

Kirchhoff prestack depth migration in velocity models with and without gradients: Comparison of triclinic anisotropy with simpler anisotropies

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Summary

We use the Kirchhoff prestack depth migration to calculate migrated sections in simple anisotropic velocity models in order to demonstrate the impact of anisotropy and simple inhomogeneity on migrated images. The recorded wave field is generated in velocity models composed of two layers separated by a non-inclined curved interface. The upper layer has triclinic anisotropy and a velocity gradient. The bottom layer is isotropic and homogeneous. We apply the Kirchhoff prestack depth migration to both heterogeneous and homogeneous single-layer velocity models with different types of anisotropy: a triclinic anisotropic medium, the transversely isotropic media with a horizontal symmetry axis and a vertical symmetry axis. We show the errors of the migrated interface caused by inaccurate velocity models used for migration. The study is limited to P-waves.

Keywords

3-D Kirchhoff prestack depth migration, anisotropic velocity model, velocity gradient

1. Introduction

We continue in the Kirchhoff prestack depth migration calculations performed by Bucha (2011, 2012). The dimensions of the velocity model, shot-receiver configuration, methods for calculation of the recorded wave field and the migration are the same as in the papers by Bucha (2011, 2012). We use the velocity model with the non-inclined curved interface only. Bucha (2011, 2012) used, for computation of the recorded wave field, a homogeneous upper layer with triclinic anisotropy, whereas in this paper, we use an inhomogeneous upper layer with triclinic anisotropy and with a vertical or horizontal velocity gradient. Our aim is to study the influence of a simple heterogeneity on the migrated image.

We generate the synthetic data using the ray theory. To calculate the synthetic recorded wave field, we use a simple anisotropic velocity model composed of two layers separated by one curved interface which is non-inclined in the direction perpendicular to the source-receiver profiles. The anisotropy in the upper layer is triclinic. The constant gradients of elastic moduli are either vertical or horizontal. The bottom layer is homogeneous and isotropic.

We migrate the synthetic data using the 3-D ray-based Kirchhoff prestack depth migration. Distortions of the imaged curved interface induced by anisotropy and heterogeneity are evaluated using several anisotropic migration velocity models. They all consist of a single layer without the interface. In the first two heterogeneous velocity models, the distribution of elastic moduli corresponds to the upper layer of the velocity

models in which the synthetic data have been calculated. The other three velocity models are homogeneous and their anisotropy is either triclinic, or transversely isotropic with a horizontal symmetry axis or a vertical symmetry axis.

We show mispositioning, distortion and defocusing of the migrated interface caused by inaccurate velocity models used for migration. We use the 3-D migration because the reflected two-point rays propagate in triclinic media in a 3-D volume. The study is limited to P-waves.

2. Anisotropic velocity models with a velocity gradient

The dimensions of the velocity models and measurement configurations are derived from the 2-D Marmousi model and dataset (Versteeg & Grau, 1991). The horizontal dimensions of the velocity model are 9.2 km x 10 km ($x_1 \times x_2$ coordinate axes) and the depth is 3 km (x_3 axis). The velocity model is composed of two layers separated by one non-inclined curved interface (see Figure 1).

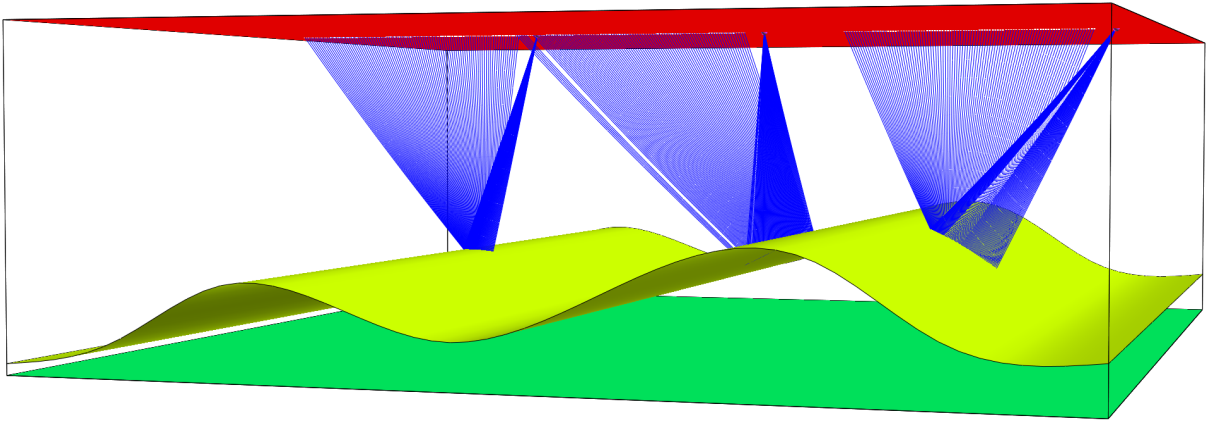


Figure 1. Velocity model with a non-inclined curved interface. The horizontal dimensions of the velocity model are 9.2 km x 10 km ($x_1 \times x_2$ axes), the depth is 3 km (x_3 axis). Velocity model consists of three interfaces: one curved interface, top and bottom velocity model planes. The interfaces are coloured according to the indices of surfaces. Two-point rays of the reflected P-wave for one selected profile line and three shot-receiver configurations (at horizontal coordinate $x_2 = 5$ km) are calculated in the velocity model with triclinic anisotropy and a vertical velocity gradient.

