Anisotropic common ray approximation of the coupling ray theory

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Summary

The common ray approximation considerably simplifies the numerical algorithm of the coupling ray theory for S waves, but may introduce errors in travel times due to the perturbation from the common reference ray. At present, the common reference rays are routinely represented by the isotropic common rays calculated in the isotropic reference model, but they would better be represented by the anisotropic common rays calculated in the anisotropic model. The errors due to the anisotropic common ray approximation of the coupling ray theory are usually considerably smaller than the errors due to the routinely used isotropic common ray approximation of the coupling ray theory.

We numerically calculate the travel–time errors due to the anisotropic common ray approximation in three 1-D models of differing degree of anisotropy, and compare them with the errors due to the isotropic common ray approximation. The differences between the isotropic and anisotropic common ray approximations of the coupling ray theory are demonstrated on synthetic seismograms.

Keywords

Coupling ray theory, common ray approximation, travel time, seismic anisotropy, inhomogeneous media.

1. Introduction

There are two different high–frequency asymptotic ray theories: the *isotropic ray theory* assuming equal velocities of both S–wave polarizations and the *anisotropic ray theory* assuming both S–wave polarizations strictly decoupled. In the isotropic ray theory, the S–wave polarization vectors do not rotate about the ray, whereas in the anisotropic ray theory they coincide with the eigenvectors of the Christoffel matrix which may rotate rapidly about the ray. Thomson, Kendall & Guest (1992) demonstrated analytically that the high–frequency asymptotic error of the anisotropic ray theory is inversely proportional to the second or higher root of the frequency if a ray passes through the point of equal S–wave eigenvalues of the Christoffel matrix.

In "weakly anisotropic" models, at moderate frequencies, the S–wave polarization tends to remain unrotated round the ray but is partly attracted by the rotation of the eigenvectors of the Christoffel matrix. The intensity of the attraction increases with frequency. This behaviour of the S–wave polarization is described by the *coupling ray theory* proposed by Coates & Chapman (1990). The coupling ray theory is applicable at all degrees of anisotropy, from isotropic models to considerably anisotropic ones. The frequency–dependent coupling ray theory is the generalization of both the zero–order

In: Seismic Waves in Complex 3-D Structures, Report 14 (Department of Geophysics, Charles University, Prague, 2004), pp. 107–122

isotropic and anisotropic ray theories and provides continuous transition between them. The numerical algorithm for calculating the frequency–dependent complex–valued S–wave polarization vectors of the coupling ray theory has been designed by Bulant & Klimeš (2002).

There are many commonly used *quasi–isotropic approximations* of the coupling ray theory (Bulant & Klimeš, 2004), which diminish the accuracy of the coupling ray theory both with increasing frequency and increasing degree of anisotropy. For example, the reference ray may be calculated in different ways (Bakker, 2002; Klimeš & Bulant, 2004), the Christoffel matrix may be approximated by its quasi–isotropic projections onto the plane perpendicular to the reference ray and onto the line tangent to the reference ray (Pšenčík, 1998a), travel times corresponding to the anisotropic ray theory may be approximated in several ways, e.g., by linear quasi–isotropic perturbation with respect to the density–normalized elastic moduli (Pšenčík, 1998a), etc. Most of these quasi–isotropic approximations can be avoided with minimal effort (Bulant & Klimeš, 2002; 2004), except for the *common ray approximation* for S waves.

In the common ray approximation, only one reference ray is traced for both anisotropic–ray-theory S waves, and both S–wave anisotropic–ray-theory travel times are approximated by the first–order perturbation expansion from the common reference ray. The common ray approximation thus considerably simplifies the coding of the coupling ray theory and numerical calculations, but may introduce errors in travel times due to the perturbation. At present, the common reference rays are routinely represented by the *isotropic common rays* calculated in the isotropic reference model, but they would better be represented by the *anisotropic common rays*, proposed by Bakker (2002). The anisotropic common rays are traced in the anisotropic model, using the averaged Hamiltonian of both S–wave polarizations. The algorithm of the dynamic ray tracing corresponding to the anisotropic common rays was proposed by Klimeš (2003).

The effects of the routinely used *isotropic common ray approximation* and of the more accurate *anisotropic common ray approximation* have been studied by Klimeš & Bulant (2004). For more detailed description of the common ray approximations of the coupling ray theory refer to Section 1.1

In this paper, we numerically calculate the travel–time errors due to the anisotropic common ray approximation in three 1-D models of differing degree of anisotropy, and compare them with the errors due to the isotropic common ray approximation. The differences between the isotropic and anisotropic common ray approximations of the coupling ray theory are demonstrated on synthetic seismograms.

1.1 Common ray approximations

The isotropic ray theory is always the limiting case of the coupling ray theory for decreasing anisotropy at a fixed frequency. On the other hand, the high–frequency limit of the coupling ray theory at a fixed anisotropy depends on the choice of the reference ray, and even on the choice of the *system* of reference rays, because the amplitudes are determined by the paraxial reference rays.

From the point of view of the high–frequency asymptotic validity, the frequency–independent reference ray is best represented by the *anisotropic–ray–theory reference ray*, provided we choose the initial condition for the polarization vector in the coupling equation given by the eigenvector of the Christoffel matrix corresponding to the reference ray. The anisotropic–ray–theory travel time corresponding to the selected polarization is then exact, and only the difference between the two anisotropic–ray–theory S–wave travel times is approximate. The coupling ray theory may then also be used at high frequencies because the approximate travel–time difference influences only the coupling due to low–frequency scattering. The coupling ray theory then correctly converges to the anisotropic ray theory for high frequencies. For other choices of reference rays, the high–frequency limit of the coupling ray theory at a fixed anisotropy is incorrect, although the differences may be small at the finite frequencies under consideration. Note that the anisotropic–ray–theory reference ray can be traced only if the eigenvectors of the Christoffel matrix vary continuously along the whole ray (Vavryčuk, 2001).

In the *anisotropic common ray approximation*, the common reference ray is traced using the averaged Hamiltonian of both anisotropic–ray-theory S waves (Bakker, 2002; Klimeš, 2003). This is probably the best common ray approximation (Klimeš & Bulant, 2004). The errors due to the anisotropic common ray approximation of the coupling ray theory are usually considerably smaller than the errors due to the routinely used isotropic common ray approximation of the coupling ray theory.

In the less accurate *isotropic common ray approximation*, the reference ray is traced in the reference isotropic model. Moreover, the reference isotropic model may be selected in different ways, yielding quasi–isotropic approximations of differing accuracies.

