RAISING THE ORDER OF MULTIVARIATE APPROXIMATION SCHEMES USING SUPPLEMENTARY DERIVATIVE DATA WITH APPLICATIONS IN RAY METHODS

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Functional approximation schemes are omnipresent in the computational sciences. Usually, the data available to approximation schemes consist of function values at discrete node locations. In many cases, such as in ray tracing applications, derivative values are available as well. The choice in schemes that are able to accommodate derivative data is rather limited. In classical interpolation theory incorporation of derivative data is well known as the Hermite interpolation problem. In the univariate case, solution procedures are well established. However, extensions to arbitrary data configurations in multiple dimensions are difficult to construct, and probably less attractive for routine application. In non-interpolating approximation schemes, such as quasi-interpolation and moving least-squares, the incorporation of derivative data appears to be largely unexplored.

We propose a generic procedure to raise the approximation order of multivariate approximation schemes using supplementary derivative data. The procedure applies to all schemes that reproduce polynomials to a certain degree, including most common types of (quasi-) interpolation and moving least-squares. For an approximation scheme of order m and a dataset that provides n supplementary orders of derivative data, the procedure results in an approximation order of $m + n$. This is achieved using a modification of the Taylor expansion, the *reduced dual Taylor expansion*, that is applied to the data prior to the evaluation of the scheme.

A particular strength of the method is that it is easily accommodated in existing schemes. The method is expected to be applicable immediately in a wide range of applications. We demonstrate some examples in seismic ray tracing.