2.1. Velocity models for the recorded wave field

The recorded wave field is computed in two velocity models composed of two layers with the triclinic anisotropy representing dry Vosges sandstone (Mensch & Rasolofosaon, 1997) and velocity gradients in the upper layer. The bottom layer is isotropic and homogeneous.

2.1.1. Velocity model with triclinic anisotropy and a vertical gradient

The TA-VG medium in the upper layer is triclinic and is specified by two matrices of density-reduced elastic moduli A_{ij} in km^2/s^2 . The matrix of density-reduced elastic moduli at depth $x_3 = 0$ km (values specified by Mensch & Rasolofosaon (1997) are multiplied by a constant 0.8) reads

$$\begin{pmatrix} 8.24 & 0.72 & 1.04 & 1.12 & 0.88 & 0.64 \\ & 8.48 & 1.68 & 0.16 & -0.16 & -0.48 \\ & & 11.28 & 0.00 & -0.40 & -0.80 \\ & & & 4.08 & 0.00 & 0.16 \\ & & & & 4.80 & 0.00 \\ & & & & & 3.92 \end{pmatrix}.$$

The matrix at depth $x_3 = 2.9$ km (values specified by Mensch & Rasolofosaon (1997)) reads

$$\begin{pmatrix} 10.3 & 0.9 & 1.3 & 1.4 & 1.1 & 0.8 \\ & 10.6 & 2.1 & 0.2 & -0.2 & -0.6 \\ & & 14.1 & 0.0 & -0.5 & -1.0 \\ & & & 5.1 & 0.0 & 0.2 \\ & & & & 6.0 & 0.0 \\ & & & & & 4.9 \end{pmatrix}.$$

The density-reduced elastic moduli inside the layer are determined by linear interpolation from the specified values of density-reduced elastic moduli. The bottom layer is isotropic and has P-wave velocity $V_p = 3.6$ km/s.

2.1.2. Velocity model with triclinic anisotropy and a horizontal gradient

The TA-HG medium in the upper layer is triclinic and is specified by two matrices of density-reduced elastic moduli. The matrix at horizontal coordinate $x_1 = 0$ km is the same as the matrix for the TA-VG medium (Section 2.1.1) at depth $x_3 = 0$ km. The matrix at horizontal coordinate $x_1 = 9.2$ km is the same as the matrix for the TA-VG medium at depth $x_3 = 2.9$ km. The horizontal gradient is parallel with the profile lines. The density-reduced elastic moduli inside the layer are determined by linear interpolation from the specified values of density-reduced elastic moduli. The bottom layer is isotropic and has P-wave velocity $V_p = 3.6$ km/s.

2.2. Velocity models for the migration

The migration is performed using single-layer (without an interface) heterogeneous velocity models with media TA-VG and TA-HG (specified in Sections 2.1.1. and 2.1.2.) and the following homogeneous velocity models with the TA, VTI-1 and HTI-2 media, used by Bucha (2011, 2012).

2.2.1. Velocity model with triclinic anisotropy

The triclinic anisotropic medium (TA) represents dry Vosges sandstone (Mensch & Rasolofosaon, 1997). The matrix of elastic moduli reads

$$\begin{pmatrix} 10.3 & 0.9 & 1.3 & 1.4 & 1.1 & 0.8 \\ & 10.6 & 2.1 & 0.2 & -0.2 & -0.6 \\ & & 14.1 & 0.0 & -0.5 & -1.0 \\ & & & 5.1 & 0.0 & 0.2 \\ & & & & 6.0 & 0.0 \\ & & & & & 4.9 \end{pmatrix}.$$

2.2.2. Velocity model with a transversely isotropic medium with a vertical symmetry axis

VTI-1 is a transversely isotropic medium with a vertical symmetry axis. The matrix of elastic moduli reads

$$\begin{pmatrix} 10.45 & 0.65 & 1.7 & 0.0 & 0.0 & 0.0 \\ & 10.45 & 1.7 & 0.0 & 0.0 & 0.0 \\ & & 14.1 & 0.0 & 0.0 & 0.0 \\ & & & 5.55 & 0.0 & 0.0 \\ & & & & 5.55 & 0.0 \\ & & & & & 4.9 \end{pmatrix}.$$

2.2.3. Velocity model with a transversely isotropic medium with a horizontal symmetry axis

HTI-2 is a transversely isotropic medium with a horizontal symmetry axis. The symmetry axis is parallel with the x_2 coordinate axis. The matrix of elastic moduli reads

$$\begin{pmatrix} 14.1 & 2.1 & 2.1 & 0.0 & 0.0 & 0.0 \\ & 10.6 & 2.1 & 0.0 & 0.0 & 0.0 \\ & & 14.1 & 0.0 & 0.0 & 0.0 \\ & & & 5.1 & 0.0 & 0.0 \\ & & & & 6.0 & 0.0 \\ & & & & & 5.1 \end{pmatrix}.$$

3. Shots and receivers

The measurement configuration is derived from the Marmousi model and dataset (Versteeg & Grau, 1991). The profile lines are parallel with the x_1 coordinate axis. Each profile line has the following configuration: The first shot is 3 km from the left-hand side of the velocity model, the last shot is 8.975 km from the left-hand side of the velocity model, the distance between the shots is 0.025 km, and the depth of the shots is 0 km. The total number of shots along one profile line is 240. The number of receivers per shot is 96, the first receiver is located 2.575 km left of the shot location, the last receiver is 0.2 km left of the shot location, the distance between the receivers is 0.025 km, and the depth of the receivers is 0 km. This configuration simulates a simplified towed streamer acquisition geometry.