The common ray approximations considerably simplify the coding of the coupling ray theory and numerical calculations, but introduce errors in travel times due to the perturbation. These travel–time errors can deteriorate the coupling–ray–theory solution at high frequencies and should be estimated. In the common ray approximations, the S–wave travel times are usually approximated by the first–order perturbation expansion from the common reference ray. The errors of S–wave travel times may then be approximated by second–order terms in the perturbation expansion. A method of estimating the errors due to the isotropic common ray approximation and the anisotropic common ray approximation has been proposed and numerically demonstrated by Klimeš & Bulant (2004). The method is based on the equations for the second–order perturbations of travel time derived by Klimeš (2002).

The accuracy of the anisotropic common ray approximation can approximately be studied along isotropic common rays, without tracing the anisotropic common rays. If the error of the isotropic common ray approximation exceeds an acceptable limit, we can immediately decide whether the anisotropic common ray approximation (Bakker, 2002; Klimeš, 2003) would be sufficiently accurate, or whether the anisotropic–ray–theory rays should be traced as reference rays for the coupling ray theory. The numerical results by Klimeš & Bulant (2004) suggest that the anisotropic common ray approximation by Bakker (2002) may be much more accurate than the routinely used isotropic common ray approximation.

2. Theory and numerical algorithm of the anisotropic common ray approximation

For the derivation of the coupling ray theory refer to Coates & Chapman (1990) and Červený (2001). For the description of the numerical algorithm of the coupling ray theory refer to Červený (2001) and Bulant & Klimeš (2002).

The algorithms of anisotropic common ray tracing in smooth models without interfaces and of corresponding dynamic ray tracing in ray–centred coordinates were described by Klimeš (2003). After that, the respective dynamic ray tracing computer code (Bucha & Bulant, 2003) has been debugged and partly numerically tested.

The numerical algorithm of the coupling ray theory by Bulant & Klimeš (2002) is independent of the reference ray. The algorithm is thus applicable to the isotropic common rays, anisotropic common rays, and anisotropic–ray–theory reference rays.

3. Numerical example

Two–point anisotropic common rays are traced in three 1-D models QI, QI2 and QI4 of differing degree of anisotropy. The first–order anisotropic common ray approximation of the anisotropic–ray–theory travel times is compared with the exact anisotropic–ray–theory travel times in order to demonstrate the accuracy of the anisotropic common ray approximation. The coupling–ray–theory synthetic seismograms calculated using the anisotropic common ray approximation are then compared with the synthetic seismograms calculated using the isotropic common ray approximation.

3.1. Model QI

A vertically heterogeneous 1-D anisotropic model QI was provided by Pšenčík & Dellinger (2001, model WA rotated by 45°) who performed the coupling–ray–theory calculations using the programs of package ANRAY (Pšenčík, 1998b) and compared the results with the reflectivity method. The density–normalized elastic moduli a_{ijkl} in $\mathrm{km^2s^{-2}}$ at the surface (zero depth) are

$$
\begin{array}{c} \\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 11 & 22 & 33 & 23 & 13 & 12 \end{array} \\
\begin{pmatrix}
14.48500 & 4.52500 & 4.75000 & 0.00000 & 0.00000 & -0.58000 \\
 & 14.48500 & 4.75000 & 0.00000 & 0.00000 & -0.58000 \\
 & & 15.71000 & 0.00000 & 0.00000 & -0.29000 \\
 & & & 5.15500 & -0.17500 & 0.00000 \\
 & & & & 5.15500 & 0.00000 \\
 & & & & & 5.04500
\end{pmatrix}
\end{array} \quad , \qquad (1)
$$

and at the depth of 1 km they are

$$
\begin{array}{c} \\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 11 & 22 & 33 & 23 & 13 & 12 \end{array} \\
\begin{pmatrix}
22.08963 & 6.90063 & 7.24375 & 0.00000 & 0.00000 & -0.88450 \\
 & 22.08963 & 7.24375 & 0.00000 & 0.00000 & -0.88450 \\
 & & 23.95775 & 0.00000 & 0.00000 & -0.44225 \\
 & & & 7.86138 & -0.26688 & 0.00000 \\
 & & & & 7.86138 & 0.00000 \\
 & & & & & 7.69363
\end{pmatrix}
\end{array} \quad . \qquad (2)
$$

Here the rows correspond to the first couple of indices of a_{ijkl}, the columns correspond to the second couple of indices. The reference isotropic model is given by

$$v_{\mathrm{P}}^2 = 15.00\,\mathrm{km^2s^{-2}} \quad , \qquad v_{\mathrm{S}}^2 = 5.10\,\mathrm{km^2s^{-2}} \qquad (3)$$

at the surface, and

$$v_{\mathrm{P}}^2 = 23.00\,\mathrm{km}^2\mathrm{s}^{-2} \quad , \qquad v_{\mathrm{S}}^2 = 7.79\,\mathrm{km}^2\mathrm{s}^{-2} \tag{4}$$

at the depth of 1 km. All the above values are interpolated linearly with depth. The density is constant.

The synthetic seismograms, corresponding to vertical force $\mathbf{F} = (0,0,100)^{\mathrm{T}}$ at position $(50,50,0)^{\mathrm{T}}$, are calculated at 29 receivers $(51,50,0.010)^{\mathrm{T}}$, $(51,50,0.030)^{\mathrm{T}}$, $(51,50,0.050)^{\mathrm{T}}$, ..., $(51,50,0.570)^{\mathrm{T}}$ located in a vertical well (distances in km). The source time function is the Gabor signal $\cos(2\pi ft)\exp[-(2\pi ft/4)^2]$ with reference frequency $f = 50$ Hz, band–pass filtered by a cosine filter given by frequencies 0 Hz, 5 Hz, 60 Hz and 100 Hz.

The data for model QI may be found on the compact disk of Bucha & Bulant (2002) together with the Fortran 77 source code of packages CRT (Červený, Klimeš & Pšenčík, 1988) and ANRAY (Gajewski & Pšenčík, 1990; Pšenčík, 1998b). For comparison with the isotropic–ray–theory and anisotropic–ray–theory seismograms in model QI and for a more detailed discussion and description of this model refer to Pšenčík & Dellinger (2001).

3.2. Models QI2, QI4 and QI8

To emphasize the effects of perturbations of travel time, new models with increased degrees of anisotropy have been derived from the QI model.

