new parametrization of the model. Let's emphasize, that the parametrization is based on the author's opinion, and is expected to be optimized during the inversion. The parametrization is as follows: for the interfaces we take irregular B–spline grid of 28 points with 26 points regularly spaced each 500 feet in the region of the salt body and 2 points at the sides of the model. For the background velocity, we take horizontally regular grid of 17 points with spacing 2500 feet and vertically irregular grid of 31 points with spacing from 200 feet at the top to 800 feet at the bottom. The parametrization may be found in file 'hes1-mod.dat', which is used as a starting model for the inversion.

For the interfaces we have the data grid of $2 \times 64 = 128$ points with spacing 200 feet and the irregular B–spline grid of $2 \times 28 = 56$ points. For the velocities we have the data grid of $202 \times 106 = 21412$ points with spacing 200 feet. The irregular B–spline grid of $17 \times 31 = 527$ points is used for the background. The velocity of the salt body is constant (1 B–spline gridpoint). The total number of data points is thus $128 + 21412 = 21540$ points, the total number of B–spline points is $56 + 527 + 1 = 584$ points. Program 'gmdmgmt.for' then requires $584 \times 21540 + 21540 + 584 \times 584 = 12941956$ storage locations in array RAM and we may perform the calculations with MRAM = 14000000, which is a value suitable for a computer with 64 Mbytes of memory.

2 Smoothing the model using the manually selected B–spline parametrization

History file 'hes-inv.h' controls the procedure of smoothing of the model HESS. It computes the quantities necessary for estimation of smoothing parameters, and performs the smoothing by means of inversion of the given data with minimization of Sobolev norms of the model.

2.1 Choosing the maximum Lyapunov exponent

To estimate the maximum sum of travel times from a source and a receiver, we compute the average slowness of the model $\bar{u}_{\mathrm{P}} = 0.120611\,\mathrm{E}{-3}\,\mathrm{s\,feet}^{-1}$. If we consider the maximum sum of ray paths about 50000 feet, we arrive at the estimation, that the maximum sum of travel times from a source and a receiver is limited by $T_{\max} = 6\,\mathrm{s}$. According to equation (4) of Klimeš (2000), we choose the maximum Lyapunov exponent λ of the new model as

$$\lambda \leq \frac{2}{T_{\max}} = 0.333\,\mathrm{s}^{-1} \quad . \tag{1}$$

The product of the maximum numbers of arrivals from a source and a receiver then should not exceed $\exp(2) \approx 7$.

2.2 Choosing the maximum Sobolev norm of the model

Taking equation (8) of Klimeš (2000) and making similar approximations, we arrive at the equation analogous to equation (12) of his paper,

$$\|v\| \leq \sqrt{8/3}\left(\frac{4(1+\ln(2))}{T_{\max}}\right)^2/\bar{v} \quad , \tag{2}$$

where 2-D Sobolev norm $\|v\|$ of the velocity field is given by file 'sob22.dat'. For the average velocity $\bar{v} = 8074.73\,\text{feet}\,\text{s}^{-1}$ of the background, we wish the maximum Sobolev norm of the velocity field in the model

$$\|v\|_{\max} = 2.6\,\text{E}{-4}\,(\text{feet}\,\text{s})^{-1} \quad . \tag{3}$$

In this example, we first try to apply the minimum reasonable smoothing of the interfaces. We thus do not prescribe the maximum Sobolev norm of the interfaces.

2.3 Inversion and smoothing

Objective function is taken in the form of

$$y = \left(\frac{\widehat{|x-x_0|}}{1\,\text{foot}}\right)^2 + (SOBMULx\|x\|)^2 + \left(\frac{\widehat{|v-v_0|}}{1\,\text{foot}\,\text{s}^{-1}}\right)^2 + (SOBMULv\|v\|)^2 \quad , \tag{4}$$

where $\widehat{|x-x_0|}$ is the standard deviation of the position of the interfaces in the smooth model from the gridded interfaces in the original model 'hess.dat', $\|x\|$ is the 2-D Sobolev norm of the curvature of the interfaces in the smoothed model given by file 'sob22.dat', $\widehat{|v-v_0|}$ is the standard velocity deviation of the model from the original gridded velocities and $\|v\|$ is the 2-D Sobolev norm of the velocity field in the smoothed model given by file 'sob22.dat'. *SOBMULx* and *SOBMULv* are numerical parameters which control the smoothing during the inversion.

