Seismic waves in inhomogeneous, weakly dissipative, anisotropic media; preliminary tests with P waves

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Summary

Program package ANRAY was generalized to calculate seismic wavefields in smooth laterally inhomogeneous, weakly attenuative, anisotropic media. Results of preliminary tests are shown and briefly discussed.

Introduction

Seismic attenuation is increasingly attracting the attention of both academia and industry. It is commonly used in the exploration of carbohydrates as presence of oil, and in particular gas, in the pore space results in high attenuation of the propagating seismic waves (Pham et al., 2002, Castielo et al., 2015). Determination of attenuation allows to investigate temporal changes in the properties of the medium (Aoki, 2015) and it also plays an important role in studies such as, for example, the inversion of the focal mechanism of earthquakes. Seismic anisotropy is another important subject. Especially recent shale gas revolution resulted in the increased focus on this property of the seismic medium, as newly explored unconventional reservoirs are often highly anisotropic (Sayers, 2005, Lonardelli et al., 2007). While velocity anisotropy is already relatively well described issue, the study of the attenuation anisotropy has not yet been discussed in such a broad way. Study of the forward problem of anisotropic attenuation is still not quite common. Guo and McMechan (2017) with their finite-difference approach represent more an exception than a rule. In this contribution, we study anisotropic attenuation using ray method applying weak-attenuation approximation proposed by Gajewski and Pšenčík (1992).

Weak attenuation approximation

Real-valued elements a_{ijkl} of the density-normalized stiffness tensor fully describe properties of the seismic elastic medium (Aki and Richards, 1980). For study of seismic wave

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propagation in attenuative media, real-valued elements are replaced by complex-valued ones, $a_{ijkl} = a^R_{ijkl} + \ \mathrm{i}\ a^I_{ijkl}$, see, for example, Carcione (2001).

In principle it means that in order to properly describe wave propagation using ray theory one should work with complex rays. Use of the complex rays is, however, complicated and the corresponding problem has been solved for a limited number of relatively simple media, see, e.g., Felsen (1982). For media with weak attenuation, Kravtsov proposed to treat attenuation as a perturbation of an elastic state, see, e.g., Kravtsov and Orlov (1990). Moczo et al. (1987) and Gajewski and Pšenčík (1992) applied the proposed *weak-attenuation approximation* successfully to isotropic and anisotropic dissipative media, respectively. Basic idea of the weak-anisotropy approximation (for more details see the above references or Červený, 2001) is assumption that imaginary parts of elements of the complex-valued density-normalized stiffness tensor are considerably smaller than their real parts:

$$a^I_{ijkl} << a^R_{ijkl} \ , \tag{1}$$

i.e., that attenuation is weak, and represents only a small perturbation of the reference elastic case. Under this assumption, the ray-theory formulae yield real-valued eikonal equation corresponding to the reference elastic medium and the effects of attenuation are shifted to the transport equation. We can thus work with real-valued rays in the reference elastic medium. The effects of the attenuation appear in the so-called global absorption factor t^*,

$$t^* = \int_{\tau_0}^{\tau} a^I_{ijkl} p_j p_l g_i g_k d\tau^R \ , \tag{2}$$

which is obtained by quadratures along a ray in the reference elastic medium, with real-valued traveltime τ^R along it. In equation (2), $\mathbf{p}$ is the slowness vector and $\mathbf{g}$ the unit polarization vector. For causal absorption, the global absorption factor affects the displacement vector $u_i(x_m, t)$ of a wave propagating in a weakly attenuative anisotropic medium in the following way (Gajewski and Pšenčík, 1992):

$$u_i(x_m, t) = U_i(x_m) \exp[-\mathrm{i}\omega(t - \tau^R + \frac{t^*}{\pi} \ln \frac{\omega}{\omega_r}) - \frac{1}{2}\omega t^*] \ . \tag{3}$$

Here $\mathbf{U}$ denotes vectorial ray amplitude, ω is a circular frequency and ω_r the reference frequency.