The differences of the elastic moduli of model QI2 from the elastic moduli of the reference isotropic model (3), (4) are exactly twice larger than the differences of model QI. The density–normalized elastic moduli a_{ijkl} of model QI2 in $\mathrm{km}^2\mathrm{s}^{-2}$ at the surface (zero depth) are

$$\begin{array}{c} \\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12 \end{array} \begin{array}{c} \begin{array}{cccccc} 11 & 22 & 33 & 23 & 13 & 12 \end{array} \\ \begin{pmatrix} 13.97000 & 4.25000 & 4.70000 & 0.00000 & 0.00000 & -1.16000 \\ & 13.97000 & 4.70000 & 0.00000 & 0.00000 & -1.16000 \\ & & 16.42000 & 0.00000 & 0.00000 & -0.58000 \\ & & & 5.21000 & -0.35000 & 0.00000 \\ & & & & 5.21000 & 0.00000 \\ & & & & & 4.99000 \end{pmatrix} \end{array} \quad , \tag{5}$$

and at the depth of 1 km they are

$$\begin{array}{c} \\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12 \end{array} \begin{array}{c} \begin{array}{cccccc} 11 & 22 & 33 & 23 & 13 & 12 \end{array} \\ \begin{pmatrix} 21.17926 & 6.38126 & 7.06750 & 0.00000 & 0.00000 & -1.76900 \\ & 21.17926 & 7.06750 & 0.00000 & 0.00000 & -1.76900 \\ & & 24.91550 & 0.00000 & 0.00000 & -0.88450 \\ & & & 7.93276 & -0.53376 & 0.00000 \\ & & & & 7.93276 & 0.00000 \\ & & & & & 7.59726 \end{pmatrix} \end{array} \quad . \tag{6}$$

Analogously, the differences of the elastic moduli of model QI4 from the elastic moduli of the reference isotropic model (3), (4) are exactly 4 times larger than the differences of model QI. The density–normalized elastic moduli a_{ijkl} of model QI4 in

km^2s^{-2} at the surface (zero depth) are

$$
\begin{array}{c}
\\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12
\end{array}
\begin{array}{c}
\begin{matrix} \quad 11 & \quad\quad 22 & \quad\quad 33 & \quad\quad 23 & \quad\quad 13 & \quad\quad 12 \end{matrix} \\
\begin{pmatrix}
12.94000 & 3.70000 & 4.60000 & 0.00000 & 0.00000 & -2.32000 \\
 & 12.94000 & 4.60000 & 0.00000 & 0.00000 & -2.32000 \\
 & & 17.84000 & 0.00000 & 0.00000 & -1.16000 \\
 & & & 5.32000 & -0.70000 & 0.00000 \\
 & & & & 5.32000 & 0.00000 \\
 & & & & & 4.88000
\end{pmatrix}
\end{array} , \qquad (7)
$$

and at the depth of 1 km they are

$$
\begin{array}{c}
\\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12
\end{array}
\begin{array}{c}
\begin{matrix} \quad 11 & \quad\quad 22 & \quad\quad 33 & \quad\quad 23 & \quad\quad 13 & \quad\quad 12 \end{matrix} \\
\begin{pmatrix}
19.35852 & 5.34252 & 6.71500 & 0.00000 & 0.00000 & -3.53800 \\
 & 19.35852 & 6.71500 & 0.00000 & 0.00000 & -3.53800 \\
 & & 26.83100 & 0.00000 & 0.00000 & -1.76900 \\
 & & & 8.07552 & -1.06752 & 0.00000 \\
 & & & & 8.07552 & 0.00000 \\
 & & & & & 7.40452
\end{pmatrix}
\end{array} . \qquad (8)
$$

Note that neither the isotropic common ray approximation nor the anisotropic common ray approximation can be applied to model QI8, in which the differences of the elastic moduli from the elastic moduli of the reference isotropic model (3), (4) are exactly 8 times larger than the differences of model QI.

The data for models QI2, QI4 and QI8 have been released on the compact disk of Bucha & Bulant (2002). Numerical examples in this paper have been calculated using the code and data by Bucha & Bulant (2004).

3.3. Effects of the common–ray approximations

Two–point anisotropic common rays have been traced from the source to the receivers using the program CRT. To check the accuracy of the first–order perturbation expansion of the anisotropic–ray–theory travel times along anisotropic common rays, the perturbation expansion is compared in Tables 4, 5 and 6 with the anisotropic–ray–theory travel times calculated by the program ANRAY along two–point anisotropic–ray–theory rays. Only the results at the 1^{st}, 8^{th}, 15^{th}, 22^{nd} and 29^{th} receivers are shown, because the variation of the quantities along the vertical profile in models QI, QI2 and QI4 is very moderate.

For convenient comparison with the accuracy of the isotropic common ray approximation, we reproduce here Tables 1, 2 and 3 by Klimeš & Bulant (2004).

The anisotropic common ray approximation of coupling–ray–theory synthetic seismograms in models QI, QI2 and QI4 is compared with the isotropic common ray approximation of coupling–ray–theory synthetic seismograms in Figures 1, 3 and 5, respectively. Refer to Tables 1 to 6 (columns ACR remaining terms and ICR quadratic terms) for the travel–time errors of both approximations.

Since Klimeš & Bulant (2004) were not able to solve the coupling equation along the anisotropic–ray–theory reference rays, they simulated the corresponding coupling–ray–theory synthetic seismograms by solving the coupling equations along isotropic common rays, with anisotropic–ray–theory travel times approximated by the second–order perturbation expansion. Their approximation of coupling–ray–theory synthetic seismograms is compared with the anisotropic common ray approximation of coupling–ray–theory synthetic seismograms in Figures 2, 4 and 6. Refer to Tables 1 to 6 (columns ICR remaining terms and ACR remaining terms) for the travel–time errors of both approximations.

Although the second–order perturbation expansion of travel time has successfully been applied to the estimation of anisotropic–ray–theory travel times in these simple 1-D models QI, QI2 and QI4 with constant gradients of the density–normalized elastic moduli, we cannot recommend approximation of travel time using the second–order perturbation expansion in more complex models, because the second–order perturbations may be infinitely large in the vicinity of caustics. The second–order perturbation expansion of travel time should be used especially for estimating and controlling the accuracy of the common ray approximations outside caustics.

4. Conclusions

Whereas the first–order isotropic common ray approximation is considerably inaccurate in model QI4, see Figure 5 and Table 3, the first–order anisotropic common ray approximation by Bakker (2002) performs well in all three models QI, QI2 and QI4, see Tables 4, 5 and 6.

Acknowledgements

The research has been supported by the Grant Agency of the Czech Republic under Contracts 205/01/D097 and 205/04/1104, by the Ministry of Education of the Czech Republic within Research Project MSM113200004, and by the members of the consortium "Seismic Waves in Complex 3-D Structures" (see "http://sw3d.mff.cuni.cz").