We use unit given velocity deviation and unit given interface deviation in the selected objective function. The unit deviations are realized by relevant selections of parameter *ERRMUL*, which should be chosen as a square root of the ratio of the whole model volume to the volume corresponding to a single data point of the considered data set.

The inversion of the interfaces and the inversion of the velocity field are mutually independent in this example. We can thus perform the inversion of the interfaces in the first step, and the inversion of the velocity in the second step.

2.3.1 Inversion of the interfaces

We choose $SOBMULv = 0\,\text{feet s}$, and perform the inversion without smoothing of the velocity, which is not of our interest at the moment.

In this example, we try to apply the minimum reasonable smoothing of the interfaces. We thus start the inversion without smoothing of the interfaces, with $SOBMULx = 0\,\text{feet}$ (the interfaces are smoothed just by the projection onto the B–splines). We obtain oscillations of both surfaces, which produce other salt body than that one given by data. In the next steps of inversion we thus include minimization of the curvature of the interfaces by increasing the value of $SOBMULx$. When we reach approximately the value of $SOBMULx = 2500\,\text{feet}$, the spurious salt body moves outside the model volume. We thus consider the value of $SOBMULx = 2500\,\text{feet}$ as the value corresponding to the minimum reasonable smoothing of the interfaces. Whether the interfaces are sufficiently smooth will be tested by ray tracing.

We compute standard interfaces deviation $\widehat{|x - x_0|} = 5.6\,\text{feet}$, stored in the file 'hes-sd1.out'. This deviation causes a travel time error dT approximately

$$dT = \widehat{|x - x_0|} \left| \frac{1}{\bar{v}} - \frac{1}{\bar{v}_{\text{salt}}} \right| \quad . \tag{5}$$

For the average velocity $\bar{v} = 8074.73\,\text{feet s}^{-1}$ of the background and the velocity of the salt $\bar{v}_{\text{salt}} = 14800\,\text{feet s}^{-1}$, we estimate the standard travel time deviation caused by the deviation of the interfaces $dT = 3.15\,\text{E}{-4}\,\text{s}$.

2.3.2 Inversion of the velocity field

We have selected the value of $SOBMULx = 2500\,\text{feet}$, which ensures proper position of the interfaces. We will keep the value during the velocity inversion.

(a) First iteration

We first perform the inversion with $SOBMULv = 0\,\text{feet s}$ (i.e., without smoothing), and obtain the standard velocity deviation of the model

$$\widehat{|v - v_0|} = 13.94\,\text{feet s}^{-1} \quad , \tag{6}$$

stored in file 'hes-vd1.out'. The Sobolev norm of the velocity field smoothed just by the projection onto the B–splines

$$\|v\| = 37\,\text{E}{-4}\,(\text{feet s})^{-1} \quad , \tag{7}$$

stored in file 'hes-vn1.out', is large, see equation (3).

(b) Second iteration

We choose

$$SOBMULv = \frac{\widehat{|v - v_0|}}{1\,\text{foot s}^{-1}\ \|v\|_{\max}} \quad , \tag{8}$$

as in the paper by Klimeš (2000, eq. 20). Then

$$SOBMULv = 13.94/2.6\,\text{E}{-4}\,\text{feet s} = 53615\,\text{feet s} \quad , \tag{9}$$

and we enter the rounded value of

$$SOBMULv = 50000\,\text{feet s} \quad , \tag{10}$$

for the second iteration. The resulting Sobolev norm

$$\|v\| = 2.9\,\text{E}{-4}\,(\text{feet s})^{-1} \quad , \tag{11}$$

of the velocity looks acceptable, see equation (3).