The 3-D measurement configuration consists of 81 parallel profile lines, see Figure 2. The distance between the parallel profile lines is 0.025 km.

4. Recorded wave field

The recorded wave field in the triclinic velocity models with velocity gradients were computed using the ANRAY software package (Gajewski & Pšenčík, 1990). 3-D ray tracing is used to calculate the two-point rays of the reflected P-wave. We then compute the ray-theory seismograms at the receivers. The two-point rays do not stay in the vertical planes corresponding to the individual profiles.

In the velocity model with the non-inclined curved interface, the recorded wave field is equal for all parallel profile lines, since the velocity gradient changes only in the profile line plane (a vertical gradient or a horizontal gradient parallel with the profile lines)

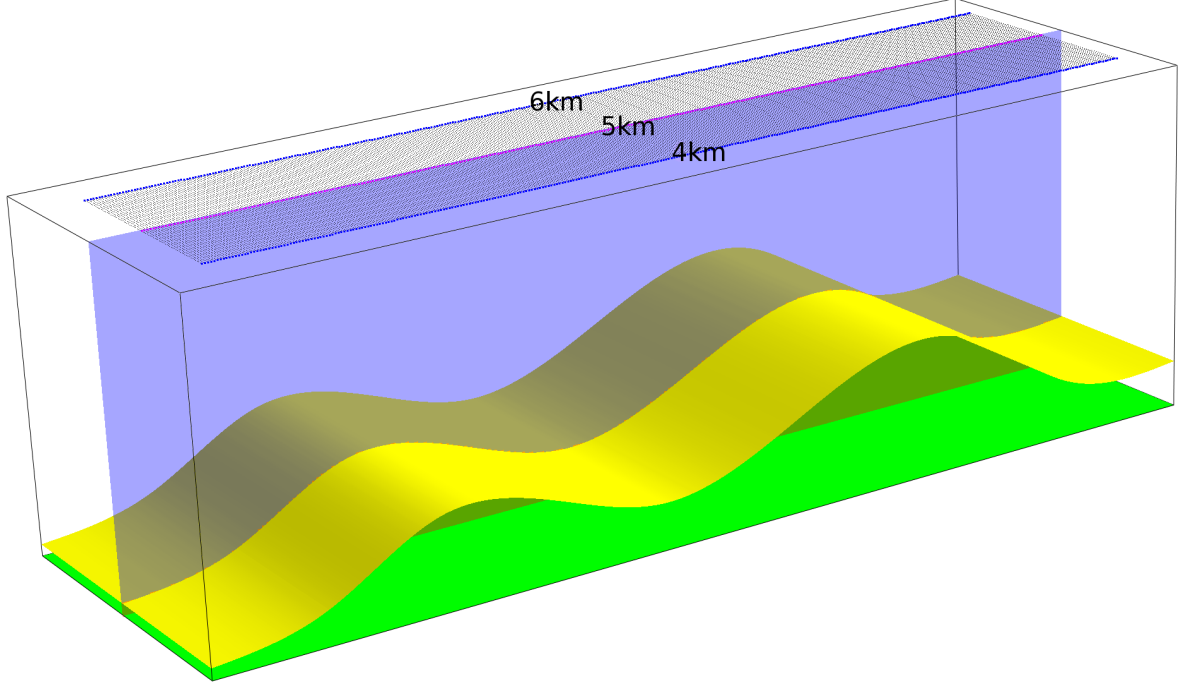


Figure 2. Slice of the velocity model with 81 parallel profile lines, the non-inclined curved interface and the bottom velocity model plane. The horizontal dimensions of the velocity model slice are 9.2 km x 3 km, the depth is 3 km. We compute and stack migrated sections in the 2-D plane located in the middle of the shot-receiver configuration (at horizontal coordinate $x_2=5$ km).

and the non-inclined curved interface is independent of the coordinate x_2 perpendicular to the profile lines (2.5-D velocity model, see Figures 1, 2).

5. 3-D Kirchhoff prestack depth migration

We use the MODEL, CRT, FORMS and DATA packages for the Kirchhoff prestack depth migration (Červený, Klimeš & Pšenčík, 1988; Bulant, 1996). The migration consists of two-parametric ray tracing from the individual surface points, calculating grid values of travel times and amplitudes, performing the common-shot migration and stacking the migrated images. The shot-receiver configuration consists of 81 parallel profile lines at intervals of 0.025 km (see Figure 2). The first profile line is situated at horizontal coordinate $x_2 = 4$ km and the last profile line is situated at horizontal coordinate $x_2 = 6$ km.

For the purpose of our analysis, we calculate only one vertical image section corresponding to the central profile line ($x_2 = 5$ km, see Figure 2). Such an image represents one vertical section of full 3-D migrated volume. We form the image by computing and summing the corresponding contributions (images) from all 81 parallel source-receiver lines. While summing the contributions, the constructive interference focuses the migrated interface and the destructive interference reduces undesirable migration artifacts (non-specular reflections). We also use a cosine taper to clear some residua.

5.1. Velocity model with triclinic anisotropy and vertical velocity gradient

We first calculate the recorded wave field in the heterogeneous velocity model with the triclinic anisotropy and the vertical gradient (TA-VG medium) in the upper layer. As we specified in Section 2 the gradient is defined by two matrices of density-reduced elastic moduli A_{ij} . The first matrix at the depth of $x_3 = 0$ km is multiplied by constant 0.8, and the second at the depth of $x_3 = 3$ km by constant 1.