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Rec. dep.	ICR time	ICR linear terms		ICR quadratic terms		ICR remaining terms		ACR q.terms
0.01	0.440993	-0.002392	0.003983	0.000253	0.000000	-0.000003	-0.000001	0.000066
0.15	0.438077	-0.002518	0.004003	0.000250	0.000000	0.000007	-0.000001	0.000066
0.29	0.443550	-0.002967	0.004117	0.000251	0.000001	0.000007	0.000001	0.000069
0.43	0.456339	-0.003661	0.004317	0.000253	0.000001	0.000008	-0.000002	0.000073
0.57	0.475205	-0.004520	0.004595	0.000251	0.000003	0.000020	0.000006	0.000076

Table 1. Linear and quadratic terms in the common ray approximations of travel time in model QI.

Rec. dep.	ICR time	ICR linear terms		ICR quadratic terms		ICR remaining terms		ACR q.terms
0.01	0.440993	-0.004746	0.011604	0.000978	0.000000	0.000001	-0.000007	0.000237
0.15	0.438077	-0.004992	0.011612	0.000967	0.000000	0.000015	0.000001	0.000233
0.29	0.443550	-0.005874	0.011837	0.000967	0.000001	0.000029	0.000005	0.000230
0.43	0.456339	-0.007234	0.012235	0.000965	0.000001	0.000047	0.000001	0.000226
0.57	0.475205	-0.008912	0.012764	0.000951	0.000002	0.000063	0.000000	0.000218

Table 2. Linear and quadratic terms in the common ray approximations of travel time in model QI2.

Rec. dep.	ICR time	ICR linear terms		ICR quadratic terms		ICR remaining terms		ACR q.terms
0.01	0.440993	-0.009341	0.041580	0.003671	0.000492	0.000092	-0.000141	0.000367
0.15	0.438077	-0.009816	0.041343	0.003616	0.000553	0.000137	-0.000147	0.000335
0.29	0.443550	-0.011517	0.041462	0.003580	0.000718	0.000224	-0.000174	0.000273
0.43	0.456339	-0.014129	0.041796	0.003520	0.000962	0.000328	-0.000212	0.000201
0.57	0.475205	-0.017335	0.042243	0.003415	0.001234	0.000434	-0.000234	0.000136

Table 3. Linear and quadratic terms in the common ray approximations of travel time in model QI4.

Tables 1, 2 and 3. *Rec. dep.* stands for the receiver depth along the vertical profile, see Figures 1 to 6. The *ICR time* is the reference travel time along the isotropic common ray. The *ICR linear terms* are the linear terms in perturbation expansion of the anisotropic–ray–theory travel times in the vicinity of the isotropic common ray, and represent the travel–time corrections considered in the isotropic common ray approximation of the coupling ray theory. The *ICR quadratic terms* stand for quadratic terms in perturbation expansion of travel time, and represent the estimates of the errors due to the isotropic common ray approximation of the anisotropic–ray–theory travel times. The *ICR remaining terms* stand for the difference between the exact anisotropic–ray–theory travel times calculated by the program ANRAY version 4.40 and the second–order perturbation expansion in order to illustrate the reliability of the error estimates. The ICR remaining terms represent both the inaccuracy of numerical ray tracing and the third–order and higher–order terms in the perturbation expansion. The *ACR quadratic terms* stand for the estimate of the equal quadratic terms in the perturbation expansion of the anisotropic–ray–theory travel times in the vicinity of the anisotropic common ray, and represent the estimate of the error due to the anisotropic common ray approximation of the anisotropic–ray–theory travel times. Note that these estimated ACR quadratic terms are approximately calculated along the isotropic common rays, i.e. without tracing the anisotropic common rays. The estimated ACR quadratic terms may be compared with the actual errors of the anisotropic common ray approximation (Tables 4, 5 and 6, column ACR remaining terms). The differences between the ACR quadratic terms calculated along the isotropic common rays and the actual errors of the anisotropic common ray approximation (ACR remaining terms) are mostly due to the neglected third–order terms in the perturbation expansion along isotropic common rays.

Rec. dep.	ACR time	ACR linear terms		ACR remaining terms		anisotropic–ray–th. travel times	
0.01	0.441850	-0.003061	0.003061	0.000062	0.000064	0.438851	0.444975
0.15	0.438880	-0.003135	0.003135	0.000071	0.000065	0.435816	0.442080
0.29	0.444183	-0.003415	0.003415	0.000073	0.000070	0.440841	0.447668
0.43	0.456721	-0.003861	0.003861	0.000079	0.000074	0.452939	0.460656
0.57	0.475296	-0.004430	0.004430	0.000090	0.000083	0.470956	0.479809

Table 4. Anisotropic common ray approximation of anisotropic–ray–theory travel times in model QI.

Rec. dep.	ACR time	ACR linear terms		ACR remaining terms		anisotropic–ray–th. travel times	
0.01	0.444667	-0.007688	0.007689	0.000247	0.000234	0.437226	0.452590
0.15	0.441634	-0.007818	0.007818	0.000251	0.000239	0.434067	0.449691
0.29	0.446788	-0.008366	0.008366	0.000250	0.000238	0.438672	0.455392
0.43	0.459093	-0.009240	0.009240	0.000263	0.000243	0.450117	0.468576
0.57	0.477392	-0.010344	0.010344	0.000259	0.000235	0.467307	0.487971

Table 5. Anisotropic common ray approximation of anisotropic–ray–theory travel times in model QI2.

Rec. dep.	ACR time	ACR linear terms		ACR remaining terms		anisotropic–ray–th. travel times	
0.01	0.458572	-0.023856	0.023856	0.000698	0.000497	0.435414	0.482925
0.15	0.455345	-0.024001	0.024001	0.000670	0.000480	0.432014	0.479826
0.29	0.460171	-0.024947	0.024947	0.000612	0.000438	0.435836	0.485556
0.43	0.472022	-0.026489	0.026489	0.000525	0.000374	0.446058	0.498885
0.57	0.489715	-0.028424	0.028424	0.000428	0.000309	0.461719	0.518448

Table 6. Anisotropic common ray approximation of anisotropic–ray–theory travel times in model QI4.