(c) Subsequent iterations

The presence of the structural interfaces and the existence of the low velocity channel at the top of the model have not been taken into account during the estimation of *SOBMULv*. We thus choose several values of *SOBMULv* for subsequent iterations in order to see the behaviour of the model in dependance on *SOBMULv*. After each inversion we perform several tests, which are described below in Chapter 4, and we compute for each model the value of average Lyapunov exponent $\bar{\lambda}$ and the maximum number of arrivals $NUM_{\max}$ from the source located in the left top corner of the model. We performed the following tests of *SOBMULv*:

model	$\widehat{\lvert v - v_0 \rvert} / (\text{ feet s}^{-1})$	$\|v\| / ((\text{feet s})^{-1})$	$\bar{\lambda} / (\text{ s}^{-1})$	$NUM_{\max}$
desired values		2.6 E−4	0.333	7
model 'hess.dat'			0.423	11
$SOBMULv = 0$	13.94	37.0 E−4	0.380	7
$SOBMULv = 17500$	15.19	4.9 E−4	0.345	6
$SOBMULv = 35000$	17.41	3.5 E−4	0.316	5
$SOBMULv = 50000$	19.24	2.9 E−4	0.300	5
$SOBMULv = 70000$	21.42	2.4 E−4	0.287	5

We can see, that for $SOBMULv = 0$ feet s we obtain the standard velocity deviation about 14 feet s^{-1}. Desired value of $\bar{\lambda} = 0.333\,\text{s}^{-1}$ corresponds to the velocity deviation about 16 feet s^{-1}. Most of the velocity deviation is thus caused by the projection onto the B–splines, and we should try the inversion with a finer parametrization of the model.

A figure of Fourier spectrum of velocity deviation should help us to decide, in which direction and how many times we should densify the parametrization of the model. We must keep in mind, that the parametrization should not be denser than the data.

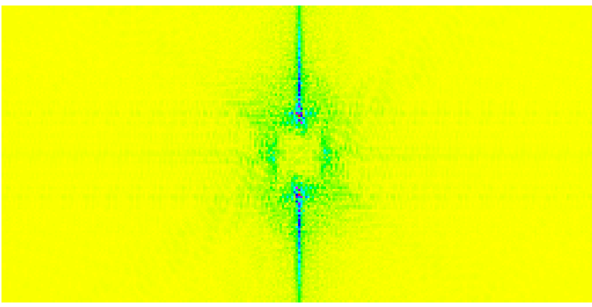

Figure 5: Fourier spectrum of velocity deviation of the smoothed model obtained with $SOBMULv =$ 50000 feet s.

3 Finer B–spline parametrization of the model

We choose the new parametrization of the model as follows: for the interfaces we keep the irregular B–spline grid of 28 points; for the background velocity we take horizontally regular grid of 26 points with spacing 1600 feet and vertically irregular grid of 58 points with spacing from 200 to 400 feet. The parametrization may be found in file 'hes2-mod.dat', which is used as a new starting model for the inversion.

We have now $56+1+26\times58 = 1565$ B–spline points, and the memory requirements of program 'gmdmgmt.for' are $1565 \times 21540 + 21540 + 1565 \times 1565 = 36180865$ storage locations. This is too much for our computer and we thus cannot use all the data. We resample the data in the horizontal direction and use the data grid of 67 points with spacing 600 feet horizontally and the original 106 points with spacing 200 feet vertically. We have now $67 \times 106 + 2 \times 64 = 7230$ data points and the memory requirements of program 'gmdmgmt.for' are $1565 \times 7230 + 7230 + 1565 \times 1565 = 13771405$ storage locations in array RAM and we may perform the calculations with MRAM = 14000000, which is a value suitable for a computer with 64 Mbytes of memory.