5.1.1. Migration using the correct velocity model with triclinic anisotropy and vertical gradient

We migrate in the correct inhomogeneous single-layer velocity model with the triclinic anisotropy and the vertical gradient (TA-VG medium). The migrated interface coincides nearly perfectly with the interface in the velocity model used to compute the recorded wave field. The migrated section in Figure 3 demonstrates that the migration algorithm works well. This migrated section may be used as a reference for comparison with the migrated sections calculated in inaccurate velocity models.

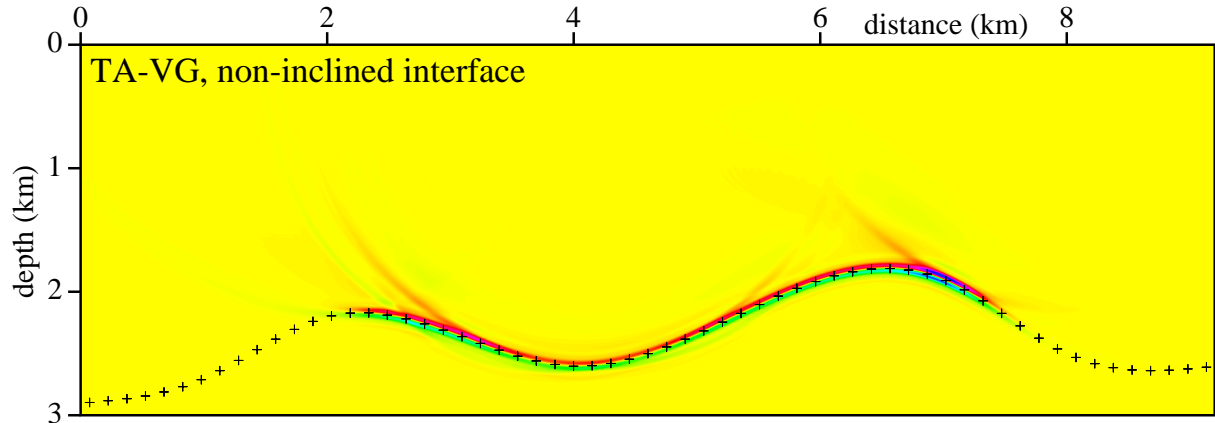


Figure 3. Stacked migrated section calculated in the velocity model with no interface. 81×240 common-shot prestack depth migrated sections, corresponding to 81 profile lines and 240 sources along each profile line, have been stacked. The triclinic anisotropy with the vertical gradient (TA-VG medium) in the upper layer of the velocity model used to compute the recorded wave field is the same as the medium in the single-layer velocity model used for migration. The crosses denote the interface in the velocity model used to compute the recorded wave field.

5.1.2. Migration using incorrect velocity models

a) Incorrect assumption of triclinic anisotropy without the gradient

The stacked migrated section is calculated in the incorrect homogeneous single-layer velocity model with the triclinic anisotropy (TA medium) without the vertical gradient. The migrated interface is shifted vertically downwards (overmigrated) and is slightly defocused (see Figure 4a).

b) Incorrect assumption of VTI symmetry

In this case we migrate in the incorrect homogeneous single-layer velocity model with a transversely isotropic medium with a vertical symmetry axis (VTI-1) without the vertical gradient. Figure 4b shows the migrated interface that is shifted vertically downwards. The segments of the interface in the horizontal ranges of approximately 2–4 km and 6–8 km are defocused and distorted (caused by a combination of inaccurate anisotropy and the velocity gradient). In comparison with Bucha (2011, 2012), we do not observe the poorly displayed interface in the horizontal range of approximately 4–6 km.

c) Incorrect assumption of HTI symmetry with the axis perpendicular to the profile lines

Here we migrate in the incorrect homogeneous single-layer velocity model with a transversely isotropic medium with a horizontal symmetry axis (HTI-1) without

the vertical gradient. The migrated interface is vertically shifted more downwards and the segments of the interface in the horizontal ranges of approximately 2–4 km and 6–8 km are more defocused and distorted (see Figure 4c in comparison with Figure 4b).