Tables 4, 5 and 6. *Rec. dep.* stands for the receiver depth along the vertical profile, see Figures 1 to 6. The *ACR time* is the reference travel time along the anisotropic common ray. The *ACR linear terms* are the linear terms in perturbation expansion of the anisotropic–ray–theory travel times in the vicinity of the anisotropic common ray, and represent the travel–time corrections considered in the anisotropic common ray approximation of the coupling ray theory. The *ACR remaining terms* stand for the difference between the anisotropic–ray–theory travel times calculated by the program ANRAY version 4.40 and their first–order perturbation expansion in the vicinity of the anisotropic common ray. The ACR remaining terms represent both the inaccuracy of numerical ray tracing and the second–order and higher–order terms in the perturbation expansion. The ACR remaining terms may be compared with their estimation calculated along isotropic common rays (Tables 1, 2 and 3, column ACR q.terms). The differences between the ACR remaining terms and their estimation are mostly due to the neglected third–order terms in the perturbation expansion along isotropic common rays. The anisotropic–ray–theory travel times are denoted here by *anisotropic–ray–th. travel times.*

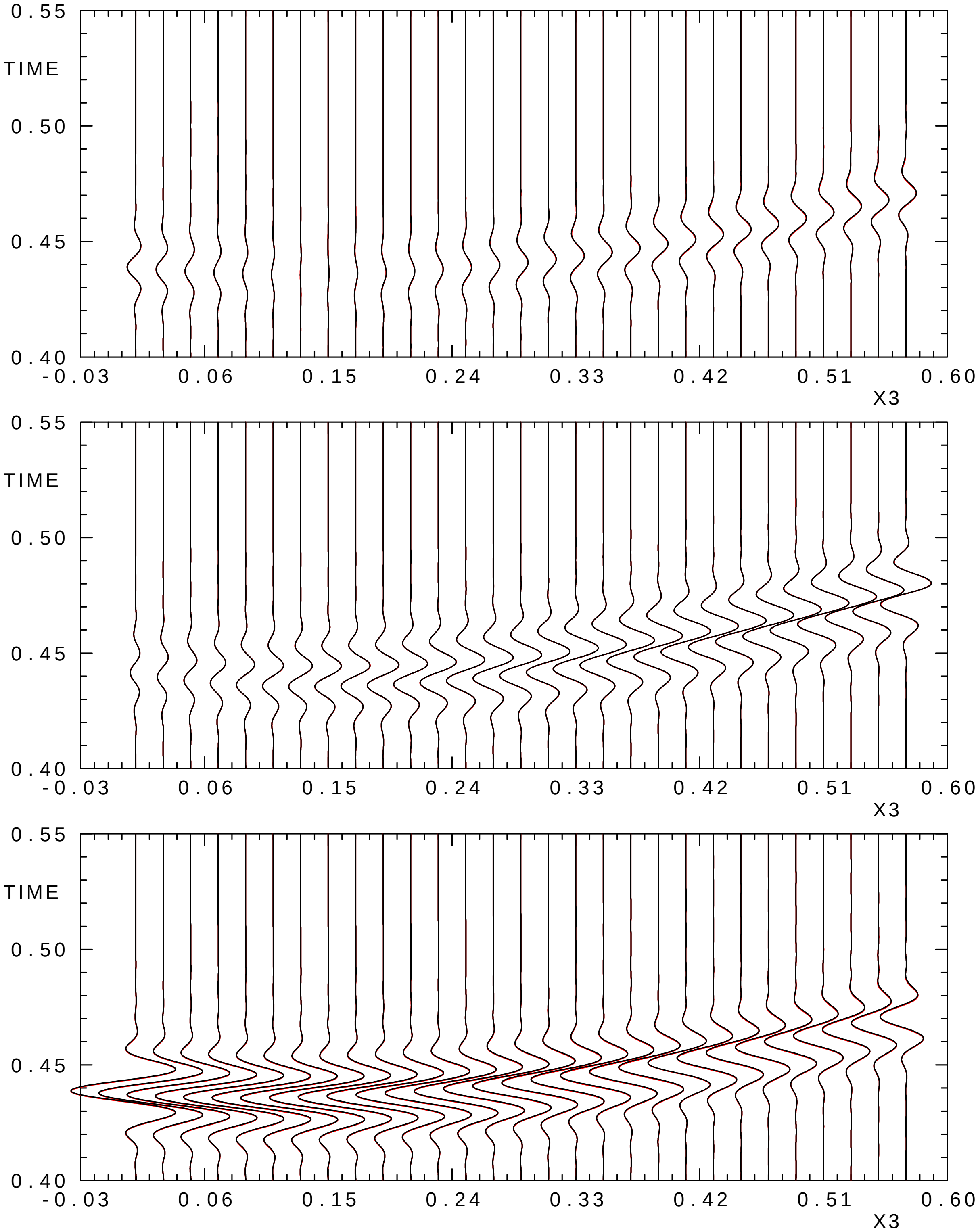

Figure 1. Comparison of the anisotropic and isotropic common ray approximations of the coupling ray theory in model QI. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Anisotropic common ray approximation. For the inaccuracy of the black seismograms refer to the *ACR remaining terms* in Table 4. **Red:** Isotropic common ray approximation. For the inaccuracy of the red seismograms refer to the *ICR quadratic terms* in Table 1. The red seismograms are mostly obscured by the black seismograms.