We performed the following tests of *SOBMULv* with the new starting model 'hes2-mod.dat':

model	$\widehat{\lvert v - v_0 \rvert}/(\mathrm{feet\,s^{-1}})$	$\lVert v \rVert/((\mathrm{feet\,s})^{-1})$	$\bar{\lambda}/(\mathrm{s^{-1}})$	$NUM_{\max}$
desired values		2.6 E−4	0.333	7
model 'hess.dat'			0.423	11
$SOBMULv = 1000$	5.52	12.3 E−4	0.398	6
$SOBMULv = 10000$	7.28	6.5 E−4	0.365	5
$SOBMULv = 17500$	9.03	5.2 E−4	0.343	5
$SOBMULv = 25000$	10.68	4.4 E−4	0.327	5
$SOBMULv = 35000$	12.75	3.7 E−4	0.313	5

Note that with the new parametrization we are not able to perform the inversion with $SOBMULv = 0$ feet s, because we have now more background velocity B–spline gridpoints in the salt body, and the inversion becomes ill-conditioned.

4 Numerical tests of the smoothed model

History file 'hes-test.h' serves for numerical tests of model 'hes-mod.out' obtained after the inversion realized by history file 'hes-inv.h'. The figures displayed below are obtained by running 'hes-inv.h' with starting model 'hes2-mod.dat' (the finer parametrization) and the values of $SOBMULx = 2500$ feet and $SOBMULv = 25000$ feet s.

4.1 Statistical properties of the model

We discretize the velocity in the smoothed model 'hes-mod.out'. We calculate the gridded velocity deviation from the velocity in the original model 'hess.dat', the gridded relative velocity deviation, the standard velocity deviation and the standard relative velocity deviation, the Fourier spectrum of the gridded velocity deviation, the gridded deviations of the interfaces, the gridded relative deviations of the interfaces, and the standard deviation of the interfaces and the standard relative deviation of the interfaces.

The standard velocity deviation stored in file 'hes-vd2.out' is 10.68 feet $\mathrm{s^{-1}}$. The standard relative velocity deviation stored in file 'hes-vr2.out' is 0.18 % (i.e. 1.8 per

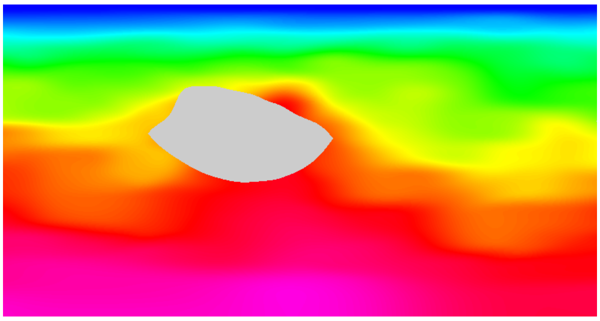

Figure 6: P-wave velocity in the smoothed model 'hes-mod.out'.

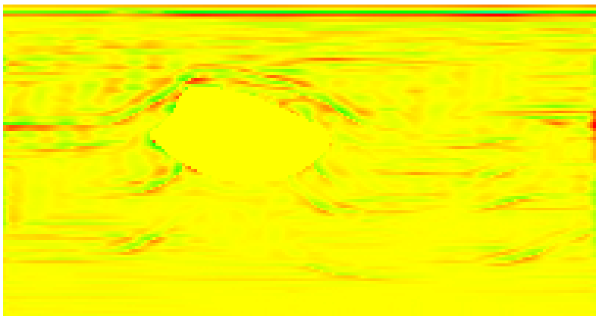

Figure 7: Deviation of the gridded velocity in the smoothed model from the gridded velocity in the original model.
Standard velocity deviation is 10.68 feet s^{-1}.

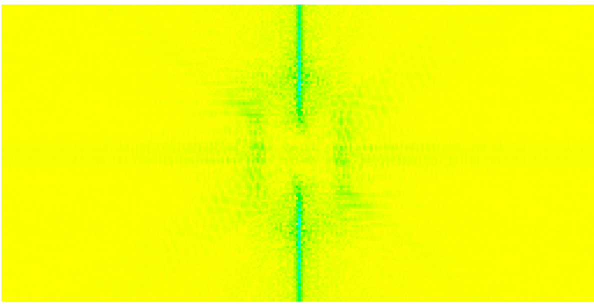

Figure 8: Fourier spectrum of the velocity deviation from Figure 7.