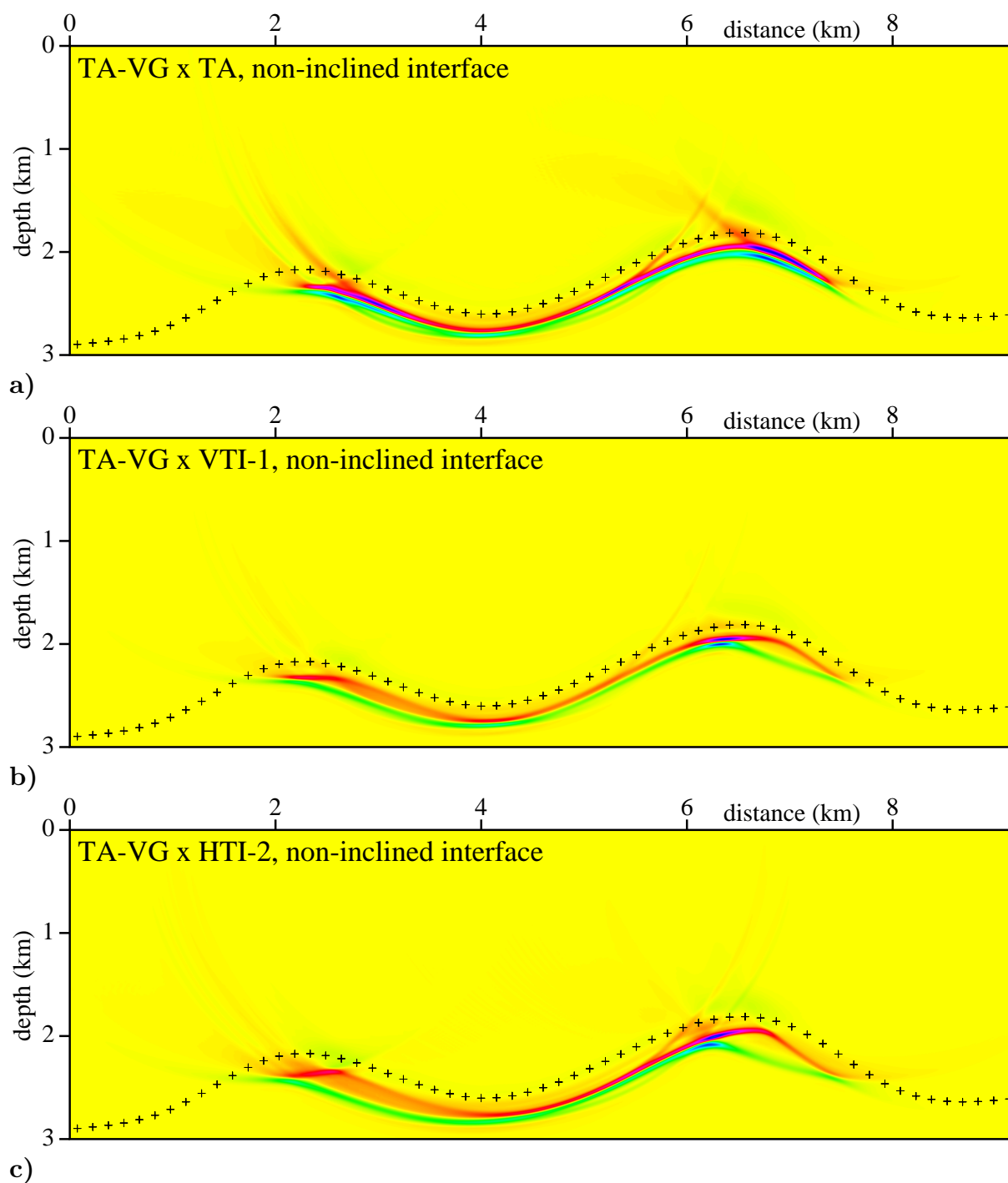


Figure 4. Image of the interface generated using incorrect homogeneous anisotropic velocity models with the TA, VTI-1 and HTI-2 media. The correct velocity model has the triclinic anisotropy and the vertical gradient (TA-VG medium). 81×240 common-shot prestack depth migrated sections, corresponding to 81 profile lines and 240 sources along each profile line, have been stacked. The crosses denote the interface in the velocity model used to compute the recorded wave field.

To explain the distorted interface in the horizontal range of 6–8 km (see Figure 4c), two single common-shot images of the interface in this horizontal range are displayed in Figure 5. Whereas the image migrated using the correct TA-VG medium is oriented correctly, the image migrated using the incorrect HTI-2 medium is shifted and rotated erroneously. When stacking the common-shot images, this shift and rotation result in distortion of the mentioned part of the interface due to the destructive interference.

