SENSITIVITY GAUSSIAN PACKETS

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We study how the perturbations of a generally heterogeneous isotropic or anisotropic structure manifest themselves in the wavefield, and which perturbations can be detected within a limited aperture and a limited frequency band. We consider a smoothly varying heterogeneous generally anisotropic background medium, with an isotropic background medium as a special case. We consider generally anisotropic perturbations of the medium, with isotropic perturbations as a special case.

We decompose infinitesimally small perturbations $\delta c_{ijkl}(\mathbf{x})$ and $\delta\varrho(\mathbf{x})$ of elastic moduli $c_{ijkl}(\mathbf{x})$ and density $\varrho(\mathbf{x})$ into Gabor functions $g^\alpha(\mathbf{x})$ indexed by α:

$$\delta c_{ijkl}\left(\mathbf{x}\right) = \sum_\alpha c^\alpha_{ijkl}\ g^\alpha\left(\mathbf{x}\right) , \qquad \delta\rho\left(\mathbf{x}\right) = \sum_\alpha \rho^\alpha\, g^\alpha\left(\mathbf{x}\right) ,$$

$$g^\alpha(\mathbf{x}) = \exp[\mathrm{i}\mathbf{k}^{\alpha\mathrm{T}}(\mathbf{x}-\mathbf{x}^\alpha) - \tfrac{1}{2}(\mathbf{x}-\mathbf{x}^\alpha)^\mathrm{T}\mathbf{K}^\alpha(\mathbf{x}-\mathbf{x}^\alpha)] .$$

Gabor functions $g^\alpha(\mathbf{x})$ are centred at various spatial positions $\mathbf{x}^\alpha$ and have various structural wavenumber vectors $\mathbf{k}^\alpha$. The wavefield scattered by the perturbations is then composed of waves $u^\alpha_i(\mathbf{x},t)$ scattered by individual Gabor functions:

$$\delta u_i\left(\mathbf{x},t\right) = \sum_\alpha u^\alpha_i\left(\mathbf{x},t\right)$$

We approximate waves $u^\alpha_i(\mathbf{x},t)$ scattered by individual Gabor functions analytically.

We assume that a short-duration broad-band wavefield with a smooth frequency spectrum, incident at the Gabor function, can be expressed in terms of the amplitude and travel time. We approximate each wave $u^\alpha_i(\mathbf{x},t)$ scattered by one Gabor function by the *first-order Born approximation* with the *paraxial ray approximation*. These approximations enable us to calculate wave $u^\alpha_i(\mathbf{x},t)$, scattered by Gabor function $g^\alpha(\mathbf{x})$, analytically (Klimeš, 2007).

Considering the above approximations, wave $u^\alpha_i(\mathbf{x},t)$ scattered by one Gabor function is composed of a few (i.e., 0 to 5 as a rule) Gaussian packets. Each of these “sensitivity” Gaussian packets has a specific frequency and propagates from point $\mathbf{x}^\alpha$ in a specific direction. We denote by P_i and E_i the slowness vector and the unit polarization vector of the incident wave, and by p_i and e_i the slowness vector and the unit polarization vector of the sensitivity Gaussian packet. Each of the sensitivity Gaussian packets scattered by Gabor function $g^\alpha(\mathbf{x})$ is sensitive to just a single linear combination $\sum_{ijkl} c^\alpha_{ijkl} E_i P_j e_k p_l - \rho^\alpha \sum_i E_i e_i$ of perturbation coefficients c^α_{ijkl} and ϱ^α corresponding to the Gabor function. This information about the Gabor function is lost if the sensitivity Gaussian packet does not fall into the aperture covered by the receivers and into the legible frequency band. The situation improves with the increasing number of differently positioned sources. If we have many sources, the sensitivity Gaussian packets propagating from a Gabor function may be lost during the measurement corresponding to one source, but recorded during the measurement corresponding to another, differently positioned source. However, the problem is not only to record the Gaussian packets from a Gabor function, but to record them in as many different measurement configurations as to resolve perturbation coefficients c^α_{ijkl} and ϱ^α.

The sensitivity Gaussian packets can enable to replace seismic migrations by true linearized inversion of reflection seismic data. For the algorithm of the linearized inversion of the complete set of seismograms recorded for all shots refer to Klimeš (2008).

References

Klimeš, L., 2007. Sensitivity of seismic waves to the structure. In: Seismic Waves in Complex 3-D Structures, Report 17, pp. 27–61, Dep. Geophys., Charles Univ., Prague, online at “http://sw3d.cz”.

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