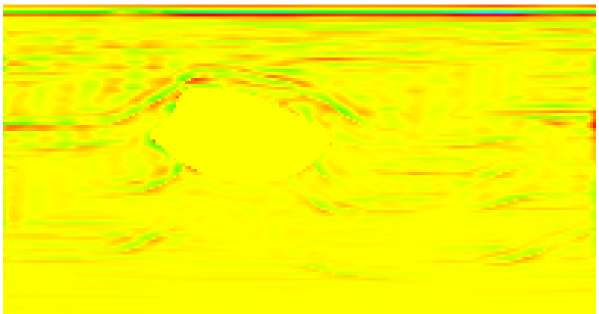

Figure 9: Gridded relative velocity deviation.
Standard relative velocity deviation is 0.18 %.

mille). The standard deviation of the interfaces stored in file 'hes-sd2.out' is 5.6 feet. The standard relative deviation of the interfaces stored in file 'hes-sr2.out' is 0.08 %.

4.2 Average Lyapunov exponent for the model

Directional Lyapunov exponents and the average Lyapunov exponent for the 2-D model, calculated by program 'modle2d.for', are shown in Figures 10 and 11.

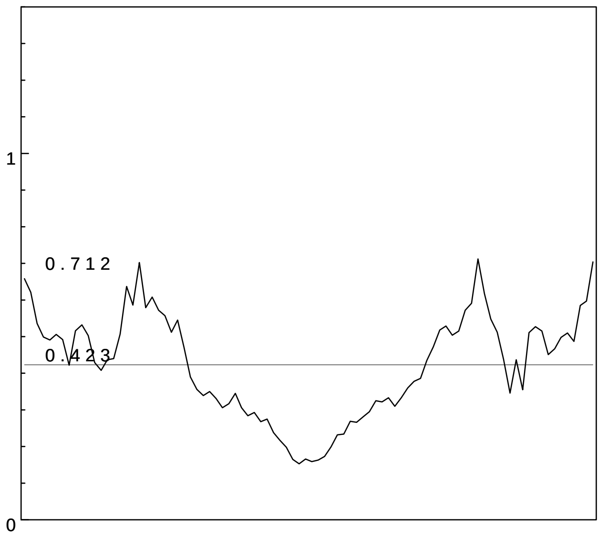

Figure 10: Angular dependence of directional Lyapunov exponents and the average Lyapunov exponent $\bar{\lambda} = 0.423\,s^{-1}$ for the original model.

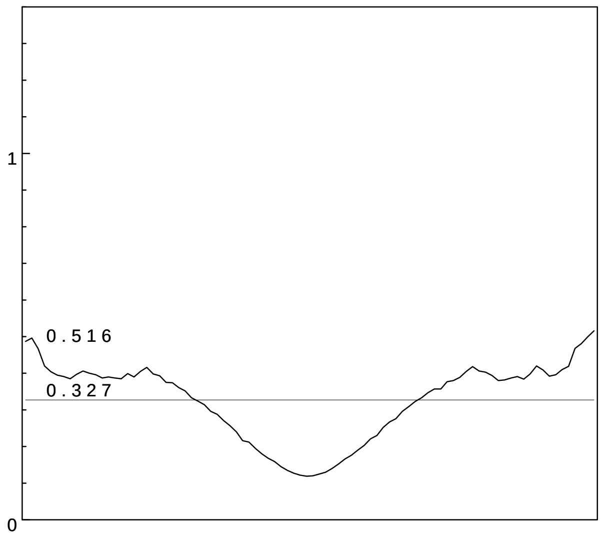

Figure 11: Angular dependence of directional Lyapunov exponents and the average Lyapunov exponent $\bar{\lambda} = 0.327\,s^{-1}$ for the smoothed model.