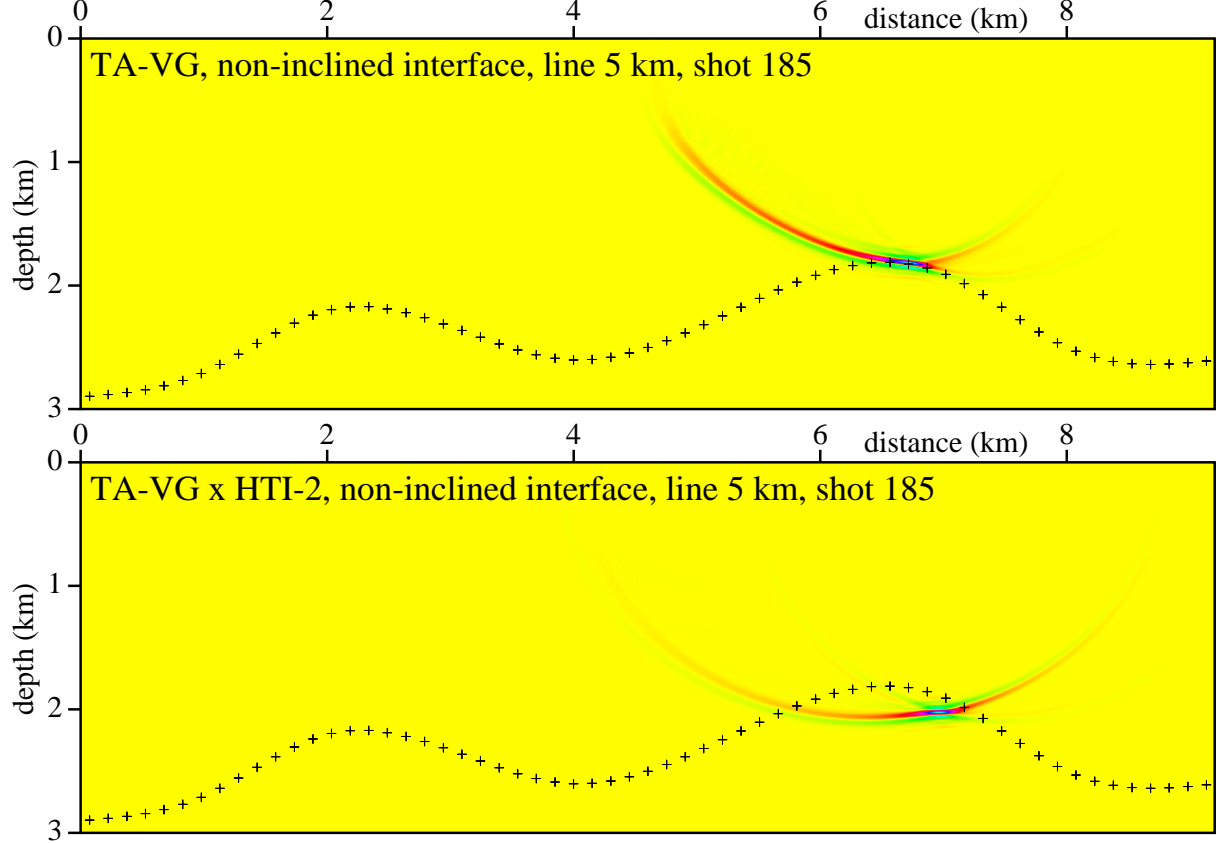


Figure 5. Prestack depth migrated images of the single common-shot gather at line $x_2 = 5$ km corresponding to shot 185 ($x_1 = 7.6$ km), migrated using the correct triclinic velocity model with the vertical gradient (TA-VG medium) and using the incorrect velocity model with the HTI-2 medium. The crosses denote the interface in the velocity model used to compute the recorded wave field.

5.2. Velocity model with triclinic anisotropy and horizontal velocity gradient parallel with the profile lines

We calculate the recorded wave field in the heterogeneous velocity model with the triclinic anisotropy and the horizontal gradient parallel with the profile lines (TA-HG medium) in the upper layer. As we specified in Section 2, the gradient is defined by two matrices of density-reduced elastic moduli A_{ij} . The first matrix at horizontal coordinate $x_1 = 0$ km is multiplied by constant 0.8, and the second at $x_1 = 9.2$ km by constant 1.

5.2.1. Migration using the correct velocity model with triclinic anisotropy and horizontal gradient parallel with the profile lines

We migrate in the correct inhomogeneous single-layer velocity model with the triclinic anisotropy and the horizontal gradient (TA-HG medium). The migrated interface coincides nearly perfectly with the interface in the velocity model used to compute the recorded wave field. The migrated section in Figure 6 shows that the migration algorithm works well. This migrated section may be used as a reference for comparison with the migrated sections calculated in inaccurate velocity models.

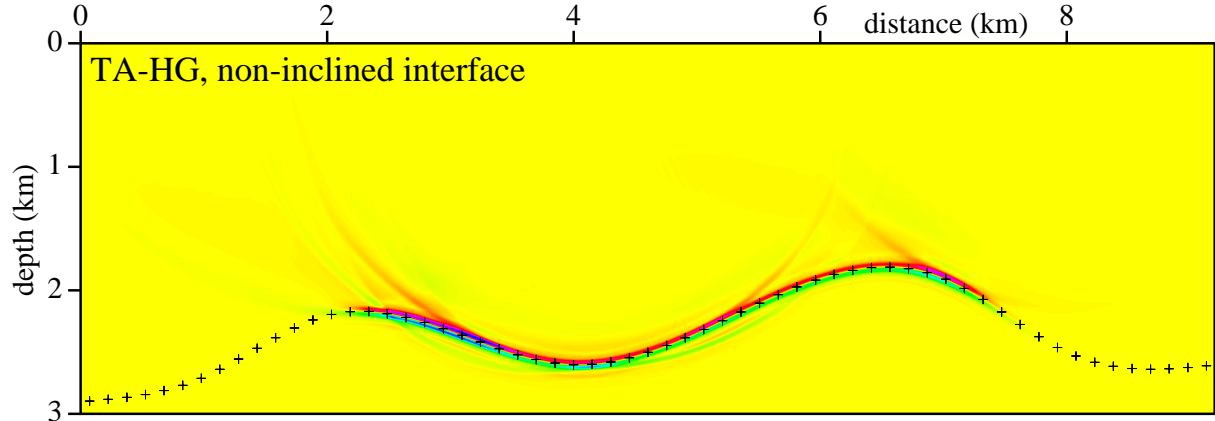


Figure 6. Stacked migrated section calculated in the velocity model with no interface. 81×240 common-shot prestack depth migrated sections, corresponding to 81 profile lines and 240 sources along each profile line, have been stacked. The triclinic anisotropy with the horizontal gradient parallel with the profile lines (TA-HG medium) in the upper layer of the velocity model used to compute the recorded wave field is the same as the medium in the single-layer velocity model used for migration. The crosses denote the interface in the velocity model used to compute the recorded wave field.