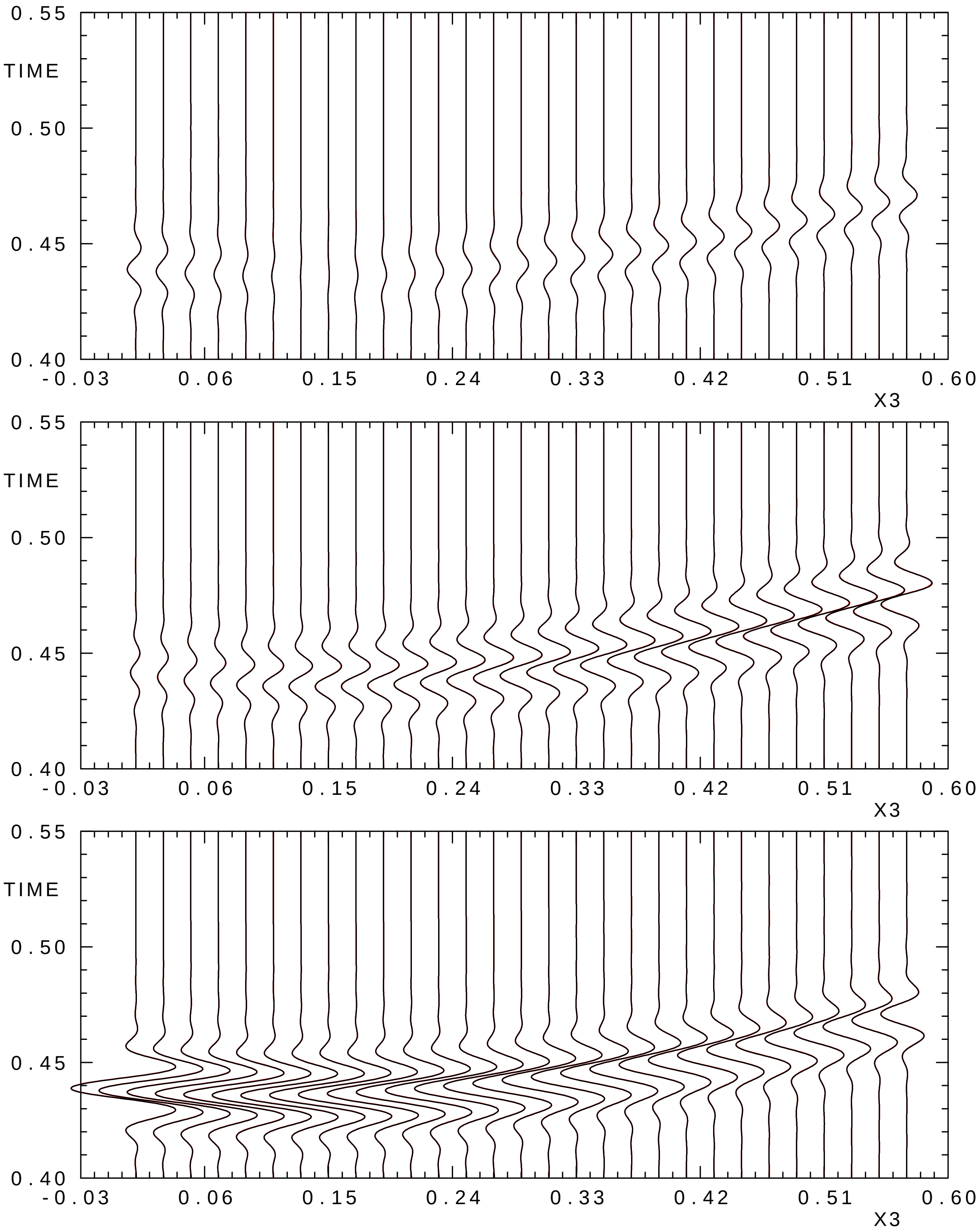

Figure 2. Comparison of the anisotropic common ray approximation with the quadratic perturbation expansion of travel time from the isotropic common rays in model QI. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Anisotropic common ray approximation. For the inaccuracy of the black seismograms refer to the *ACR remaining terms* in Table 4. **Red:** Second–order perturbation expansion of travel time calculated along isotropic common rays. This approximation was used by Klimeš & Bulant (2004) to simulate the coupling–ray–theory synthetic seismograms without common ray approximation. For the inaccuracy of this simulation refer to the *ICR remaining terms* in Table 1. The red seismograms are obscured by the black seismograms.

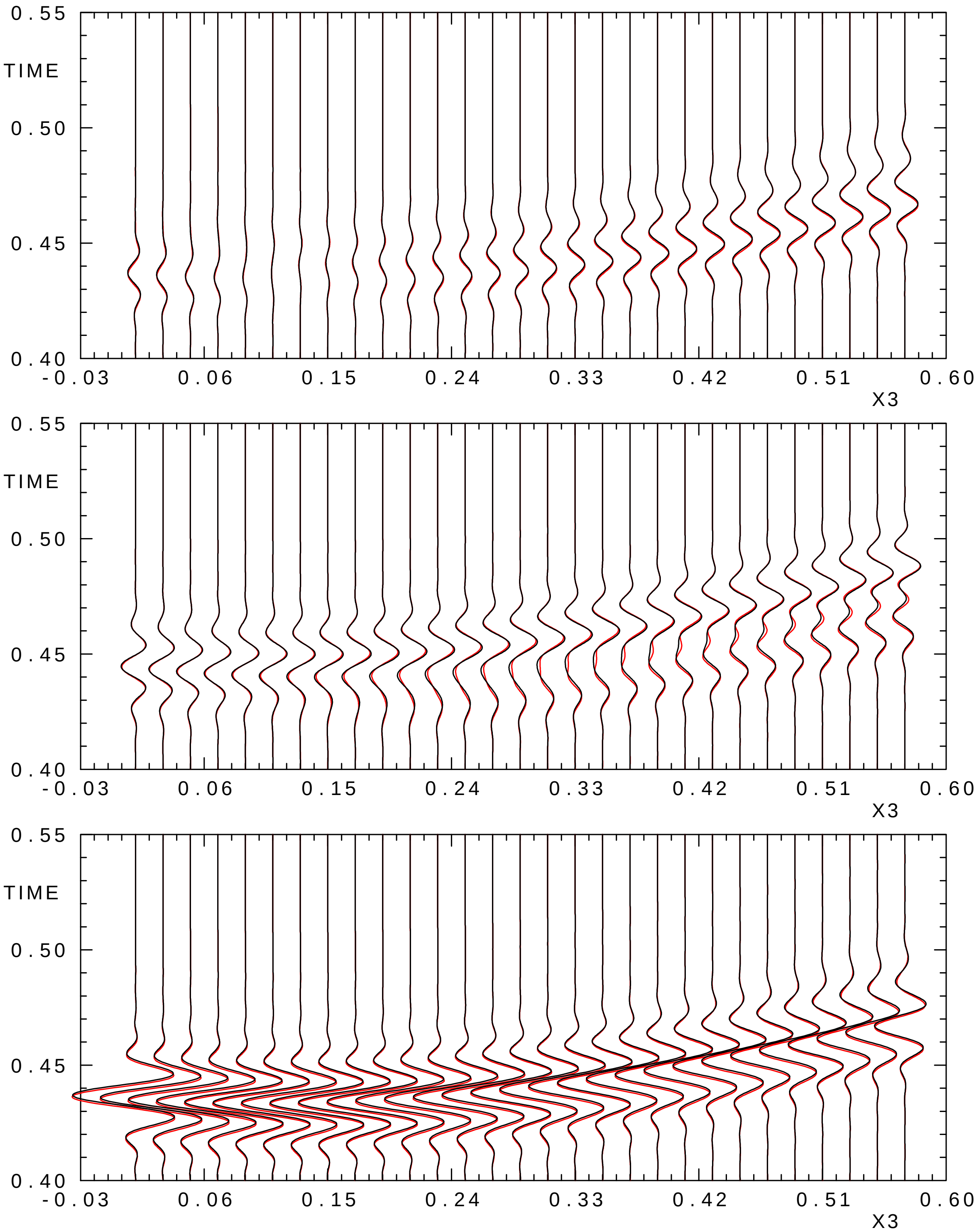

Figure 3. Comparison of the anisotropic and isotropic common ray approximations of the coupling ray theory in model QI2. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Anisotropic common ray approximation. For the inaccuracy of the black seismograms refer to the *ACR remaining terms* in Table 5. **Red:** Isotropic common ray approximation. For the inaccuracy of the red seismograms refer to the *ICR quadratic terms* in Table 2. The differences between the seismograms are small but already clearly visible.