Numerical tests

The above theory was implemented to the ANRAY software package (Gajewski and Pšenčík, 1987, 1990). Modified ANRAY allows now to calculate ray synthetic seismograms of waves propagating in smooth, laterally varying, weakly attenuative, anisotropic media. We present here results of preliminary tests obtained with the modified version of the ANRAY package. For test, we use the configuration shown in Figure 1. The configuration is based on the commonly used geometry of routinely conducted monitoring surveys of the microseismicity induced by the hydraulic fracturing. Array consisting of 13 receivers is placed in the vertical borehole with the upper-most receiver at the depth of 0.4 km and the step between receivers equal 0.05 km. Explosive source is located in the vertical plane containing receiver array at the depth of 1 km, and in the distance of 0.3 km from the

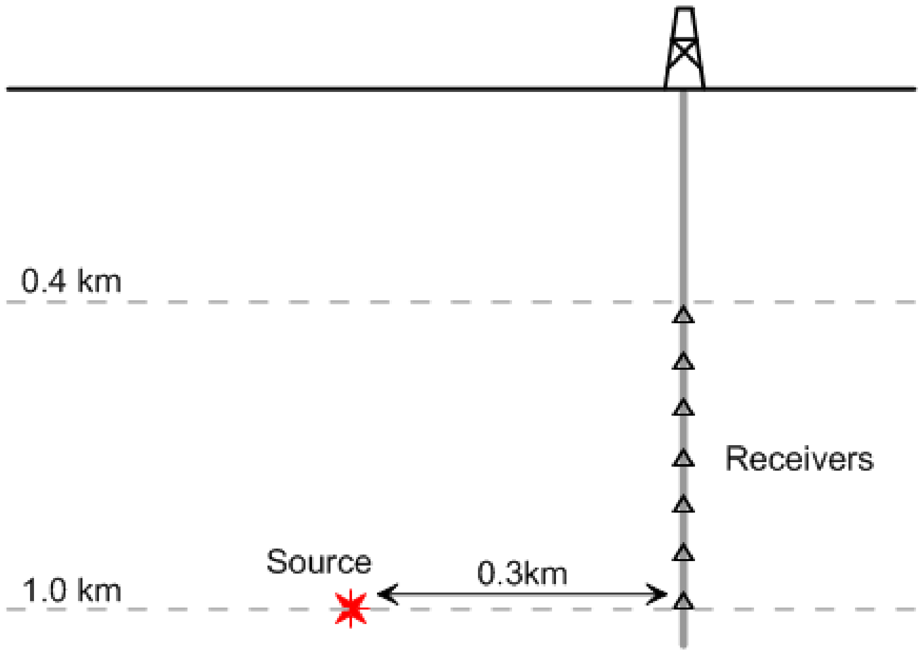

Figure 1: Configuration used in the calculation of synthetic seismograms.