4.3 Tests of ray tracing

Three tests of ray tracing of a refracted wave followed by interpolation between rays are performed using programs 'crt.for' and 'mtt.for'. The three sources are located at the top of the model in the corners and in the middle of the model.

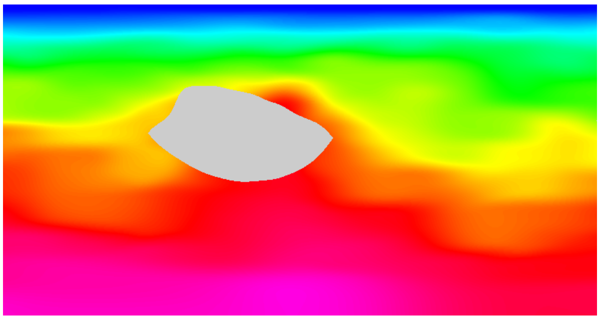

Figure 6 (repeated): Velocity in the smoothed model 'hes-mod.out'.

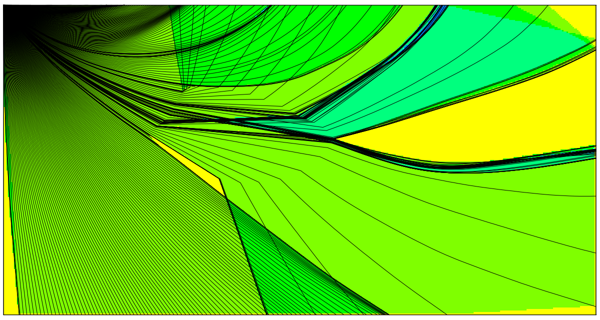

Figure 12: Numbers of travel times and traced rays computed for the source located in the left top corner of the model. The colour scale is the same as in Fig. 2.

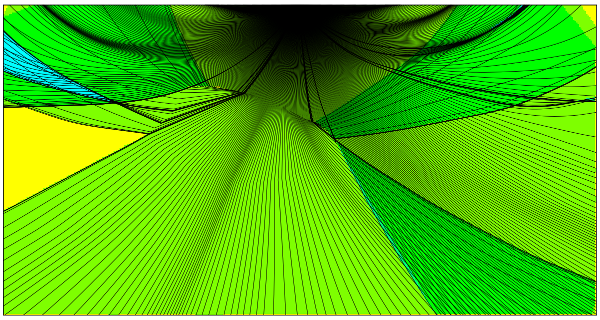

Figure 13: Numbers of travel times and traced rays computed for the refracted wave in the smoothed model. The point source is located in the middle of the top of the model. The colour scale is the same as in Fig. 2.

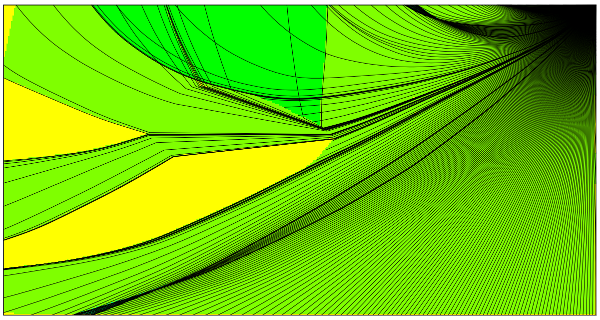

Figure 14: Numbers of travel times and traced rays computed for the source located in the top right corner of the model. The colour scale is the same as in Fig. 2.

References

Klimeš, L. (1999): Lyapunov exponents for 2-D ray tracing without interfaces. In: Seismic Waves in Complex 3-D Structures, Report 8, pp. 83–96, Dep. Geophys., Charles Univ., Prague.

Klimeš, L. (2000): Smoothing the Marmousi model for Gaussian-packet migrations. In: Seismic Waves in Complex 3-D Structures, Report 10, pp. 63–74, Dep. Geophys., Charles Univ., Prague.