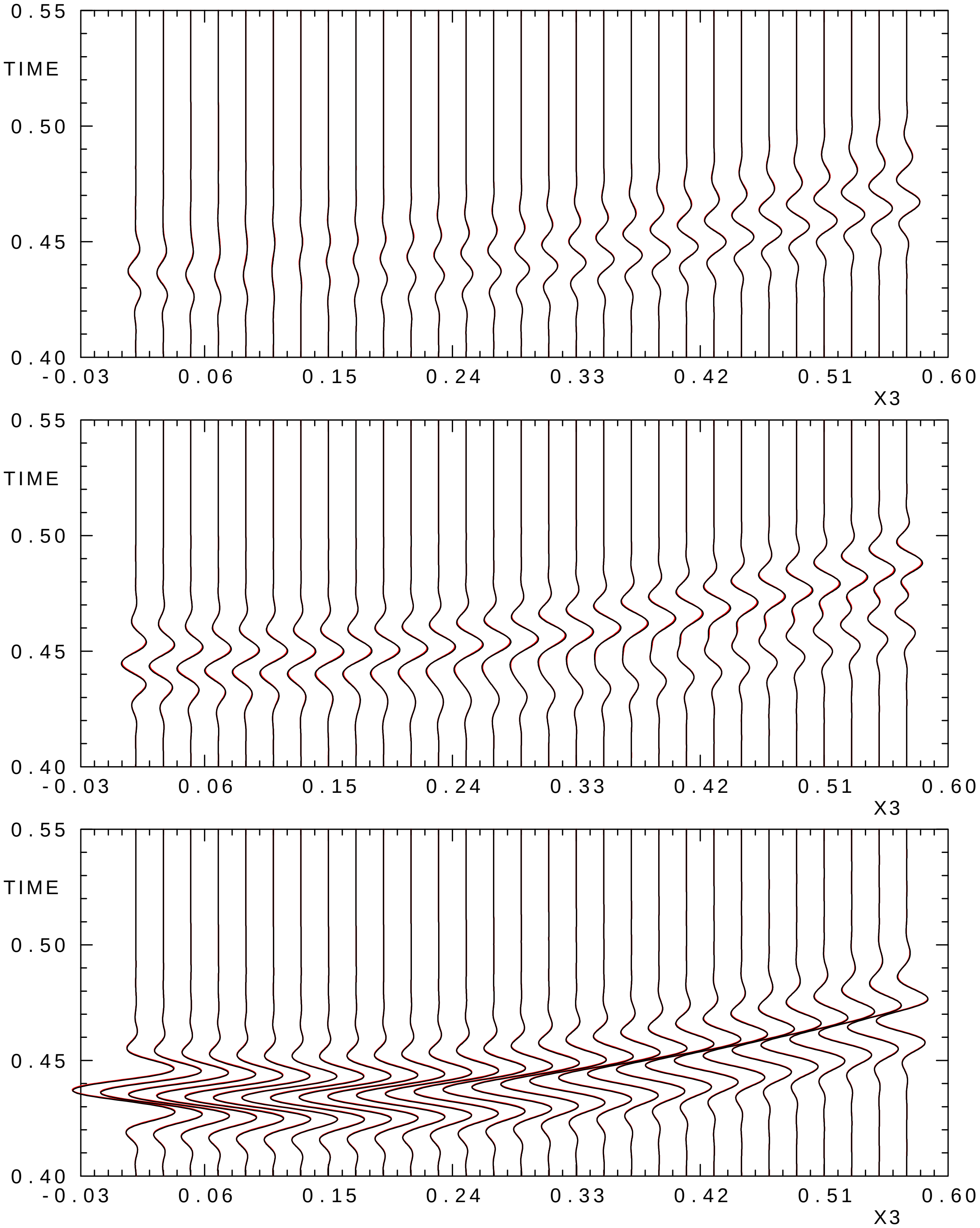

Figure 4. Comparison of the anisotropic common ray approximation with the quadratic perturbation expansion of travel time from the isotropic common rays in model QI2. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Anisotropic common ray approximation. For the inaccuracy of the black seismograms refer to the *ACR remaining terms* in Table 5. **Red:** Second–order perturbation expansion of travel time calculated along isotropic common rays. This approximation was used by Klimeš & Bulant (2004) to simulate the coupling–ray–theory synthetic seismograms without common ray approximation. For the inaccuracy of this simulation refer to the *ICR remaining terms* in Table 2. The red seismograms are mostly obscured by the black seismograms.