5.2.2. Migration using incorrect velocity models

a) Incorrect assumption of triclinic anisotropy without the gradient

In this case we migrate in the incorrect homogeneous single-layer velocity model with the triclinic anisotropy (TA medium) without the horizontal velocity gradient. The migrated interface is shifted asymmetrically downwards and is slightly defocused (see Figure 7a).

b) Incorrect assumption of VTI symmetry

We migrate in the incorrect homogeneous single-layer velocity model with a transversely isotropic medium with a vertical symmetry axis (VTI-1) without the horizontal gradient. Figure 7b shows the migrated interface that is shifted asymmetrically downwards (caused by the horizontal velocity gradient). The segments of the interface in the horizontal ranges of approximately 2–4 km and 6–8 km are defocused and distorted (caused by a combination of inaccurate anisotropy and the horizontal velocity gradient).

c) Incorrect assumption of HTI symmetry with the axis perpendicular to the profile lines

The stacked migrated section is computed in the incorrect homogeneous single-layer velocity model with a transversely isotropic medium with a horizontal symmetry axis (HTI-1) without the horizontal velocity gradient. The migrated interface is shifted more vertically downwards and the segments of the interface in the horizontal ranges of approximately 2–4 km and 6–8 km are more defocused and distorted (see Figure 7c in comparison with Figure 7b).

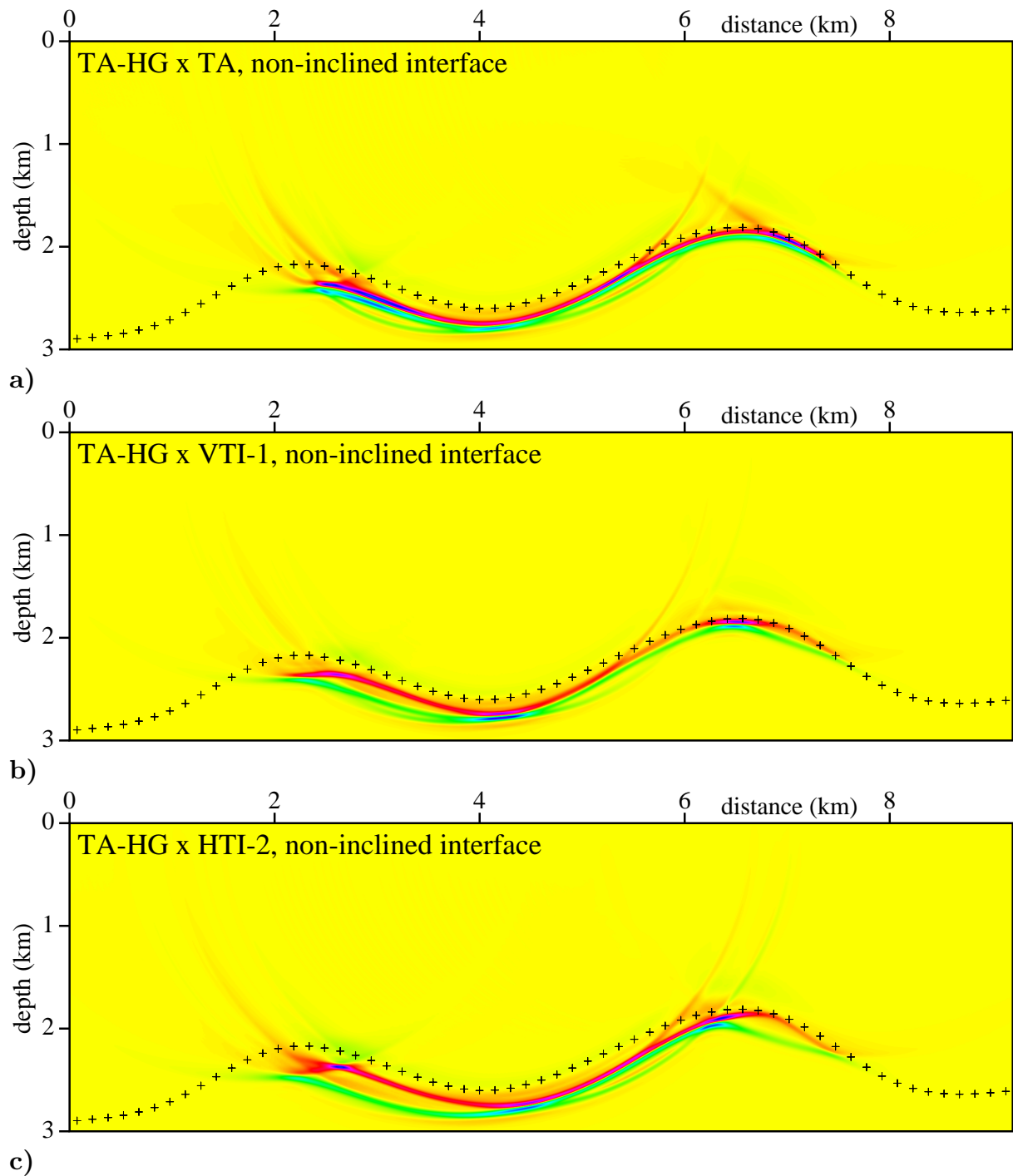


Figure 7. Image of the interface generated using incorrect homogeneous anisotropic velocity models with the TA, VTI-1 and HTI-2 media. The correct velocity model has the triclinic anisotropy and the horizontal gradient parallel with the profile lines (TA-HG medium). 81×240 common-shot prestack depth migrated sections, corresponding to 81 profile lines and 240 sources along each profile line, have been stacked. The crosses denote the interface in the velocity model used to compute the recorded wave field.

