# RECENT DEVELOPMENTS IN PERTURBATION AND PARAXIAL RAY METHODS

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We refer to the partial derivatives of travel time with respect to perturbation parameters as perturbation derivatives. Klimeš (2002) derived the equations for calculating both spatial and perturbation derivatives of travel time of arbitrary orders along unperturbed rays in both isotropic and anisotropic media without interfaces. The explicit equations for transforming both spatial and perturbation derivatives of travel time of arbitrary orders at a general smooth interface between two general media have recently been derived using the Hamilton's (1837) approach (Klimeš, 2010). The equations are applicable to both real-valued and complex-valued travel time. The equations are expressed in terms of a general Hamiltonian function, and are applicable to the transformation of travel time derivatives in both isotropic and anisotropic media. The interface is specified by an implicit equation. No local coordinates are needed for the transformation.

The eikonal equation in an attenuating medium has the form of the complex-valued Hamilton-Jacobi equation and yields the complex-valued travel time. Since the material properties are known in real space only, we cannot calculate complex-valued rays. We need to calculate the complex-valued travel time by the perturbation from the reference travel time calculated along real-valued reference rays. For this perturbation, we need a reference Hamiltonian function yielding real-valued reference rays, and a perturbation Hamiltonian function (family of Hamiltonian functions parametrized by a perturbation parameter) smoothly connecting the reference Hamiltonian function with the given Hamiltonian function corresponding to the given complex-valued Hamilton-Jacobi equation. A practical general method of constructing the reference and perturbation Hamiltonian functions, especially suitable for this perturbation, has recently been proposed (Klimeš and Klimeš, 2010).

The second-order derivatives of travel-time can be determined by solving the equations of geodesic deviation (dynamic ray tracing), which may be performed either in Cartesian coordinates or in ray-centred coordinates. The transformation relations between the second derivatives of travel time in Cartesian and ray-centred coordinates have been derived (Červený and Klimeš, 2010).

The relation between the propagator matrix of geodesic deviation in general coordinates and the second-order spatial derivatives of two-point travel time has been derived (Klimeš, 2009).

References

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