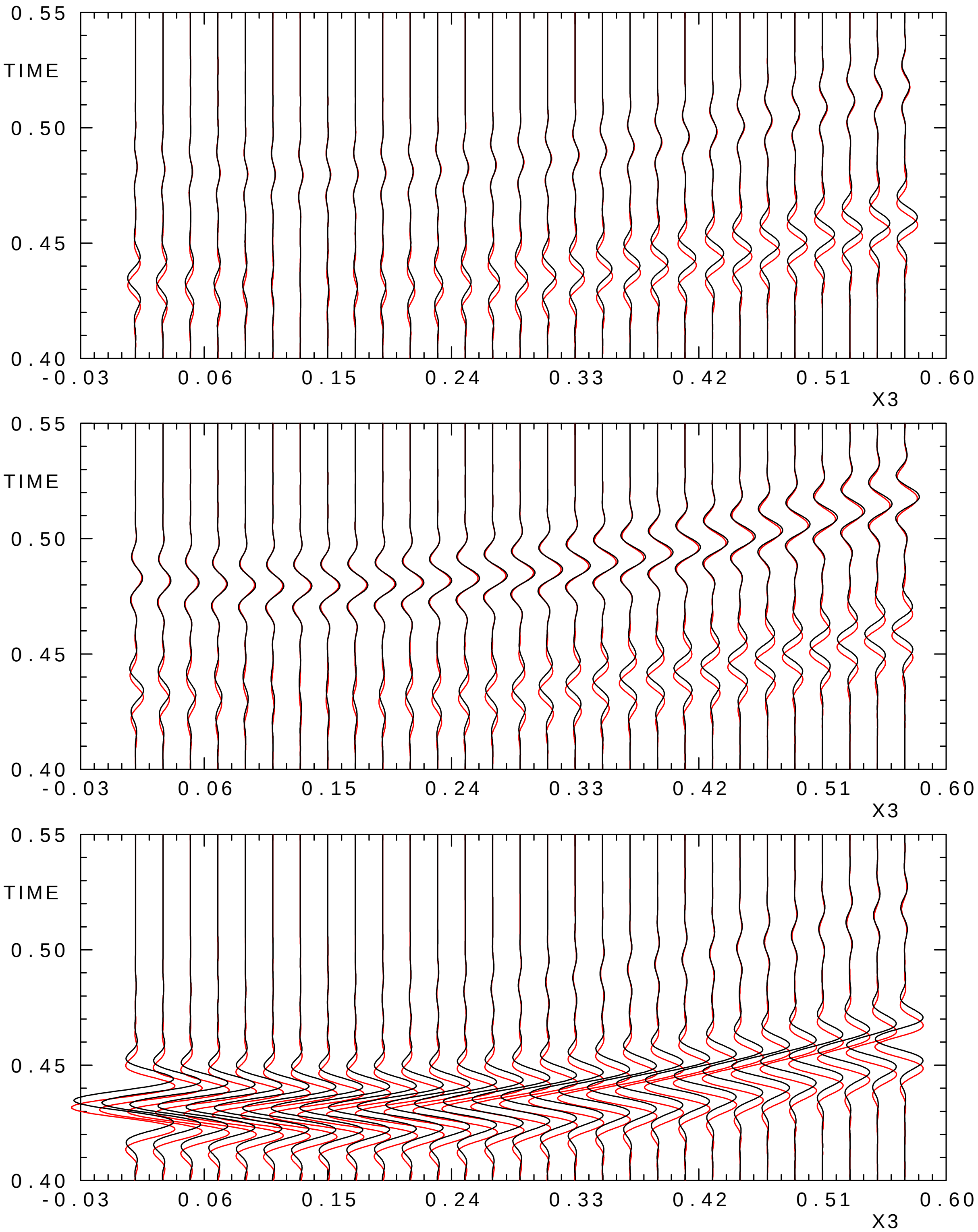

Figure 5. Comparison of the anisotropic and isotropic common ray approximations of the coupling ray theory in model QI4. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Anisotropic common ray approximation. For the inaccuracy of the black seismograms refer to the *ACR remaining terms* in Table 6. **Red:** Isotropic common ray approximation. For the inaccuracy of the red seismograms refer to the *ICR quadratic terms* in Table 3. The differences between the seismograms are considerable in this quite strongly anisotropic model QI4.

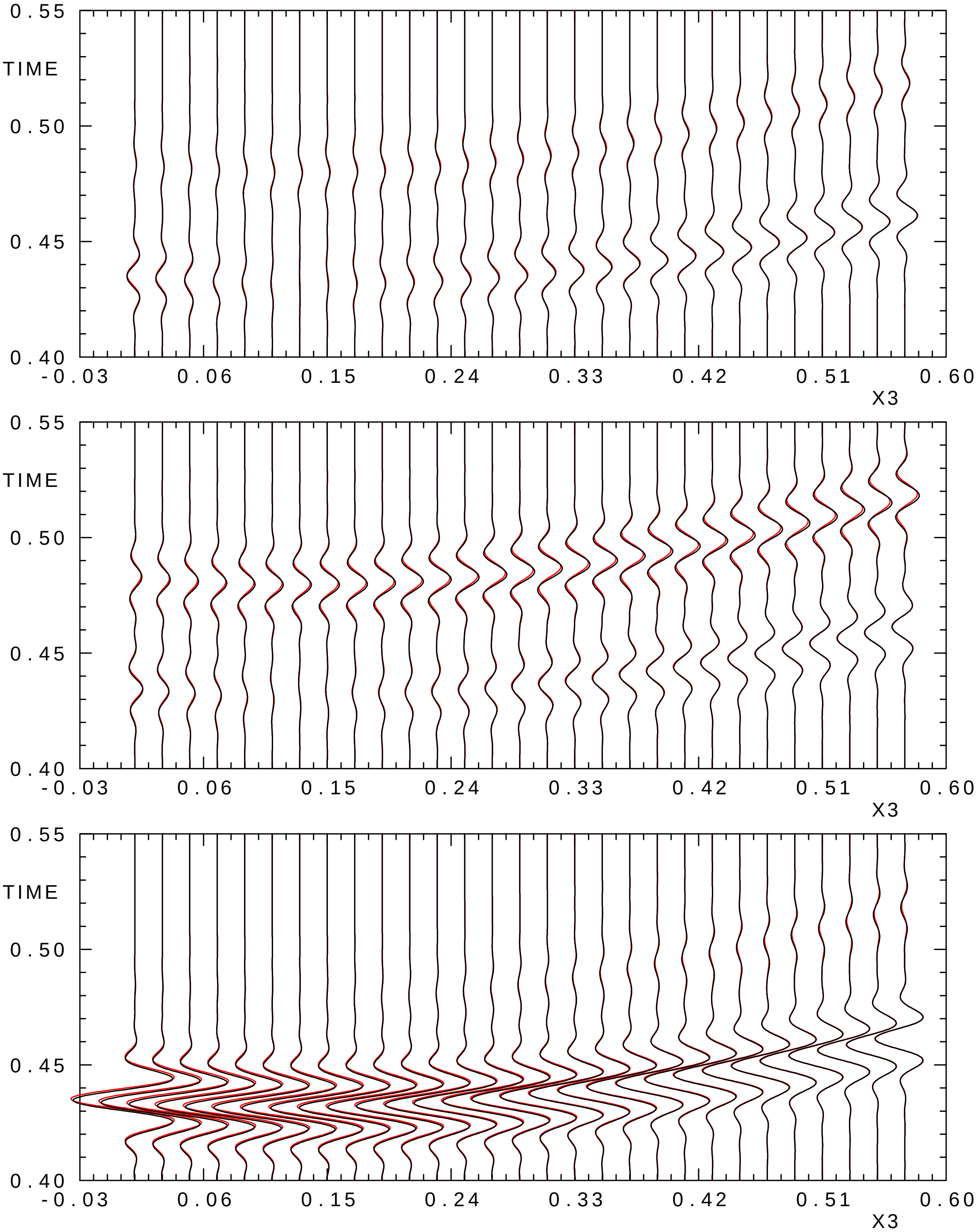

Figure 6. Comparison of the anisotropic common ray approximation with the quadratic perturbation expansion of travel time from the isotropic common rays in model QI4. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Anisotropic common ray approximation. For the inaccuracy of the black seismograms refer to the *ACR remaining terms* in Table 6. **Red:** Second–order perturbation expansion of travel time calculated along isotropic common rays. This approximation was used by Klimeš & Bulant (2004) to simulate the coupling–ray–theory synthetic seismograms without common ray approximation. For the inaccuracy of this simulation refer to the *ICR remaining terms* in Table 3.

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