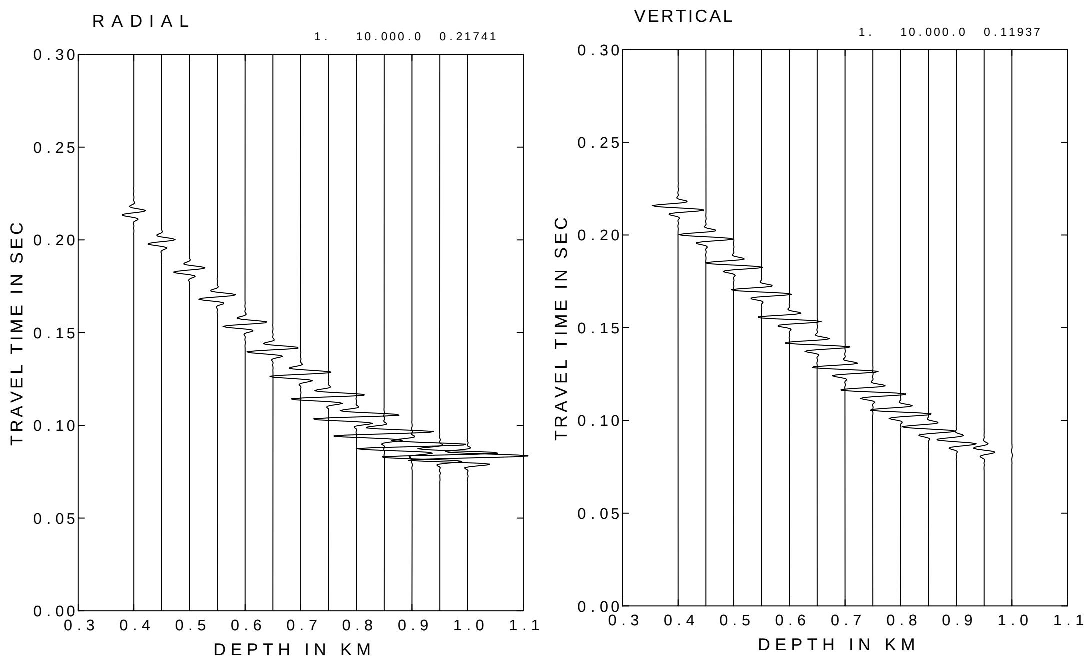

Figure 2: Synthetic seismograms (radial and vertical components) at receivers from the configuration shown in Figure 1. Elastic case.

borehole with the receiver array. The phase-shifted (by $\pi/2$) Gabor signal with prevailing frequency of 200 Hz is used as the source-time function.

Media used for tests are vertically inhomogeneous and transversely isotropic with vertical axis of symmetry (VTI). Two types of attenuation are considered. In the first, imaginary part of the complex-valued stiffness matrix in Voigt notation is chosen so that it specifies attenuation, which has isotropic character. This model was chosen to make possible a comparison with results of the FD code of BP company, which we plan to use to benchmark the result obtained with the modified ANRAY package. For isotropic Q, imaginary parts of elements of stiffness tensor are related linearly to the real parts as $a^I_{ijkl} = Q^{-1} a^R_{ijkl}$, see Gajewski and Pšenčík, 1992. The use of the isotropic quality factor Q is suitable for a simple demonstration of the effect of attenuation on waveforms, as shown later. In the second type of attenuation, the complex-valued stiffness matrix specifies angularly varying attenuation.

Ray synthetic seismograms in media with isotropic Q

First we consider vertically inhomogeneous VTI media with constant vertical gradient and constant and isotropic Q factor. P-wave anisotropy is around 20%. In Figure 2, synthetic seismograms of radial and vertical components of the P-wave displacement vector in a perfectly elastic (i.e., infinite Q) medium are shown. Due to the configuration of the experiment, transverse component is identically zero, and, therefore, it is not shown.

The results shown in Figure 3 correspond to varying values of the Q factor. The same scaling is used for all plots so that true variation of amplitudes can be observed not only within individual plots, but also between them. From the top left to the bottom right, the plots correspond to Q=200, 100, 50 and 25. The results corresponding to $Q = 25$ must be considered with care. For this value of the Q factor the weak-attenuation approximation may yield inaccurate results. To find the limit of applicability of the weak-attenuation concept will be the goal of the comparison of ANRAY results with results of BP's FD code.

In Figure 3, we can see clearly pronounced effects of varying Q on seismograms of radial components of displacement. Maximum amplitude for $Q = 25$ is six times smaller than in the elastic case. On seismograms with $Q = 50$ and 25 we start to see changes of waveforms' shape and decrease of the prevailing frequency as higher frequencies are attenuated more rapidly.

