AN EIKONAL AND WAVEFIELD BASED FORMULATIONS FOR PERTURBATION WITH RESPECT TO THE SOURCE LOCATION

Tariq Alkhalifah

KAUST, Thuwal, 23955-6900, Saudi Arabia

Traveltime calculations amount to solving the nonlinear eikonal equation for a given source location. We analyze the relationship between the eikonal solution and its perturbations with respect to the source location and develop a partial differential equation that relates the traveltime field for one source location to that for a nearby source. This linear first-order equation in one form depends on lateral changes in velocity and in another form is independent of the velocity field and relies on second-order derivatives of the original traveltime field. For stable finite-difference calculations, this requires that the velocity field be smooth in a sense similar to the ray-tracing requirements. The formulation for traveltime perturbation formulation has three potential applications. First, it can be used for fast traveltime calculation by source-location perturbation. Second, it can be used for velocity-independent interpolation including datuming as well as velocity estimation. Third, higher-order expansions provide parameters necessary for Gaussian-beam computations.

In addition, the direct relation between the shape of the wavefield and the source location can provide insights useful for velocity estimation and interpolation. As a result, I derive partial differential equations that relate changes in the wavefield shape due to perturbations in the source location, especially along the Earth's surface. These partial differential equations have the same structure as the wave equation with a source function that depends on the background (original source) wavefield. The similarity in form implies that we can use familiar numerical methods to solve the perturbation equations, including finite difference and downward continuation. In fact, we can use the same Green's function to solve the wave equation and its source perturbations by simply incorporating source functions derived from the background field. The solutions of the perturbation equations represent the coefficients of a Taylor's series type expansion for the wavefield. As a result, we can speed up the wavefield calculation as we can approximate the wavefield shape in the vicinity of the original source. The new formula introduces changes to the background wavefield only in the presence of lateral velocity variation or in general terms velocity variations in the perturbation direction. The accuracy of the representation, as demonstrated on the Marmousi model, is generally high, with some amplitude shortcomings due to its approximation nature and its dependence on derivatives of the velocity field. Another form of the perturbation partial differential wave equation is independent of direct velocity derivatives, and thus, provide possibilities for wavefield continuation in complex media. The caveat here is that the medium complexity information is embedded in the wavefield and thus the wavefield shape evolution as a function of shift in the velocity or source can be extracted from the background wavefield and produce wavefield shapes for nearby sources.