Perturbation of the directivity function in inhomogeneous anisotropic structures

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Perturbation formulae for the directivity function in an arbitrary inhomogeneous anisotropic medium are presented. These formulae can find applications in both forward and inverse seismic modeling. For example, Tsvankin (1995) points out an importance of study of effects of source radiation on the AVO in anisotropic media. The presented formulae could be a useful tool for such studies. Another possible application of the formulae can be in inverse problems to determine the parameters of a source situated in an arbitrary anisotropic medium, or to determine parameters of the anisotropic medium surrounding the source.

The directivity function $D(x_{om}, \gamma_1, \gamma_2)$ in inhomogeneous anisotropic media has the form, see Pšenčík & Teles (1996), depending on the type of the point source. In the case of the force source, it has the form

$$D(x_{om}, \gamma_1, \gamma_2) = \frac{g_n(x_{om}) f_n}{4\pi\rho(x_{om})c(x_{om})} \tag{1a}$$

and in the case of the moment tensor source, it has the form

$$D(x_{om}, \gamma_1, \gamma_2) = \frac{g_n(x_{om}) M_{nl} p_l(x_{om})}{4\pi\rho(x_{om})c(x_{om})} \ . \tag{1b}$$

The symbols $\rho(x_{om})$ and $c(x_{om})$ denote the density and the phase velocity, $g_i(x_{om})$ is the unit polarization vector of the considered wave and $p_l(x_{om})$ is the slowness vector at the source. The symbols f_n and M_{nl} denote the vectorial force and the moment tensor. Let us note that the directivity function for the moment tensor source can also be used for the explosive source if the moment tensor is specified as $M_{nl} = M_o\delta_{nl}$, where M_o is a constant factor.

It may be useful to find expressions for the perturbation ΔD of the directivity function D_o specified in an unpertubed background medium due to a perturbation of the medium. In the following, we consider only perturbations of the density-normalized elastic parameters and keep unchanged the density. Additional introduction of density perturbations into the formulae given below is straightforward. We consider a given direction of the phase normal, which can also be specified by the corresponding slowness vector, and assume that the force vector and/or the moment tensor are given.

Due to the perturbation Δa_{ijkl} of the tensor of density-normalized elastic parameters a^o_{ijkl} of the background medium, the phase velocity c_o, the slowness vector p^o_i and the polarization vector g^o_i in the background medium are perturbed by Δc, $\Delta p_i = -(p^o_i/c_o)\Delta c$ and Δg_i, respectively. The perturbation of the directivity function (1a) has then the following form:

$$D(\gamma_1,\gamma_2) = \frac{(g^o_n + \Delta g_n)f_n}{4\pi\rho(c_o+\Delta c)} = D_o + \Delta D = \frac{g^o_n f_n}{4\pi\rho c_o} + \frac{\Delta g_n f_n - (g^o_n f_n)\frac{\Delta c}{c_o}}{4\pi\rho c_o} \quad . \tag{2a}$$

The perturbation of the directivity function (1b) can be written as follows:

$$D(\gamma_1,\gamma_2) = \frac{(g^o_n + \Delta g_n)M_{nl}(p^o_l - p^o_l\frac{\Delta c}{c_o})}{4\pi\rho(c_o+\Delta c)} = D_o + \Delta D$$

$$= \frac{g^o_n M_{nl} p^o_l}{4\pi\rho c_o} + \frac{\Delta g_n M_{nl} p^o_l - 2 g^o_n M_{nl} p^o_l \frac{\Delta c}{c_o}}{4\pi\rho c_o} \quad . \tag{2b}$$

The above formulae can be immediately applied in the case that the unperturbed background medium is nondegenerate. This means that the formulae can be used universally for qP and qS waves if the background medium is anisotropic and if we avoid singular directions. Moreover, the formulae can be used for qP waves even if the background medium is isotropic. In case of qS waves and a background isotropic medium, degenerate formulation must be considered for Δc and Δg_i. The expressions for the perturbations Δc and Δg_i, no matter if the background medium is degenerate or nondegenerate, can be found in Jech & Pšenčík (1989).

An important application of the above formulae will be probably in weakly anisotropic media, in which the unperturbed medium is isotropic. We shall concentrate on this situation in the near future.

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References

Jech,J. & Pšenčík,I.,1989. First-order perturbation method for anisotropic media. Geophys. J. Int., **99**, 369-376.

Pšenčík,I. & Teles,T.N.,1996. Point source radiation in inhomogeneous anisotropic structures, PAGEOPH, **148**, in print.

Tsvankin,I., 1995. Body-wave radiation patterns and AVO in transversely isotropic media. Geophysics, **60**, 1409-1425.