Ray synthetic seismograms in media with anisotropic Q

In the previous section we considered the influence of the isotropic attenuation on the waveforms. It is not likely, however, that the rocks, which manifest anisotropy in velocities, do not manifest anisotropy in attenuation, especially when differences in attenuation are usually much higher in comparison to velocities. Červený and Pšenčík (2005, 2008) showed that changes of attenuation due to anisotropy may be, in fact, quite pronounced and may exceed in their variability multiple times changes of seismic velocities. In Figure 4, we

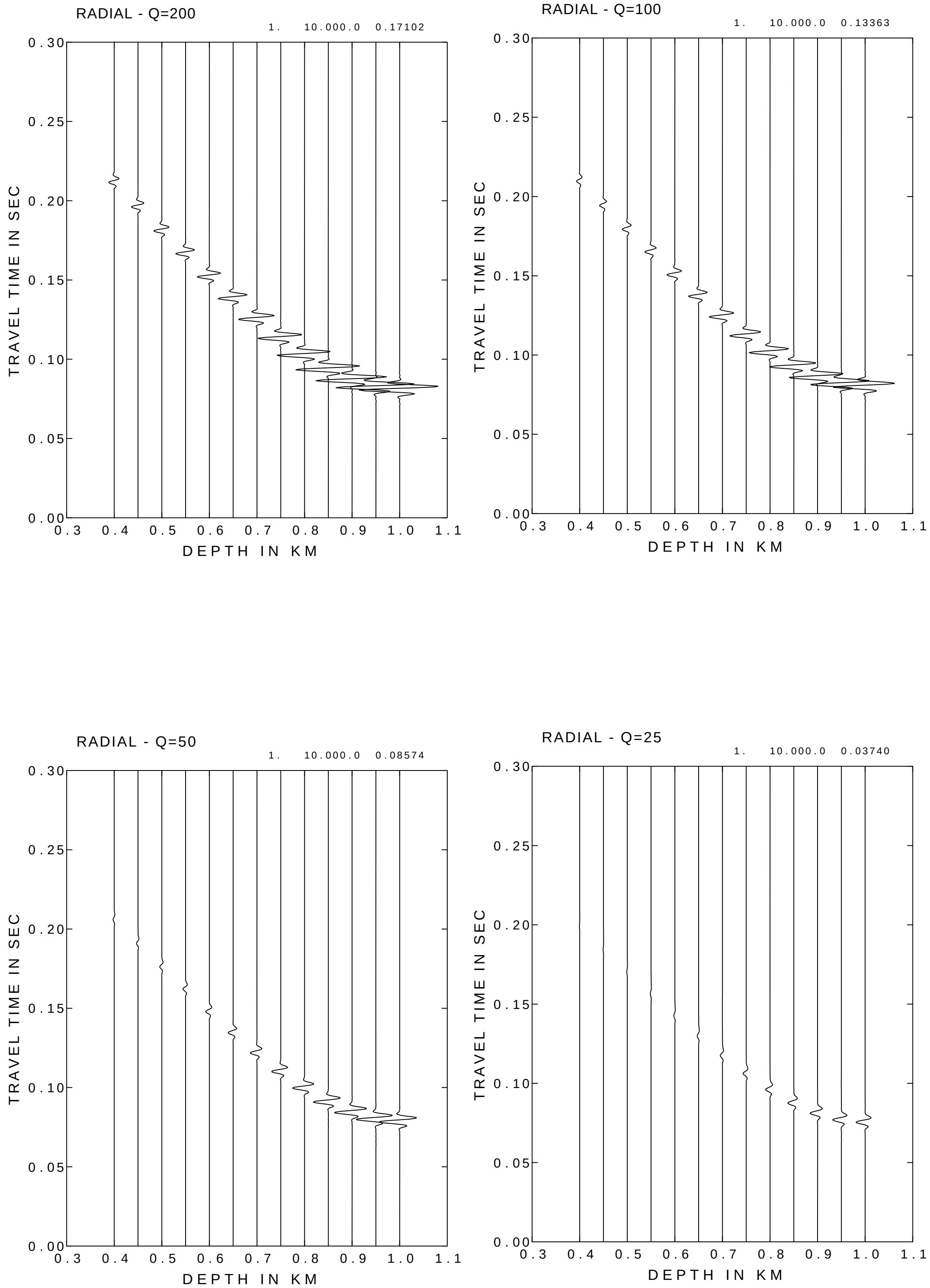

Figure 3: Synthetic seismograms (radial components) at receivers from the configuration shown in Figure 1. Anelastic case. Isotropic Q varying from $Q = 200$ to $Q = 25$.

show polar diagrams of P-wave ray velocities (top) and of inverse quality factors Q^{-1} (bottom) for two VTI models. First model corresponds to quartz grain rock with two sets of pores (spherical and randomly oriented flat pores) with relatively high concentration of nearly fully aligned flat cracks (Jakobsen et al., 2003). Second model is a model of sedimentary rock. The model is a modification of the model proposed by Zhu and Tsvankin (2006) and Vavryčuk (2007), and used by Červený and Pšenčík (2008).

We can see that the attenuation may, in fact, significantly change its strength for different directions in an anisotropic medium. Note that for the model of quartz grains rock, behavior of attenuation resembles the radiation pattern of a single force with distinct lobes of strong attenuation. It may have a pronounced effect on the character of seismograms that are registered by the monitoring arrays.