To explain the distorted and defocused interface in the horizontal range of 2–4 km (see Figure 7c), two single common-shot images of the interface in this horizontal range are displayed in Figure 8. Whereas the image migrated using the correct TA-HG medium is oriented correctly, the image migrated using the incorrect HTI-2 medium is shifted and rotated erroneously. When stacking the common-shot images, this shift and rotation result in distortion and defocusing of the mentioned part of the interface due to the destructive interference.

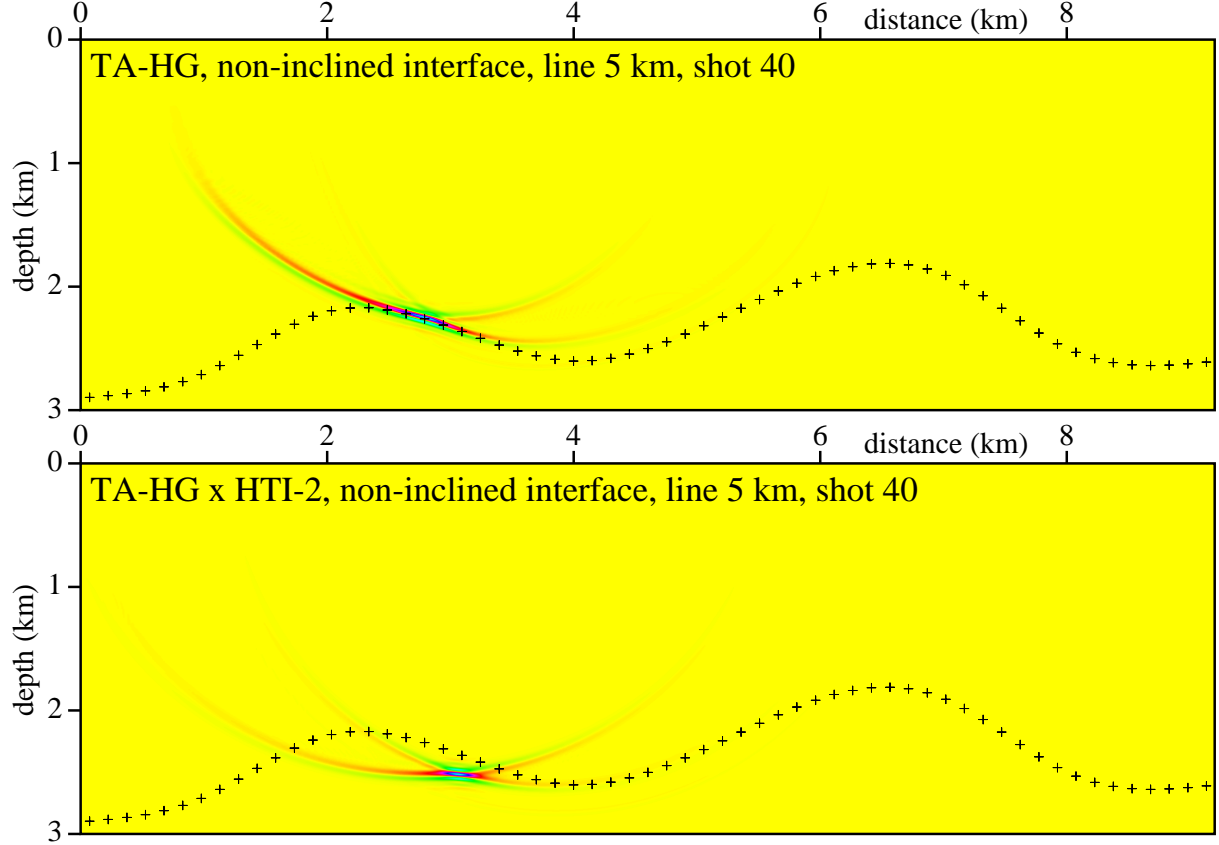


Figure 8. Prestack depth migrated images of the single common-shot gather at line $x_2 = 5$ km corresponding to shot 40 ($x_1 = 3.975$ km), migrated using the correct triclinic velocity model with the horizontal gradient (TA-HG medium) and using the incorrect velocity model with the HTI-2 medium. The crosses denote the interface in the velocity model used to compute the recorded wave field.

6. Conclusions

We continued in the Kirchhoff prestack depth migrations reported by Bucha (2011, 2012). The dimensions of the velocity model, shot-receiver configuration, methods for calculation of the recorded wave field and the migration were the same as in the papers by Bucha (2011, 2012). We used the velocity model with the non-inclined curved interface only. Bucha (2011, 2012) used, for computation of synthetic data, a homogeneous upper layer with triclinic anisotropy, while in this paper, we used an inhomogeneous upper layer with triclinic anisotropy and a vertical or horizontal velocity gradient.

We first migrated the synthetic data in the correct migration velocity models, with the triclinic anisotropy and the gradients. In this case the migrated interface in the final stacked image coincides nearly perfectly with the interface in the velocity model used to compute the recorded wave field.

We then applied the 3-D Kirchhoff prestack depth migration to three single layer homogeneous migration velocity models. The first homogeneous migration velocity model has triclinic anisotropy analogous to the correct one. The other two homogeneous velocity models have incorrect simpler anisotropies: transversely isotropic media with a horizontal symmetry axis and with a vertical symmetry axis.

When migrating in the velocity models with the correct anisotropy without a velocity gradient, we observed the shift and defocusing of the migrated interface. In the case of incorrect anisotropy and without a velocity gradient, we also observed the distortion of the migrated interface.

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