ELASTIC CORRECTIONS TO ACOUSTIC FINITE-DIFFERENCE SIMULATIONS

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For many applications in imaging and reservoir characterization (reverse time migration, waveform inversion, etc.), we require accurate simulations of seismic wave propagation. To realistically model the Earth, these are needed for elastic, anisotropic and anelastic models. The finite-difference method is widely used in this context as it is robust, simple to implement, and offers a good balance between accuracy and efficiency. However, it is still a computational challenge to perform elastic, anisotropic finite-difference simulations in three dimensions, so approximate calculations are often performed in an equivalent acoustic model. Even for *P* waves, the amplitudes of the first arrivals in the acoustic medium differ from those in the elastic medium. The objective of this paper is to describe a scheme whereby the acoustic wavefield can be partially corrected for elastic effects without incurring the cost of the full elastic computation.

The computational costs (memory and CPU) of elastic finite-difference simulations exceed those of acoustic simulations because of increases in i) the number of model parameters required, ii) the number of field variables that must be stored, iii) the number of operations required to solve the wave equation, and iv) the number of cells and time steps required to avoid numerical dispersion and maintain stability in the presence of typical *S*-wave velocities.

Normally the final item above is the most significant. Even if the elastic medium is a Poisson solid ($V_P = \sqrt{3}\,V_S$), the increase in cost is 9, and for more realistic models with low shear velocity sediments (e.g. near the ocean floor), the ratio is often between one and two orders of magnitude. The combination of all four effects makes elastic modeling a real challenge, often raising the cost by between two and three orders of magnitude and introducing memory limitations.

Consider two models, one acoustic and the other elastic, designed so that the density and acoustic/*P*-wave velocity fields match. For a pressure source, the solutions in the acoustic medium and for *P* waves in the elastic medium are expected to be very similar. The most significant differences will occur in the amplitudes of reflected and transmitted *P* waves from interfaces (or pseudo-interfaces where properties vary rapidly). In the regions away from interfaces, properties are either homogeneous or varying slowly and smoothly and the coupling between *P* and *S* waves is insignificant. The objective is to correct the acoustic solution for elastic effects at interfaces, without incurring the cost of the full elastic solution. We do not discuss here how anisotropic kinematic properties can be introduced into the acoustic equation (e.g. Fletcher et al., 2009), although this is a logical extension of the method.

We denote the acoustic model and field variables by a superscript $^{\mathrm{A}}$, e.g. $\mathbf{v}^{\mathrm{A}}$, and those in the elastic model by a superscript $^{\mathrm{E}}$, e.g. $\mathbf{v}^{\mathrm{E}}$. Suppose we have the solution of the acoustic wave equation, i.e. $\mathbf{v}^{\mathrm{A}}$ satisfying

$$\rho \frac{\partial \mathbf{v}^{\mathrm{A}}}{\partial t} = -\nabla P^{\mathrm{A}} + \mathbf{f} \quad , \tag{1}$$

$$-\frac{\partial P^{\mathrm{A}}}{\partial t} = \kappa^{\mathrm{A}} \nabla \cdot \mathbf{v}^{\mathrm{A}} \quad . \tag{2}$$

Using the acoustic solution $\mathbf{v}^{\mathrm{A}}$ as an approximation for the elastic solution, we can calculate the traction solutions for the acoustic wavefield by applying Hooke's Law in the elastic medium

$$\frac{\partial \mathbf{t}_k^{\mathrm{A}}}{\partial t} = \mathbf{c}_{kj}^{\mathrm{E}} \frac{\partial \mathbf{v}^{\mathrm{A}}}{\partial x_j} \quad . \tag{3}$$

The acoustic solution will not exactly satisfy the elastic equation of motion and we can calculate the error in the equation of motion

$$\mathscr{E} = \rho \frac{\partial \mathbf{v}^{\mathrm{A}}}{\partial t} - \frac{\partial \mathbf{t}_j^{\mathrm{A}}}{\partial x_j} - \mathbf{f} \quad . \tag{4}$$

An effective source $\mathscr{E}$ in the acoustic equation

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla P + \mathbf{f} - \mathscr{E} \;\; , \tag{5}$$

will modify the acoustic solution by scattering new waves from each point where the acoustic solution does not satisfy the elastic equation. If this is introduced as a source field in a second, independent solution of the acoustic equation, the simulated wavefield provides a correction to the wavefield obtained in the original, zeroth-iteration solution of the acoustic equation. Thus the solution (1) is called the zeroth iteration. This solution ($\mathbf{v}^{(0)}$) is used to calculate the error (4) in the elastic equation ($\mathscr{E}^{(0)}$), which defines effective sources that are used to compute the first iteration correction wavefield ($\delta\mathbf{v}$). The corrected solution is then given by

$$\mathbf{v} \approx \mathbf{v}^{(1)} = \mathbf{v}^{(0)} + \delta\mathbf{v} \;\; . \tag{6}$$

In addition to the cost of two independent acoustic solutions, the equivalent elastic tractions (3) and error terms (4) need to be calculated, but only on the coarser acoustic grid and at the small percentage of points where the *P–S* coupling is significant. In addition, the derivatives of the equivalent elastic tractions in equations (3) and (4) are with respect to different variables, and when calculating the errors (4), particle accelerations are needed which are not normally saved. Nevertheless the cost (memory and CPU) of computing and storing these extra field variables is not great as they are only needed in regions where *P–S* coupling is important. For this algorithm to be efficient, it is important to identify these regions (the interfaces and pseudo-interfaces in the model) *a priori*, either during model building or by other means (e.g. from a single full elastic simulation).

References

Fletcher, R., Du, X., and Fowler, P.J., 2009. Reverse time migration in titled transversely isotropic (TTI) media. *Geophysics*, **74**, 179–187.