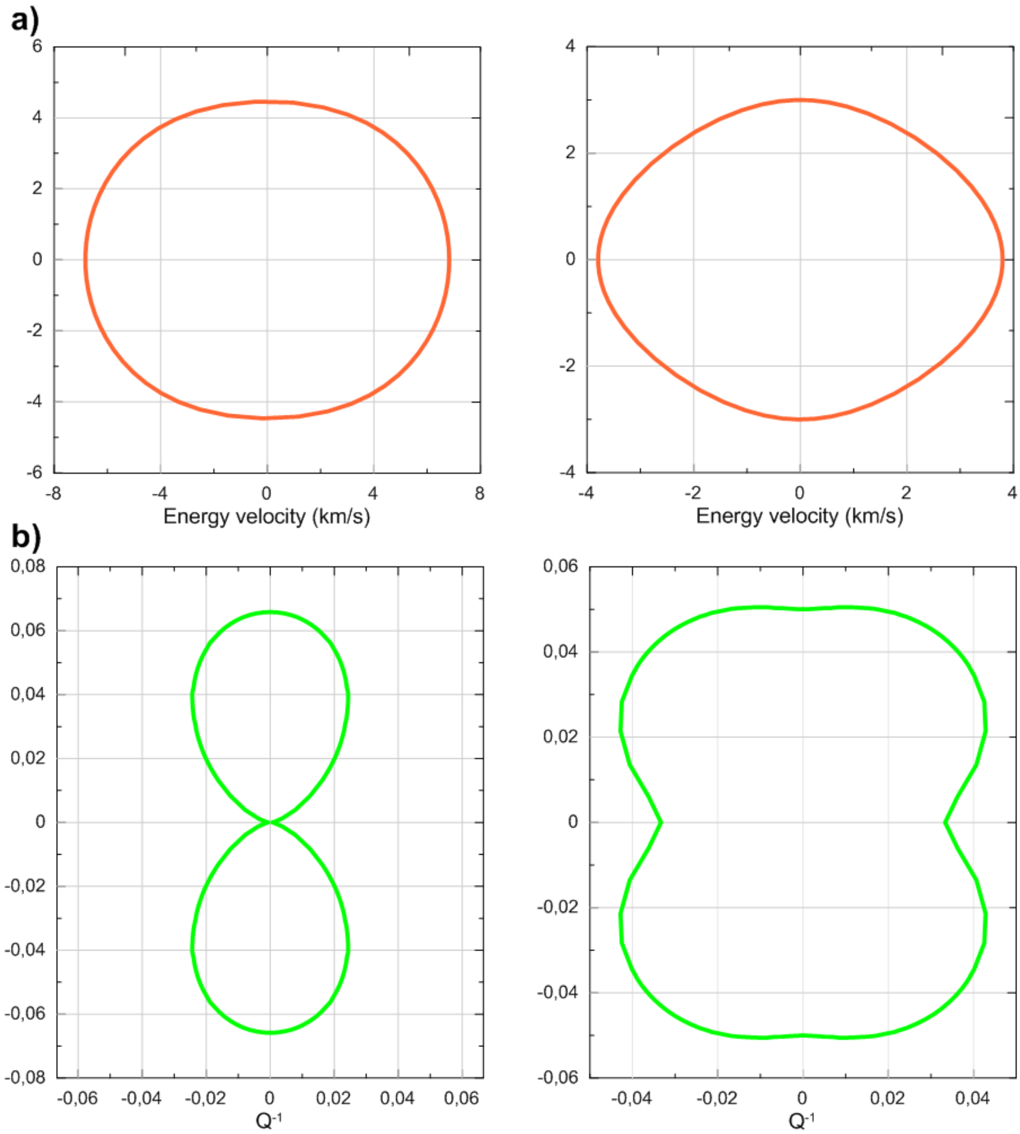

Figure 4: a) Polar diagrams of P-wave ray velocities for the quartz grain rock (left) and sedimentary rock (right); b) polar diagrams of Q^{-1} for the corresponding rocks.

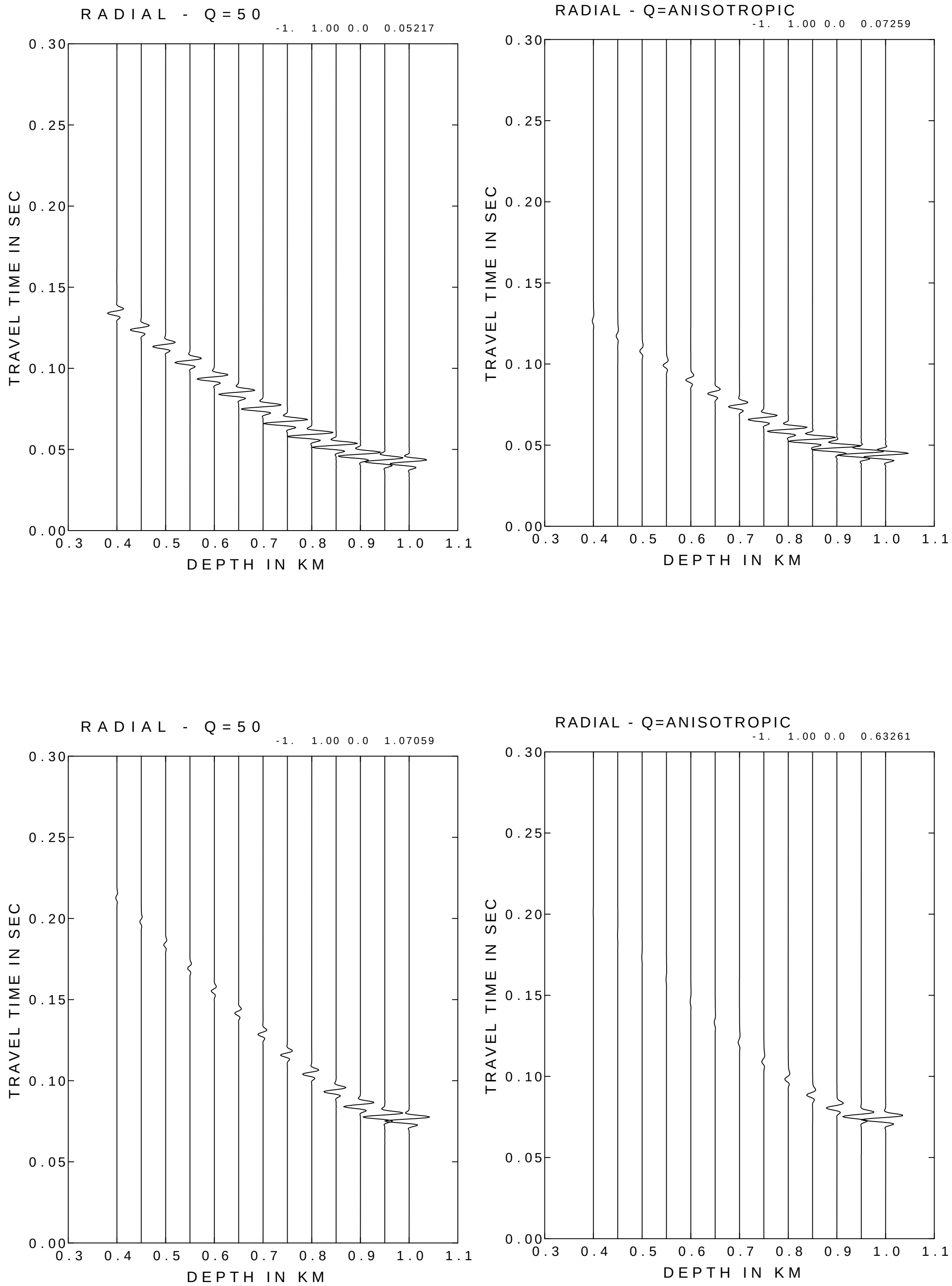

Figure 5: Synthetic seismograms (radial components) for the quartz grain rock (top) and the sedimentary rock (bottom). Left panels: isotropic attenuation, right panels: anisotropic attenuation. Seismograms in each frame are normalized with respect to the maximum amplitude.

In Figure 5, we show effects of anisotropic attenuation on P-wave ray synthetic seismograms for the configuration of Figure 1. Explosive source is used as its radiation pattern is nearly uniform even in anisotropic media (Shekar and Tsvankin, 2014). Seismograms calculated in models composed of the quartz grain rock (top) and the sedimentary rock (bottom) are shown. As a benchmark, we use seismograms calculated with the use of the isotropic Q, $Q = 50$, see left panels of Figure 5. Right panels show results for angularly varying Q. Seismograms in each frame are scaled to the maximum amplitude. Different behavior of seismograms in models with isotropic and anisotropic attenuation may lead to confusion during, for example, source-mechanism inversion processes. Effects of the Q variations could be misinterpreted as the effect of radiation pattern. It may be specifically important in the case of borehole monitoring of the microseismicity induced by hydraulic fracturing, as the anisotropy in shale rocks is high, and furthermore, as the mechanism of cracking is still not well understood. Therefore, determination of the Q anisotropy in such reservoirs may be of high importance and may help in the study of both: development of created cracks and the description of the rock medium itself.

Conclusions

Weak-attenuation approximation is useful and effective tool in the study of wave propagation in anisotropic, weakly attenuative media. It simplifies considerably ray-theory calculations as it avoids necessity to work with complex rays. The weak-attenuation approximation was implemented into the software package ANRAY, which allows to calculate synthetic seismograms in smooth, inhomogeneous, anisotropic and attenuative media. Seismograms calculated with the modified ANRAY package clearly show the importance of the attenuation effect on amplitudes and frequency content of signals. When applying the weak-attenuation approximation, one must remember that the approximation has its limitations, which have not been well examined yet. Among other tasks, modified ANRAY package will be used to test these limitations.

Models that we used to calculate synthetic seismograms show that anisotropy of attenuation may be quite strong. It may exceed significantly the strength of the corresponding seismic velocity anisotropy. Influence of the attenuation anisotropy may play an important role in the solution of the seismic source inversion. It may also be used in the determination of crack orientation and other properties of the seismic media.

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